SIMPLIFIED ANALYSIS OF SKEW SINGLE-LANE SHEAR-CONNECTED CONCRETE PLANK BRIDGES

by

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EXECUTIVE SUMMARY

The proposed simplified method for skew bridges involves the following two steps.

(a) By assuming that the bridge is right with a span equal to the skew span of the original skew bridge, obtain the maximum longitudinal shear per plank by the simplified method proposed by Bakht (2004) for right bridges.

(b) Calculate the magnifier C_v from the following equation.

$$C_e = 1 + \frac{L\psi}{8000}$$

where the span length L is in metres and skew angle ψ is in degrees. Multiply the longitudinal shear per plank obtained in (a) with C_v. The shear thus obtained will be the longitudinal shear per plank in the skew bridge.

1. INTRODUCTION

A simplified method has been presented by Bakht (2004) to determine longitudinal moments and shears due to a variety of design live loads in single-span shear-connected concrete plank bridges with zero angle of skew (i.e. in right bridges).

The British Columbia Ministry of Forests wanted the above simplified method to be extended to skew bridges through the use of the kind of multipliers that are specified in the Clause CA5.1 (b)(i) of the Commentary to the CHBDC (2001). It is recalled that the CHBDC multipliers are applicable to only slab-on-girder bridges.

This report provides the details of the simplified method for skew shear-connected bridges with one lane, and subjected to a design truck, in which the centres of the two lines of wheels are 1.8 m apart and the loads between the two lines of wheels are divided 50:50; this truck is identified as Truck A2 by Bakht (2004).

2. BACKGROUND TO CHBDC METHOD

The CHBDC method, referred to above, provides values of the skew multipliers based on two dimensionless parameters, ε and η, which are defined as follows; these parameters, relating to the idealisation of the bridge as an orthotropic plate, were derived by Jaeger et al. (1988), and are described by Jaeger and Bakht (1989).

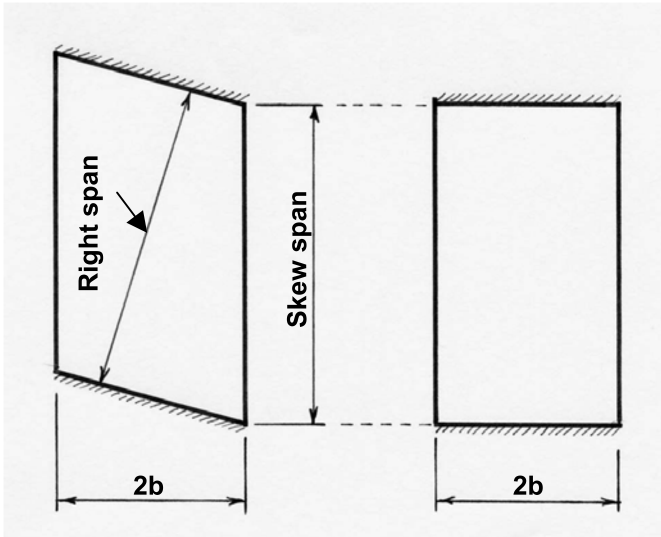
[1] $$\varepsilon = \frac{S \tan\psi}{L}$$

[2] $$\eta = 0.5\left(\frac{D_y}{D_x}\right)\left(\frac{L}{S}\right)^4$$

where S is the girder spacing, ψ is the angle of skew, L is the span length, D_y is the transverse flexural rigidity per unit length, and D_x is the longitudinal flexural rigidity per unit width.

As discussed by Bakht (2004), the shear-connected bridges under consideration are analyzed as articulated plates, a special case of the orthotropic plate in which D_y is equal to zero. From Equation [2], it can be seen that for articulated plates, in which $D_y = 0$, η is always zero. It is concluded that the longitudinal shear is likely to depend only on the angle of skew.

Bakht (1988) has shown that when skew bridges are analysed as right bridges by assuming that the equivalent span of the right bridge (Fig. 1 b) is the same as the skew span of the skew bridge (Fig. 1 a), the analysis always gives conservative (i.e. safe) results for longitudinal moments. The longitudinal shears obtained by the simplified method, however, are smaller than the same response in the skew bridge. It is for this reason that the CHBDC (2001) multipliers, which are always greater than 1.0, are applied to only longitudinal shears. It can be seen from Fig. 1 (a) that the skew span is always greater than the right span.



(a) (b)

Figure 1 Analysing skew bridge as right: (a) skew bridge; (b) equivalent right bridge

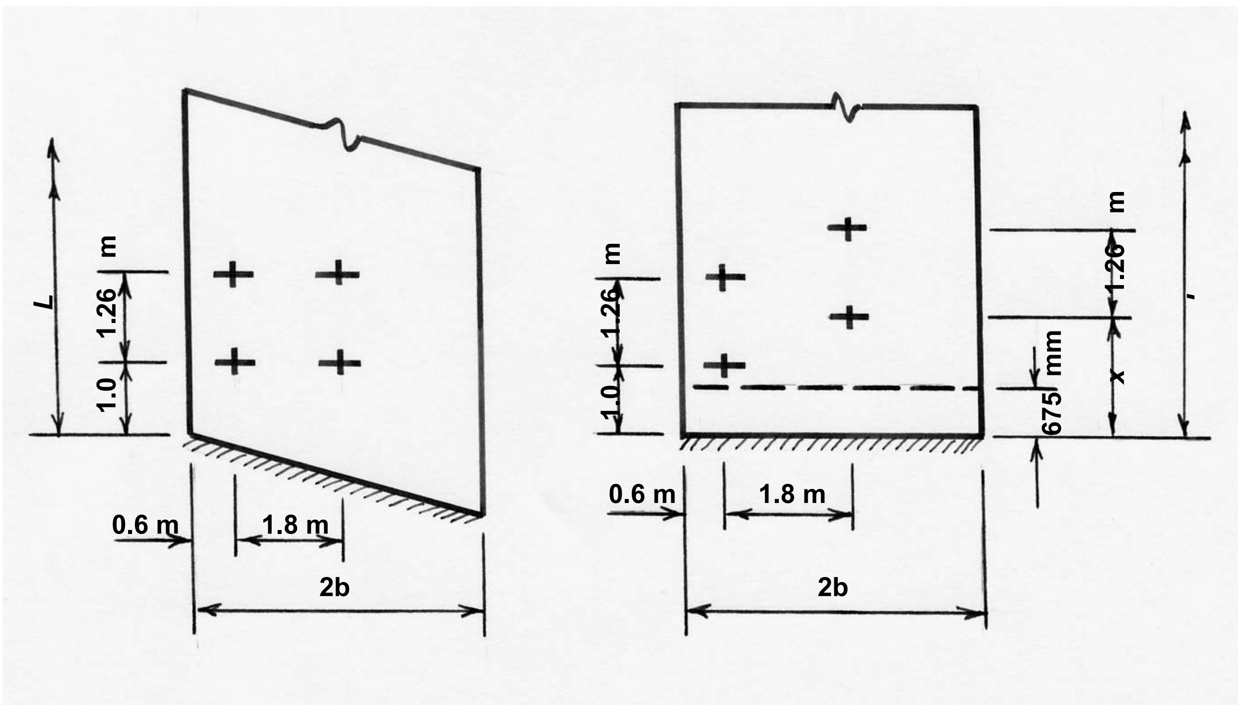
3. ANALYSIS OF SKEW BRIDGE AS RIGHT

Bakht (1988) has shown that the effect of vehicles with an orthogonal pattern of wheel loads of a truck on a skew bridge (Fig. 2 a) can be analysed realistically by analysing the skew bridge as right in which the orthogonal pattern of wheel loads is made skew so that longitudinal positions of the loads on the equivalent right bridge with respect to the transverse reference section are the same as those on the original skew bridge (Fig. 2 b).

Table 1 Parameters of idealized bridges

Designation	**Span, m**	D_x, **kN·mm^2**	D_x, **kN·mm^2**	β
6N	6.0	125,052	24,439	5.04
8N	8.0	283,897	64,242	3.52
10N	10.0	434,224	104,958	2.72
12N	12.0	540,146	150,006	2.11
14N	14.0	630,000	210,600	1.65
6W	6.0	125,052	40,838	5.04
8W	8.0	283,897	106,583	3.52
10W	10.0	434,224	175,303	2.72
12W	12.0	540,146	250,011	2.11
14W	14.0	630,000	351,540	1.65

In the previous study (Bakht 2004), it was shown that the maximum intensities in bridges under consideration are induced in the outer-most plank, when the design truck is placed as eccentrically as possible. Accordingly, it was decided to use the same governing longitudinal and transverse load position of the dual-axle tandem of the A2 Truck with respect to the closer longitudinal and transverse free edges of the articulated plate; this position is shown in Fig. 2 (a) for the skew bridges, and in Fig. 2 (b) for the equivalent right bridges. As shown in the latter figure, the longitudinal shears were investigated at transverse section that is 765 mm from the closer supported edge. Similar to the previous study, the span length *L* was varied from 6 to 14 m, but in steps of 4.0 m. Two bridge widths were considered: 4.26 and 5.50 m. The orthotropic plate properties for the 10 idealised bridges were the same as used in the previous study. These properties are listed in Table 1 for easy reference.



(a) (b)

Figure 2 Analysing a skew bridge as right: (a) original skew bridge with orthogonal load pattern; (b) equivalent right bridge with skew load pattern

Each wheel load, represented by a + sign in Figs. 2 (a) and (b) represents a rectangular patch load measuring 300 mm in the longitudinal direction and 600 mm in the transverse direction.

Four skew angles were considered in the analyses. As shown in Fig. 3, these skew angles were 0º, 15º, 30º and 45º. Thus for each of the idealised bridges listed in Table 1, four load cases were considered corresponding to each of these skew angles. Since the orthotropic plate program PLATO (Bakht et al., 2002) can handle only similar longitudinal lines of wheels, each load case involved two sets of analyses, one for each line of loads. The results for dissimilar lines of loads (Fig. 2 b) were obtained by summing the results due to the separate lines of wheels.

It is noted that *L* in Fig. 3 was 6, 10 and 14 m, and two values of width 2*b* were considered, these being 4.26 m and 5.50 m.

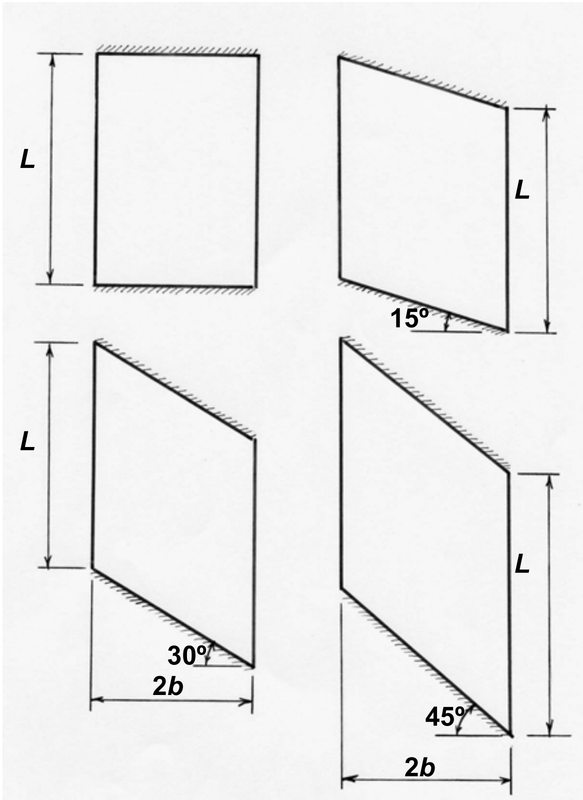


Figure 3 Four skew angles considered in the analyses

4. DETAILS OF ANALYSES

Numerical results of analyses described above are presented in spreadsheet format in Appendix A. For each idealised bridge, the absolute values of longitudinal shear intensity is calculated, in kN/m, for skew angle = 0º, 15º, 30º and 45º, respectively. Following the notation of CHBDC Commentary, the magnifier for longitudinal shear is denoted herein as C_v. The value of C_v for a bridge with given angle of skew is obtained by dividing the maximum longitudinal shear/plank for the skew bridge with the corresponding value in the right bridge having the same span length, width and relative position of the design truck. From Appendix A, it can be seen that the values of C_v for nearly all analysed skew bridges are greater than 1.0. The reasons for some values of C_v being smaller than 1.0 are discussed in the following.

The variation of C_v with respect to the angle of skew can be studied readily when the results are presented graphically, as in Fig. 4. It can be seen in this figure that C_v increases most rapidly with increase in the skew angle when the span length is the largest, being 14 m. The increase become less rapid for the smaller span length of 10 m. However, for the smallest span of 6 m, the magnifier rises initially with increase in the angle of skew, but drops just below 1.0 for higher angles of skew. A study of the three C_v-ψ angle curves in Fig. 4 shows a systematic change with respect to both the span length and skew angle. This observation confirms that no arithmetical errors were committed in the analyses.

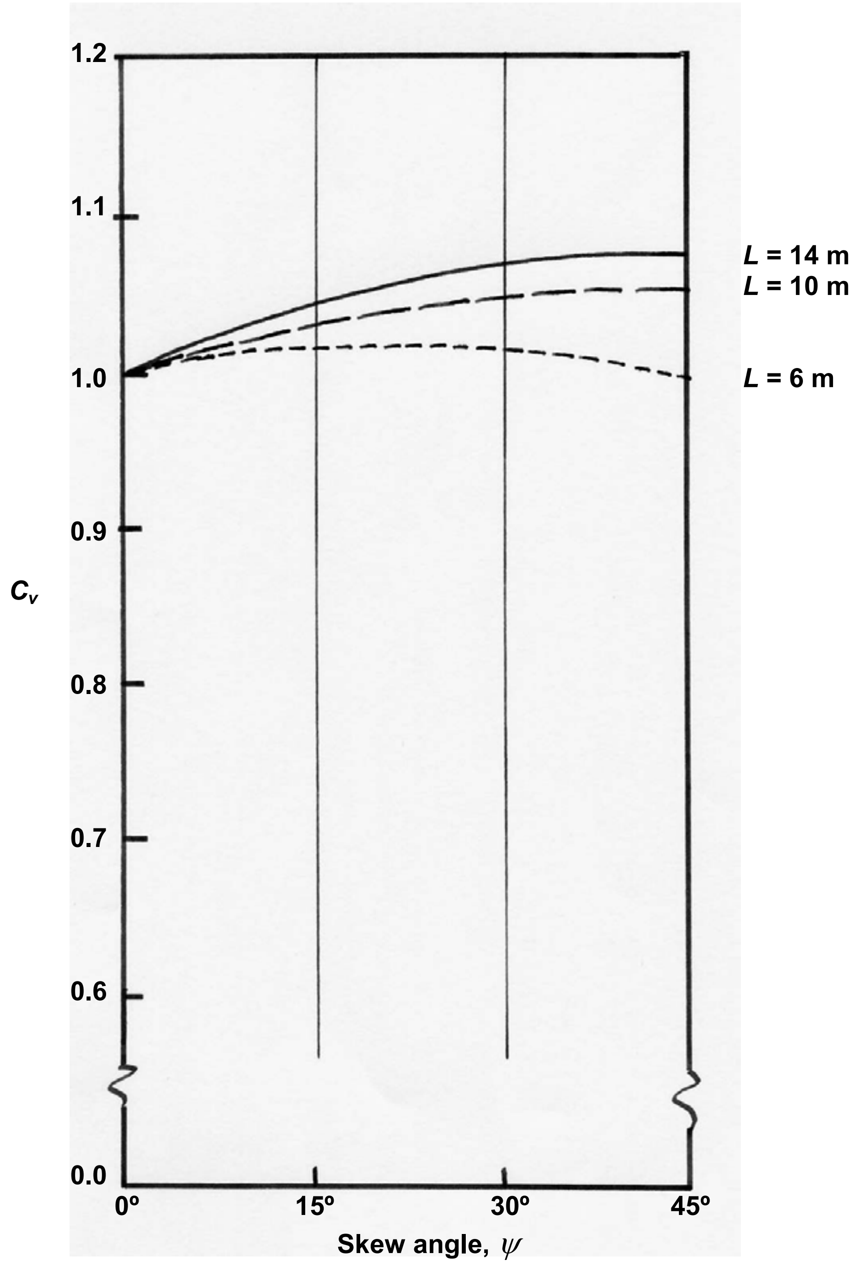


Figure 4 C_v plotted against angle of skew

The values of C_v for outer and inner planks in some of analysed the shear-connected bridges are listed in Table 2 for both narrow (N) and wide (W) bridges, having widths of 4.26 and 5.50 m, respectively. It can be seen in this table that the magnifier always has a larger value for the outer planks, and that small changes in the bridge width have negligible effect on C_v.

The results shown in Table 2 clearly show that the effect of bridge width can be neglected in developing the magnifiers. Further, it is also obvious that similar to the simplified method for right bridges, the magnifiers need be developed only for the outer planks.

Table 2 Values of C_v for some cases

Bridge	**C_v for outer planks for skew angle =**				**C_v for inner planks for skew angle =**			
	0º	**15º**	**30º**	**45º**	**0º**	**15º**	**30º**	**45º**
6N	1.000	1.013	1.015	0.998	1.000	1.018	1.018	0.990
6W	1.000	1.017	1.016	0.989	1.000	1.021	1.016	0.973
14N	1.000	1.039	1.065	1.076	1.000	1.047	1.076	1.084
14W	1.000	1.034	1.052	1.052	1.000	-	-	-

While the trends of three C_v-ψ curves are well defined, it can be seen that the maximum value of the magnifier is nearly 1.08. An 8% increase in the maximum longitudinal shear intensity is very small and can be neglected. The 3rd edition of the Ontario Highway Bridge Design Code (OHBDC, 1991), the predecessor of the CHBDC (2000), specified that the simplified analysis for live loads could be applied to a skew slab-on-girder bridge provided that the value of the skew parameter, defined by Equation [1], is less than 1/18. The commentary to the OHBDC (1991) states that this limit ensures that the shear values obtained by the simplified method are not in unsafe error by more than 5%.

Since an unsafe error of up to 5% is considered acceptable by a state-of-the-art bridge design code, a case can also be made for increasing this limit to 8%. It is noted that, as explained later, only a few bridges will have an unsafe error of more than 5%.

The curves drawn in Fig. 4 have a relatively small vertical scale, making it difficult to visualise minute variations. In order to study them microscopically, the curves are redrawn in Fig. 5 with an exaggerated vertical scale, in which each division represents a 0.01 step in C_v.

It can be seen from Fig. 5 that C_v is larger than 1.05 only for large span bridges having skew angles greater than about 20º. For all other skew bridges, the degree of unsafe error in analysing them as right bridges will be 5% or smaller. Notwithstanding this observation, a simplified method is now developed so that no theoretical error is involved in the simplified method.

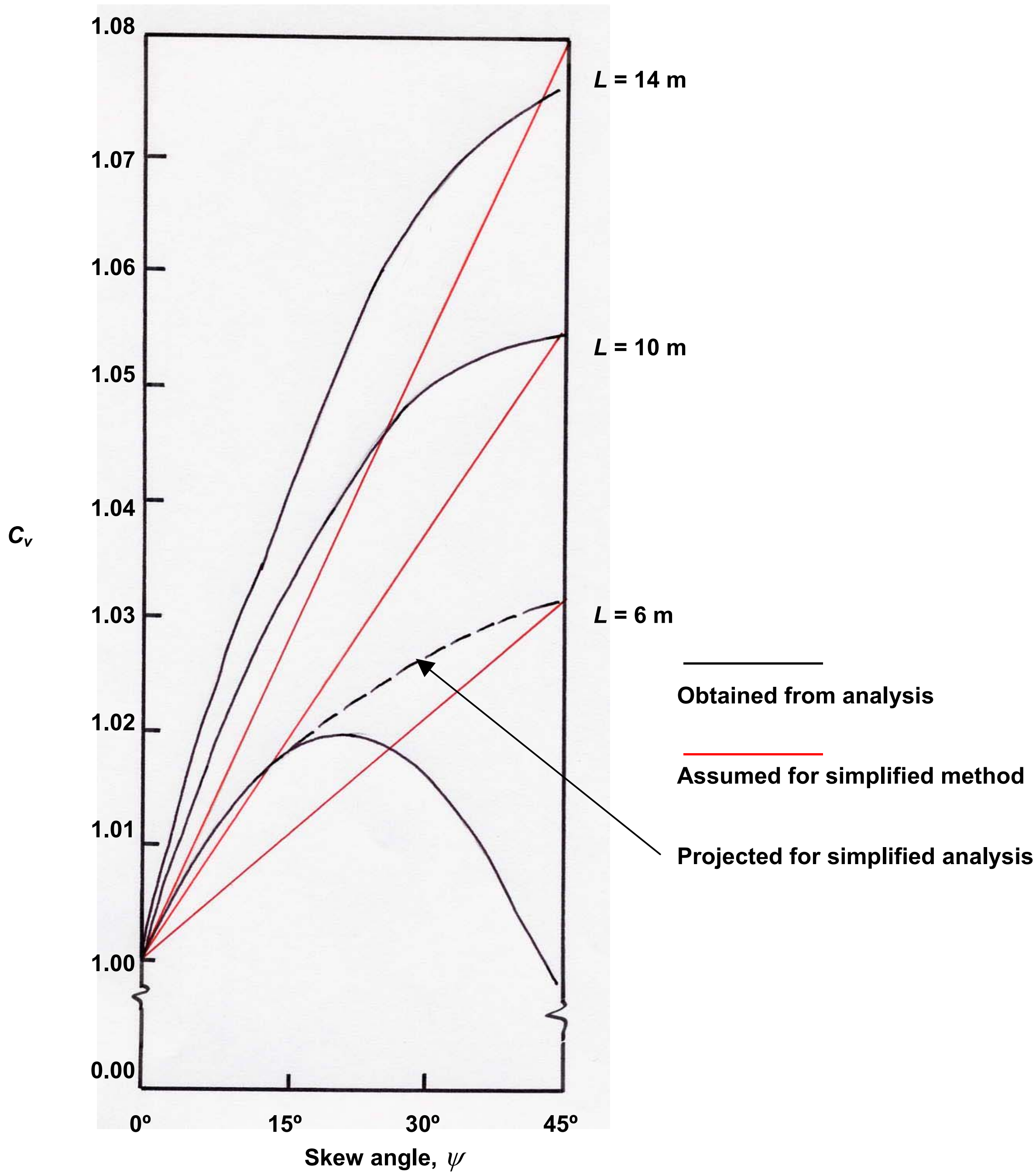


Figure 5 C_v plotted against angle of skew with an exaggerated scale for C_v

5. PROPOSED METHOD

In the interest of keeping the simplified method really *simple* three simplifying assumptions are made regarding the C_v- ψ curves, two of which are illustrated in Fig. 5: (a) C_v varies linearly with respect to the angle of skew; (b) for L = 6 m, C_v does not drop with increase in the skew angle, but keeps rising as shown in Fig. 5; and (c) C_v varies linearly with span length. As shown later, these assumptions lead to miniscule errors. By adopting these assumptions, the curves of Fig. 5 can be represented by the following equation.

$$[3] \qquad C_e = 1 + \frac{L\psi}{8000}$$

where the span length L is in metres and skew angle ψ is in degrees. The application of the magnifier C_v is quite simple: Obtain the maximum intensity of longitudinal shears by the simplified method proposed by Bakht (2004), and multiply this intensity by C_v obtained from Equation [3].

6. ACCURACY OF PROPOSED METHOD

The values of C_v obtained from rigorous analysis (Appendix A) are compared in Table 3 with those obtained from Equation [3].

Table 3 Comparison of values of C_v obtained from rigorous analysis and Equation [3]

***L*, m**	**Method**	**C_v for skew angle =**		
		15°	**30°**	**45°**
14.0	Rigorous	1.03	1.07	1.08
	Equation [3]	1.03	1.05	1.08
10.0	Rigorous	1.03	1.05	1.05
	Equation [3]	1.02	1.04	1.06
6.0	Rigorous	1.02	1.02	1.00
	Equation [3]	1.01	1.02	1.03

It can be seen in Table 3 that the differences in values of C_v given by rigorous analysis and obtained by Equation [3] are less than 0.01 in all cases except one, in which the difference is 0.03 on the safe side. It is thus concluded that the proposed method, although based on simplifying assumptions, is fairly accurate.

7. CONCLUSIONS

A simplified method has been developed for skew shear-connected bridges with one design lane to correct the design values of longitudinal shear obtained by the simplified method proposed by Bakht (2004). Similar to the method specified in the Commentary to the CHBDC (2001), the proposed method utilises a multiplier, always greater than 1.0, that depends upon the span length and angle of skew (Equation 1). It has been shown that the maximum unsafe error involved in predicting the design values of longitudinal shear in the bridges under consideration is likely to be under 8%. If this degree of error is deemed to be acceptable, then the effect of skew angle need not be considered.

8. REFERENCES

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