Smalian's Formula

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4.1 Smalian's Formula

From the inception of cubic scaling the Smalian formula, which calculates logs on the basis of a parabolic frustum, was adopted as the cubic scale rule. This scale requires measurement of the two inside bark diameters and the length.

$$V = \frac{A_1 + A_2}{2} \times L$$

where: V is the volume of the log in m^3.

A$_1$ is the area of the small end of the log in m^2.

A$_2$ is the area of the large end of the log in m^2.

L is the length of the log in m.

Figure 4.1 The Smalian Formula.

Smalian's formula states that the volume of a log can be closely estimated by multiplying the average of the areas of the two log ends by the log's length. The units used for the areas and the length must be the same, (e.g., square metres and metres in British Columbia), in order to arrive at the volume in cubic metres.

4.1.1 Smalian's Formula in Detail

Smalian's formula uses the formula for the area of a circle to find A_1 (area of the top end in m^2) and A_2 (area of the butt end in m^2):

$$A_1 = \frac{\pi T^2}{10000}, \text{ and}$$

$$A_2 = \frac{\pi B^2}{10000}$$

Fully expressed, Smalian's formula becomes:

$$V = \frac{\frac{\pi T^2}{10000} + \frac{\pi B^2}{10000}}{2} L$$

where:

V	=	volume of the log in cubic metres,
π	=	3.141 592 (to 7 significant figures or 6 decimal places),
T	=	radius of the small end in centimetres (or the diameter in rads as illustrated in Figure 4.5),
B	=	radius of the large end in centimetres (or the diameter in rads as illustrated in Figure 4.5), and
L	=	length of the log in metres as shown in Figure 4.5.

(Division of the top and butt areas by 10 000 converts square centimetres to square metres. Division of the sum of the top and butt areas by 2 determines the average end area.)

4.2 Log Forms and the Formula

A tree tapers from the base to the tip but the rate of taper is not uniform throughout all parts of the stem. Logs approach the form of truncated cones, neiloids, paraboloids, and occasionally, cylinders. These truncated sections, illustrated in Figure 4.2, are referred to as frustums of the geometric figure they resemble. The scaler can identify the log shapes by noting the position of the average of the diameter at both ends. If the average diameter occurs exactly equidistant from both ends the log form is conoid. If the average diameter is found in the butt half, the form is neiloid, and if in the upper half, the form is parabolic.
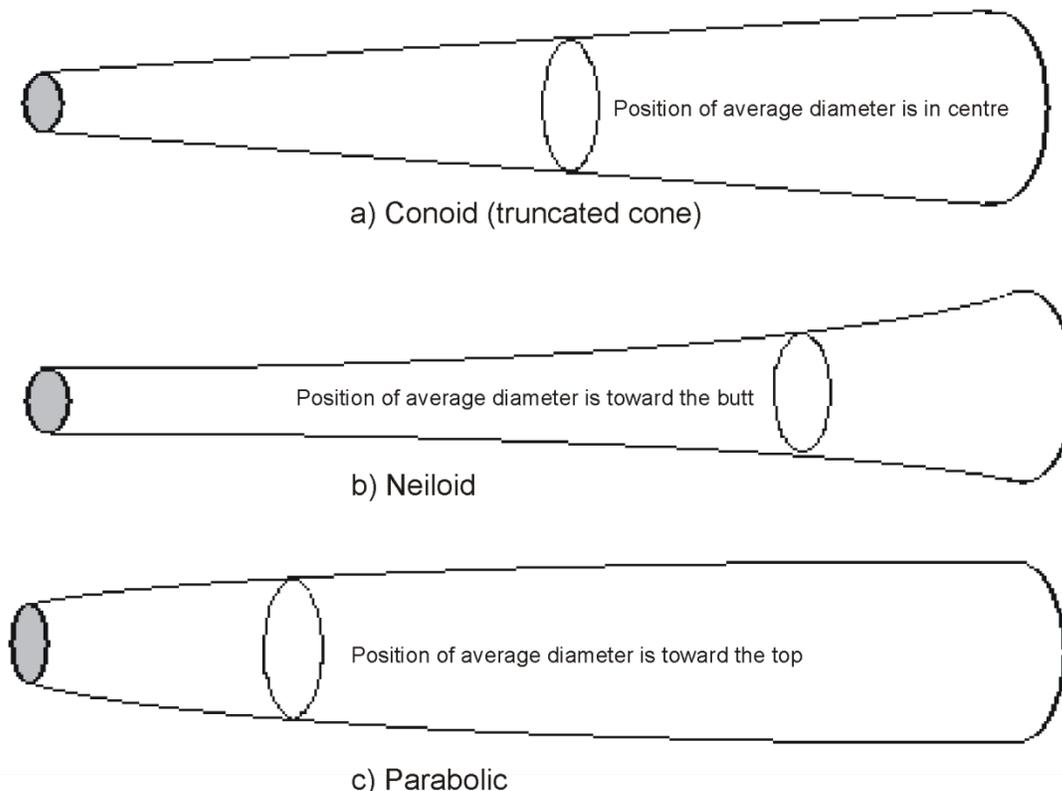
Position of average diameter is in centre

a) Conoid (truncated cone)

Position of average diameter is toward the butt

b) Neiloid

Position of average diameter is toward the top

c) Parabolic

Figure 4.2 Depictions of Three Typical Log Forms.

Smalian's formula arrives at average area by averaging the areas of the top and butt ends of a log. The Smalian formula is the official log scaling rule for the Province of British Columbia because it is practical and works well where logs are bucked prior to scaling.

The effect of bucking is to reduce taper, and experience has shown that where the difference between the top and butt diameters exceeds 30 percent, (where the butt diametre exceeds 150 percent of the top diameter) the use of Smalian's formula is not an accurate reflection of log volume. This is demonstrated in Figure 4.3, where a one cubic metre cylinder is gradually reduced in diameter at one end, becoming a cone, which has a volume of 0.333 m^3. Calculated with Smalian's formula, the volume is overstated as

0.500 m^3, and if calculated by averaging the diameters, the volume is understated as 0.250 m^3.
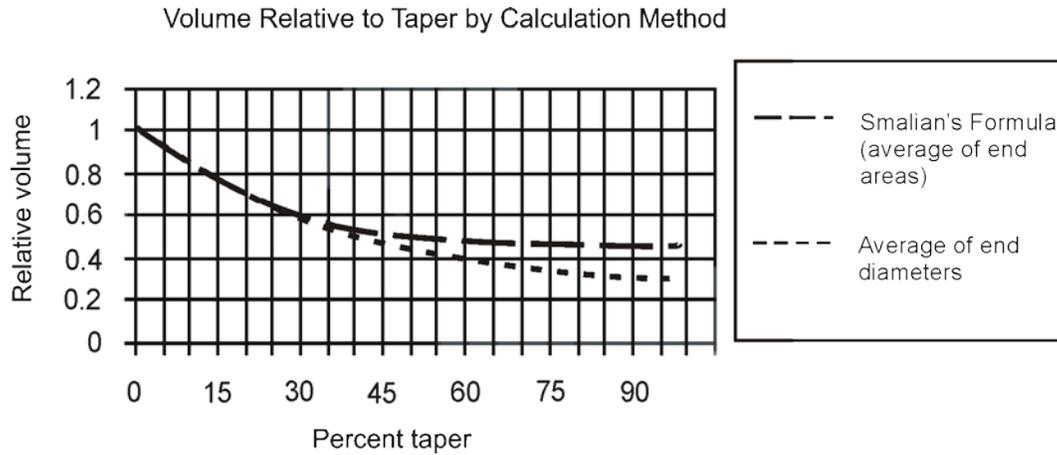
Volume Relative to Taper by Calculation Method

Figure 4.3 Graph of the Effects of Taper on Log Volume.

Normally, the difference between the top and butt diametres of typical logs is less than 30 percent, and Smalian's formula is an accurate reflection of the true log volume. However, the greater the difference in diametre between the log ends, the less reliable will be the volumes obtained using Smalian's formula. Where excessive taper exists, the Forest Service may order that logs be bucked prior to scaling to improve accuracy.

Figure 4.4 illustrates how Smalian's formula may be visualized as either one cylinder projecting the average of the basal areas of the large and small ends for the full log length, or as two cylinders; one projecting the area of the small end for exactly one half the log length and the other projecting the area of the large end for exactly one half the log length. Either view will produce the same results; a volume equivalent of the log represented.

A_2 Average of the two end areas $\longrightarrow$ $\left(\dfrac{A_1 + A_2}{2}\right) \times L$ A_1

L
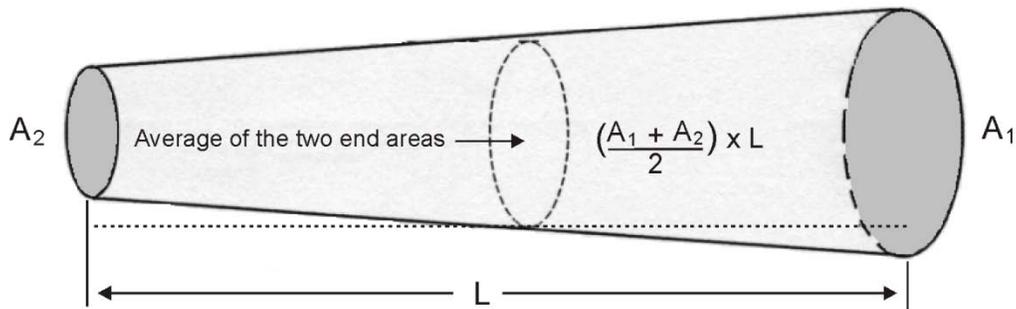
Here, the dashed outline represents a cylinder with a cross-sectional area equal to the average of the top and butt areas.

$\dfrac{L}{2}$ $\dfrac{L}{2}$

A_2 $A_1 \times \dfrac{L}{2} \div A_2 \times \dfrac{L}{2}$ A_1
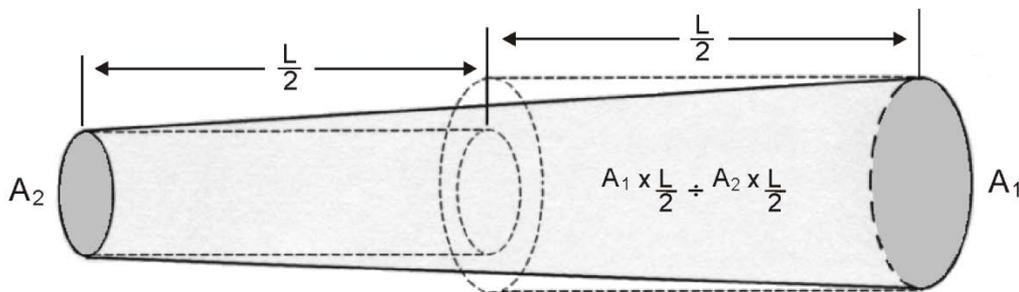
In this example, the solid represented by the dashed outline shows two cylinders - one with an eand area equal to the top end area, the other equal to the butt end area; both running exactly half the log length. This is how the scale stick calculates Smalian volumes (using half-volume tables).

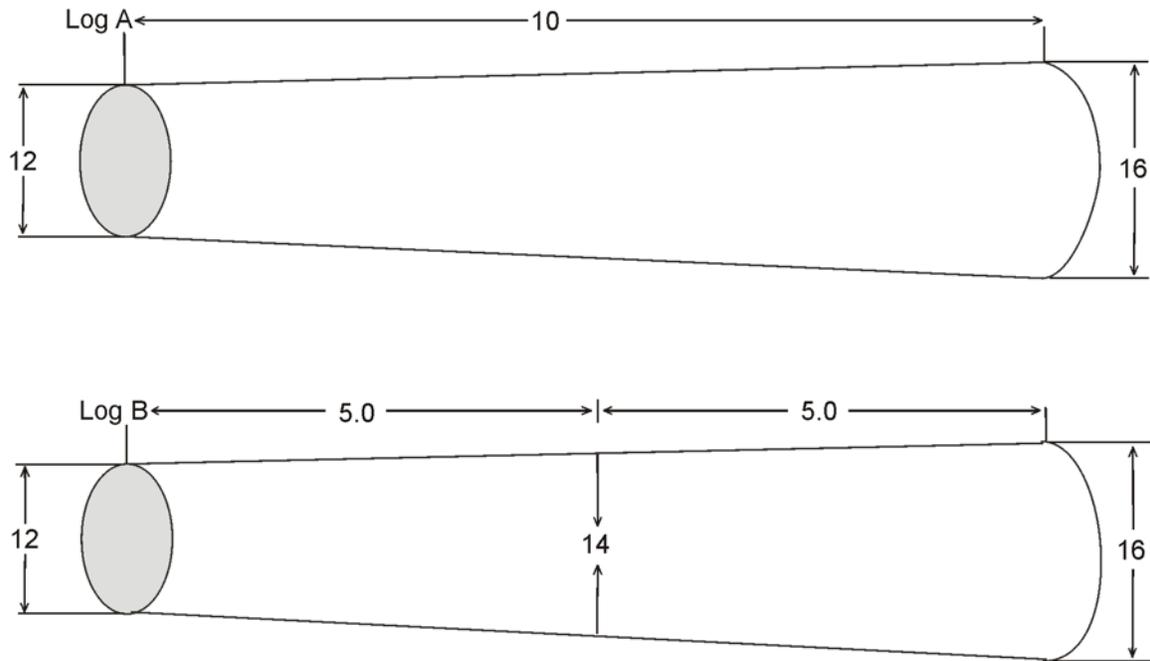*Figure 4.4 Two Ways to Visualize Smalian's Formula.*

Figure 4.5 Measurements for Finding the Values "T", "B" and "L" in Smalian's Formula.

If a log is measured across its diametre in rads using a scale stick, the recorded measurement will also represent the radius of the log in centimetres.

4.2.1 Smalian's Formula - Example Calculation

If the log illustrated in Figure 4.5 is given a top radius "T " of 9 cm (9 rads in diametre), a butt radius "B" of 11 cm (11 rads in diametre), and a length "L" of 9 m.

Fully expressed, Smalian's formula states:

$$V \quad = \quad \frac{\dfrac{\pi T^2}{10000} + \dfrac{\pi B^2}{10000}}{2} \; L$$

Substitute the variables "T", "B" and "L" with the logs dimensions:

$$V \quad = \quad \frac{\dfrac{\pi 9^2}{10000} + \dfrac{\pi 11^2}{10000}}{2} \; 9.0$$

Calculate the area of the two ends in square centimetres:

$$V \quad = \quad \left(\frac{\dfrac{254.468\,952}{10\,000} + \dfrac{380.132\,632}{10\,000}}{2} \right) 9.0$$

Divide each area by 10 000 to change square centimetres to square metres:

$$V \quad = \quad \frac{0.025\,446\,895\,2 + 0.038\,013\,263\,2}{2} \; 9.0$$

Sum the areas of the two ends and divide by two to average the areas:

$$V \quad = \quad 0.031\,730\,079\,2 \; x \; 9.0$$

Multiply the average end area by the length in metres to yield the volume:

$$V \quad = \quad 0.285\,570\,712\,8 \; m^3,$$

Round the volume of the log to 3 decimal places:

$$\text{Volume} \quad = \quad 0.286 \; m^3$$

4.2.2 Smalian's Formula - Simplified

The fully expressed Smalian's formula described in Section 4.2. contains three constants; "π", 10 000, and "2". For field use, the formula may be simplified by reducing these constants to one constant, expressed as "C ". Smalian's formula is:

$$V \quad = \quad \frac{\dfrac{\pi\,(T^2 + B^2)}{10000}}{2} = L$$

By rearranging the terms to consolidate the constants, the formula becomes:

$$V \quad = \quad \left(T^2 + B^2\right)\frac{\dfrac{\pi}{10000}}{2}\,L = 0.0001570796$$

The value of "C" is therefore:

$$C \quad = \quad \frac{\dfrac{\pi}{10000}}{2} = \frac{\dfrac{3.141592}{10000}}{2}$$

$$C \quad = \quad 0.000\ 157\ 079\ 6$$

Using the constant "C" provides for Smalian's simplified formula:

$$V \quad = \quad \left(T^2 + B^2\right) x\,L\,x\,C$$

where: V = the volume of the log in cubic metres,

T = the top radius in centimetres (or the diameter in rads),

B = the butt radius in centimetres (or the diameter in rads),

L = the length in metres, and

C = 0.000 157 079 6 (the constant).

4.2.3 Smalian's Simple Formula - Example Calculation

Using the simplified Smalian's field formula developed in Section 4.2.2 and substituting the log dimensions from Figure 4.5 into the formula, the volume of the log may be calculated in fewer steps than with the long formula demonstrated in Section 4.2.1. Many pocket calculators will not carry this number of decimals, so one may use 1.570 796 and divide by 10000 after multiplication.

$$V \quad = \quad \left(T^2 + B^2\right) x\, L\, x\, C$$

$$V \quad = \quad \left(09^2 + 11^2\right) x\, 9.0\, x\, 0.0001570796$$

$$V \quad = \quad \left(81 + 121\right) x\, 9.0\, x\, 0.0001570796$$

$$V \quad = \quad 202\, x\, 9.0\, x\, 0.0001570796$$

$$V \quad = \quad 1818\, x\, 0.0001570796$$

$$V \quad = \quad 0.28557071$$

Round the volume of the log to 3 decimal places:

$$\text{Volume} \quad = \quad 0.286\ m^3$$

4.2.4 The Application of Smalian's Formula

Most scalers derive log volumes from computer software, from the BC Metric Scale Stick, or from the FS 546 Table of the Half Volume of Cylinders in Cubic Decimetres. All of these methods, while still based upon Smalian's formula, save unnecessary arithmetic but scalers must know and be able to use the formula in situations where these tools are not available.

Scalers must be able to demonstrate their knowledge and comprehension of the principles underlying scaling by stating the full version of Smalian's formula and by calculating volumes with either the full or simplified version on the written portion of the Licenced Scaler's Exam.

All persons designing software for scaling purposes must employ Smalian's formula and have the software approved by the Ministry. Information on approval procedures can be obtained from a regional office of the Ministry or at:

MFLNRO Public Website

4.3 Other Formulae

Roundwood and defect formulae all use Smalian's formula to determine volume in cubic metres. Other formulae allow scalers to determine volumes on shapes other than round. These scaling formulae include the quadrilateral (4 sided), triangular (3 sided), sector and 'parent log'. Some of these formulae are used in defect measurement and for out-of-round slab shapes.

They are all thoroughly explained in the *Gross Measurement Procedures* chapter of this manual.

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