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PAMPHLET NO. # 50 DATE: September 29, 1995

SUBJECT: Power Analysis and Sample Sizes for Randomized Block Designs with Subsampling

This pamphlet will extend the results of the previous pamphlet to the Randomized Complete Block Design. The effectiveness of blocking to increase the power of treatment tests is also discussed. The creation of the graphs in terms of programming is almost identical to that of the Completely Randomized Design, except for the calculation of the denominator degrees of freedom of the F-test for treatments. This is described in the next pamphlet (BI #51).

As before, two components of variation requiring estimates are identified from the ANOVA table. For discussion purposes we shall use a design similar to that of the previous pamphlet. This is obtained by randomly assigning the treatments (factor T with t = 4 levels) to one plot (factor P with p = 1 levels) within each of several blocks (factor B with b = 4 levels). Plots are the experimental units and each is subsampled e = 10 times to obtain an estimate of the plot response (subsamples will be factor E as before). Since blocks are usually considered random, this design has only one fixed factor, namely the treatment T. The general ANOVA table is:

Source of Variation		Degrees of freedom	Expected Mean Squares	Mean Square	Error
Block	B	b-1	$\sigma_e^2 + e\sigma_{p(b)}^2 + \quad + tpe\sigma_B^2$ [1]	MSB	MSP
Treatment	T	t-1	$\sigma_e^2 + e\sigma_{p(b)}^2 + pe\sigma_{BT}^2 + bpe\phi_T$	MST	MSBT
Error	B x T	(b-1)(t-1)	$\sigma_e^2 + e\sigma_{p(b)}^2 + pe\sigma_{BT}^2$	MSBT	MSP
Plots	P(BT)	bt(p-1)	$\sigma_e^2 + e\sigma_{p(b)}^2$	MSP	MSE
Subsamples	E(PBT)	pbt(e-1)	σ_e^2	MSE	-

This table explicitly includes the source of variation for plots nested within blocks. This source is traditionally left out of textbook discussions since it has zero degrees of freedom. I prefer to include it so that the effect of plot variability within blocks can be seen. Further, designs with replication of plots within blocks (that is p > 1) are occasionally established and it is clear from the above table how that design would be analyzed. In that case, there is a legitimate test for blocks (assuming that blocks are not confounded with some other source of variation for which the test is really of interest, see BI #34 for a discussion). Nevertheless, the traditional design has only one plot per block/treatment and the above table can be modified to reflect this fact. The resulting table is:

[1] Notice that the controversy around this EMS centers around whether the block by treatment interaction should also be included as part of the EMS. In any case, the test for treatment is clear.

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Source of Variation		Degrees of freedom	Expected Mean Squares	Mean Square	Error
Block	B	3	$\sigma_e^2 + e\sigma_{p(b)}^2 + \quad + te\sigma_B^2$	MSB	MSP
Treatment	T	3	$\sigma_e^2 + e(\sigma_{p(b)}^2 + \sigma_{BT}^2) + be\phi_T$	MST	MSBT
Error	B x T	9	$\sigma_e^2 + e(\sigma_{p(b)}^2 + \sigma_{BT}^2)$	MSBT	MSE [2]
Plots	P(BT)	0	$\sigma_e^2 + e\sigma_{p(b)}^2$	MSP	MSE
Subsamples	E(PBT)	144	σ_e^2	MSE	-
Total		ptbe - 1 = 159			

We can see from this table that there is a suitable error term for Treatment, namely the Error or B x T source of variation (estimated by MSBT). This EMS contains three components of variation, only one of which can be separately estimated; namely, σ_e^2, estimated by MSE. Since MSP is not estimable (with zero degrees of freedom) we cannot obtain separate estimates for $\sigma_{p(b)}^2$ and σ_{BT}^2 from an ANOVA table for a *post hoc* analysis[3]. Nevertheless, it is only necessary to obtain an estimate for the combined components of variation $\sigma_{p(b)}^2 + \sigma_{BT}^2$.

To determine sample sizes or power for the Randomized Block Design, as before we must consider: 1) the alternate hypothesis, H_A ; 2) our choices for α and β; 3) the number of subsamples, e and σ_e^2; and 4) the number of blocks, b (instead of plots, p) and $\sigma_{p(b)}^2 + \sigma_{BT}^2$ (instead of σ_p^2)[4]. The number of blocks or plots and the components of variation $\sigma_{p(b)}^2 + \sigma_{BT}^2$ and σ_p^2 play equivalent mathematical roles in the determination of power and in the use of Cox's rule of thumb.

Cox's rule of thumb also applies but must be modified to state that ***there is not much increase in power when e is greater than 4(σ_e^2/($\sigma_{p(b)}^2 + \sigma_{BT}^2$))*** (from Cox, 1958, page 181). The values of σ_e^2 and σ_p^2 used in the graphs for the completely randomized design (see BI #49) and the corresponding values of the ratio are shown below. If $\sigma_{p(b)}^2 + \sigma_{BT}^2$ for the Randomized Block Design (RBD) has the same value as σ_p^2 for the Completely Randomized Design (CRD) then the ratio is identical for the two designs:

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[2] This error term is only appropriate if it can be assumed that $\sigma_{p(b)}^2$ is zero.

[3] It is possible that separate estimates for these components of variation can be obtained from other sources.

[4] Note that σ_p^2 and $\sigma_{p(b)}^2$ have slightly different meanings. σ_p^2 is the variability between plots while $\sigma_{p(b)}^2$ is the variability between plots nested within blocks. If blocking is effective then we would expect $\sigma_{p(b)}^2$ to be smaller than σ_p^2.

Cox's Ratio for the two designs:

σ_e^2	σ_p^2 or $\sigma_{p(b)}^2 + \sigma_{BT}^2$	CRD: $4(\sigma_e^2/\sigma_p^2)$	RBD: $4(\sigma_e^2/(\sigma_{p(b)}^2 + \sigma_{BT}^2)$
100	100	4	4
100	500	4/5 ≈ 1	4/5 ≈ 1
1000	100	40	40
1000	500	8	8

Under the assumption of equal variances ($\sigma_p^2 = \sigma_{p(b)}^2 + \sigma_{BT}^2$), the power for the Randomized Block Design will be similar to that of the Completely Randomized Design. There is a slight reduction in power because the treatment error term has fewer degrees of freedom. To see this consider these two designs with the same number of plots or blocks per treatment, that is b = p. Then the degrees of freedom for the error term of the randomized block design is (b-1)(t-1) while that of the completely randomized design is t(p-1). The difference between these two is t(b-1) - (p-1)(t-1) = t(b-p) + (p-1) = p-1 or b-1 (since b = p), the degrees of freedom for the blocks. The reduction in power is generally small as shown by the graph on the left below.

a) b)

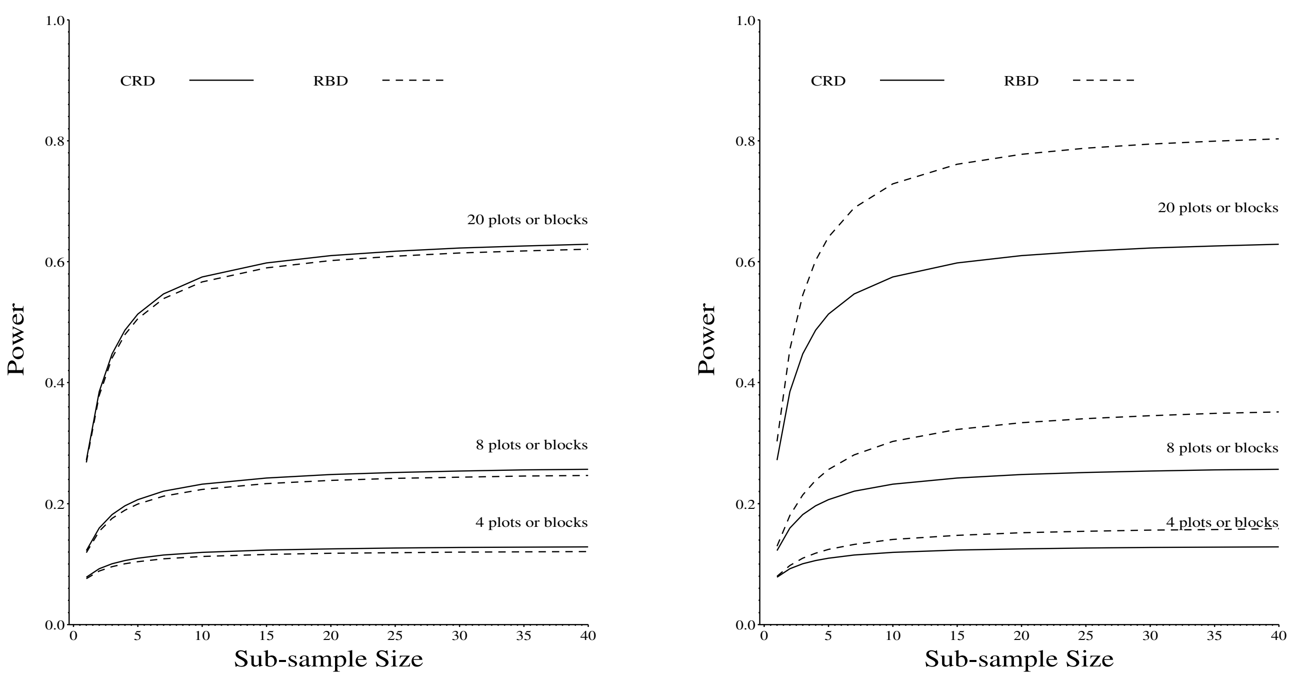

Figure 1. Comparing power for the randomized block and completely randomized designs: $\alpha = 0.05$, SSM = 125, $\sigma_e^2 = 500$, $\sigma_p^2 = 300$ and a) $\sigma_{p(b)}^2 + \sigma_{BT}^2 = 300$ and b) $\sigma_{p(b)}^2 + \sigma_{BT}^2 = 200$.

Mathematically, the removal of b-1 degrees of freedom from the treatment error term corresponds to a transfer of sums of squares from the error sums of squares to that of the block source of variation. If this transfer is 'large' then blocks 'explains' a substantial amount of the variability in the data (that is, σ_B^2 is greater than zero). Further, the size of the denominator mean square is reduced resulting in a more powerful test for treatment differences because the Mean Square for treatment will have a larger observed F-value. This increase in power is shown by the graph on the right above where it is assumed that blocking has reduced $\sigma^2_{p(\ b)} + \sigma^2_{BT}$ from 300 to 200. Clearly the power has been increased, although not necessarily by a large amount. The power would be further increased if blocking had been more effective and reduced $\sigma^2_{p(\ b)} + \sigma^2_{BT}$ to an even greater extent.

Other discussions comparing the Completely Randomized Design and the Randomized Block Design can be found in Nemec (pgs 18-20) and in texts like Snedecor and Cochran, Kuehl, and Hinkelmann and Kempthorne.

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