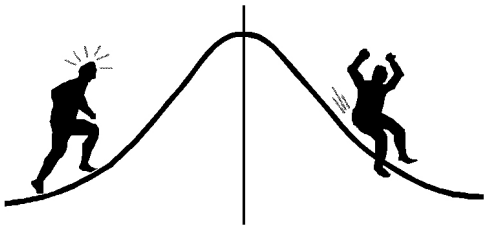

BIOMETRICS INFORMATION

(You're 95% likely to need this information)

PAMPHLET NO. # 27 DATE: May 17, 1990

SUBJECT: When the t-test and the F-test are equivalent

In general, for every t-test there is a matching F-test with 1 numerator degree of freedom. When this is the case, the t-test will have degrees of freedom equal to the F-test's denominator degrees of freedom and the observed F-value will be the square of the observed t-value. That is:

$$F_{1,df} = t^2_{df}$$

Some common statistical tests that can be conducted with either the t-test or the F-test are:

1. Difference between two means:

   a) Independent samples: An independent sample t-test can be used or a one-way ANOVA with only two levels for the treatment. The degrees of freedom (df) for the t-test will be $n_1 + n_2 - 2$, while the df for the F-test will be 1, $n_1 + n_2 - 2$ (where n_1 and n_2 are the sample sizes for the two means).

   b) Paired samples: A paired t-test can be used to test for differences between paired samples when one member of each pair was assigned a treatment level and the other member assigned the other treatment level (often a control). A randomized block ANOVA can also be used to do this test, if block is given a different value for each pair and the treatment in the ANOVA has just two levels. If there are *b* pairs, then the df for the t-test is *b*-1 and the df for the F-test are 1, *b*-1.

2. Specific differences between several means (*i.e.* contrasts):

Biometrics Information pamphlet #16 discusses contrasts viewed as t-tests, while many textbooks discuss them as F-tests. `PROC GLM` in SAS provides F-values to test contrasts. One degree of freedom contrasts can be tested using either a t-test or an F-test. If two or more contrasts are pooled together for testing, then an F-test is required since the numerator df will be greater than 1. In fact, each source of variation in an ANOVA can be thought of as either a contrast if the source has 1 df or as a pooled set of orthogonal contrasts if the source has 2 or more df.

3. Slope in a simple regression:

The fitted slope (parameter estimate) of a line can be tested using either a t-test or an F-test. In fact, the overall F-test for the regression is a direct test of whether the slope is zero. Hence the square of the observed t-value for the parameter estimate will be the same. This can be seen

clearly in any output from PROC REG of SAS. As an example look at **Output 28.38** in the SAS/STAT guide (Version 6.03) on page 856. The overall F-value is 201.873 which has a square root of 14.208. The t-value for YEAR is 14.208 (under the column titled 'Y for H0: Parameter=0').

4. Slopes or parameter estimates in multiple regression:

Multiple regression estimates several 'slopes' or parameters, one for each variable in the model and one for the intercept. The overall F-value is used to test whether the set of parameters **as a whole** (but excluding the intercept) have true values different from zero. Each parameter estimate can be individually tested with a t-test. Nevertheless, the interpretation of these individual t-tests is not completely straightforward. This is because the t-test for a variable's parameter depends on which other variables are in the model[1]. The interpretation of the parameter t-tests is particularly important when selecting variables to include in a multiple regression[2]. For instance, suppose that we were considering the variables X_1, X_2, and X_3. The parameter estimate for X_1 and its corresponding t-value will have a different value depending on whether X_2 or X_3 or both X_2 and X_3 were included. So what does the t-test really test? It tests the contribution of X_1 to the multiple regression if it were the **last** variable added to the model[3]. In other words, two multiple regressions are fitted, one with just X_2 and X_3 and the second with all three variables. The test for the contribution of X_1 to the full model is done by comparing these two models. One version of this test is described in BI #18 and it is an F-test (see part ii on page 3). The F-test is general and can be used to test whether any group of variables should be included in a regression. When the group contains only one variable then the F-test is also a t-test, and the square root of the F-value is the observed t-value usually found on computer printouts.

These are the most common instances that come to mind. You might consider other instances where t-tests can also be viewed as F-tests.

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———————————PROBLEM FROM BI #26———————————

The equation can be rewritten as $Y = 5(X-12)^2$. This equation has a trough (not a peak) with the lowest point at $X = 12$ and $Y = 0$.

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[1]Only in the unusual circumstance that all the independent variables are uncorrelated with each other, would the t-tests be the same regardless of which other variables were in the model.

[2]see BI #18--Multiple Regression: Selecting the best subset

[3]I call this a **last-in** test.