# BIOMETRICS INFORMATION

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SUBJECT: ANOVA: Contrasts viewed as t-tests

Contrasts are a fundamental concept in analysis of variance and can be examined from several different viewpoints. In this pamphlet I will discuss how they are a type of t-test.

Very simply, a t-test is an estimate of some quantity ($\gamma$) divided by it's standard error. If the quantity is normally distributed with a mean of zero then the t-value has a (central) t-distribution. Thus, hypotheses regarding the quantity can be tested.

Readers will be most familiar with the t-test for the one sample case (discussed in pamphlet #11) and the independent sample t-test, where the t-value is calculated by:

$$t = \frac{\gamma}{SE(\gamma)} = \frac{\overline{Y}_1 - \overline{Y}_2}{SE(\overline{Y}_1 - \overline{Y}_2)}, \quad df = 2(n-1)$$

and, assuming homogeneity of variance,

$$SE(\overline{Y}_1 - \overline{Y}_2) = [2\ MSE/n]^{\frac{1}{2}}$$

with $\quad MSE = [\Sigma\ (Y_{1i}-\overline{Y}_1)^2 + \Sigma\ (Y_{2i}-\overline{Y}_2)^2]\ /\ 2(n-1).$

MSE is the mean square error obtained from an ANOVA and n is the number of experimental units sampled for each treatment mean. This t-test can be used to test if the true value estimated by $\gamma$ is zero.

We can expand our definition of $\gamma$ to include any linear combination or weighted sum of means (where each mean is normally distributed with similar variance). As an example, suppose that an experiment has three treatment levels, a control and 2 treatments. An obvious question to ask of the results is whether the control mean is different from the average of the two treatments. If the null hypothesis is true then the control mean ($\overline{Y}_c$) is equal to the average of the treatment means ($(\overline{Y}_1+\overline{Y}_2)/2$), or twice the control mean ($\overline{Y}_c$) is equal to the sum of the treatment means ($\overline{Y}_1 + \overline{Y}_2$). That is:

$$\gamma = 2\overline{Y}_c - \overline{Y}_1 - \overline{Y}_2 = 0$$

The null hypothesis can be tested by:

$$t = \frac{\gamma}{SE(\gamma)} = \frac{2\overline{Y}_c - \overline{Y}_1 - \overline{Y}_2}{SE(2\overline{Y}_c - \overline{Y}_1 - \overline{Y}_2)}, \quad df = 3(n-1)$$

with $\quad SE(\gamma) = [(2^2+1^2+1^2)(MSE)/n]^{\frac{1}{2}} = [6MSE/n]^{\frac{1}{2}}$

and MSE is obtained from the ANOVA.

The general formula for a linear combination or weighted sum of k means is:

$$\gamma = c_1\overline{Y}_1 + c_2\overline{Y}_2 + \cdots + c_k\overline{Y}_k = \sum c_i\overline{Y}_i$$

where $c_1 + c_2 + \cdots + c_k = \sum c_i = 0$ so that $\gamma = 0$ if the null hypothesis is true. (The weights, $c_i$, are called contrast coefficients). Then the standard error is:

$$SE(\gamma) = [(\sum c_i^2)(MSE)/n]^{\frac{1}{2}} \text{ with } df = k(n-1).$$

Suppose that the experiment described above had the following results:

| Treatment | Mean | n | Contrast Coefficients Q1 | Contrast Coefficients Q2 | MSE |
|---|---|---|---|---|---|
| Control | 10 | 12 | c1 = 2 | 0 | |
| Treatment 1 | 25 | 12 | c2 = -1 | 1 | |
| Treatment 2 | 15 | 12 | c3 = -1 | -1 | 800 |

The contrast coefficients are designed to answer the following questions:

Q1: Is the control mean response the same as that of the two treatments?
Q2: Is there a difference in response between the two treatments?

The tests are conducted as follows:

Q1: $\gamma = 2(10) - 25 - 15 = -20$;
$SE(\gamma) = [[2^2 + (-1)^2 + (-1)^2]\,[800/12]]^{\frac{1}{2}} = 20$

thus $t = -20/20 = -1.0$, $df = 33$, not significant

Q2: $\gamma = 25-15 = -10$;
$SE(\gamma) = [1^2 + (-1)^2]\,[800/12]^{\frac{1}{2}} = 11.5470$

thus $t = -10/11.5470 = -0.87$, $df = 33$, not significant

CONTACT: Wendy Bergerud
387-5676