BIOMETRICS INFORMATION

(You're 95% likely to need this information)

PAMPHLET NO. # 13 DATE: November 28, 1988

SUBJECT: Analysis of Covariance: Comparing Adjusted Means

Comparisons between adjusted means from an analysis of covariance (ANCOVA) are often desired if the overall F-test for treatment effects is significant. As in ANOVA, specific questions about the means should be tested with contrasts, while a general question of "where do the differences lie?" can be tested by multiple pair-wise comparisons (similar to Duncan's MRT or Fisher's protected LSD test for ANOVA).

Suppose that the response variable is Y, the treatment variable is A, and the covariate is X. The calculations require two SAS runs:

i) ANOVA on the covariate, X with treatment A

ii) ANCOVA on Y with treatment A and covariate X

which produces output for the following tables:

i) ANOVA on X

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Square
Treatment A	a-l	SSBX	MSBX
Error	$\sum n_i$-a	SSEX	MSEX

ii) ANCOVA on Y

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Square
Treatment A	a-l	SSBY	MSBY
Covariate X	l	SSX	MSX
Error	$\sum n_i$-a-l	SSEY	MSEY

Where a is the number of levels for treatment A and n_i is the sample size per treatment level.

l. CONTRASTS--DUNN-BONFERRONI TESTS

Contrasts are calculated the usual way but on adjusted means (obtained with the `LSMEANS` statements in `PROC GLM`) instead of the ordinary, unadjusted means. That is, a contrast $\hat{\gamma} = \sum c_i \overline{Y}_{iadj}$, where c_i are the usual contrast coefficients and $\overline{Y}_{iadj}$ are the adjusted means. The standard error of the contrast is calculated by:

$$SE_{DB} = \left[MSEY\ [l + MSBX/SSEX][\ \sum c_i^2/n_i]\ \right]^{\frac{1}{2}}$$

The Dunn-Bonferroni t-test is calculated by:

$$t_{DB} = \hat{\gamma}/SE_{DB} \text{ with df } = \sum n_i - a - 1$$

NOTE: The critical values for this test are NOT those of the usual t-distribution. Table A.6 of Huitema is one source of the appropriate values.

2. MULTIPLE PAIRWISE COMPARISONS--PAIRWISE LSD TEST

This test is ONLY conducted if the F-test for the treatment, A, is significant. The numerator of the t-test is simply the difference between a specific pair of means, $\overline{Y}_{iadj} - \overline{Y}_{jadj}$ while the denominator is the standard error of the difference given by:

$$SE = \left[MSEY\ [1/n_i + 1/n_j) + (\overline{X}_i - \overline{X}_j)^2/SSEX] \right]^{\frac{1}{2}}$$

The (ordinary) t-value is calculated by $t = (\overline{Y}_{iadj} - \overline{Y}_{jadj})/SE$ with df = $\sum n_i - a - 1$. The α-level for the t-test should be the same as that of the overall F-test.

EXAMPLE: Data from Table 3.1 (page 38) of Huitema.

A=1				A=2				A=3			
X=29	Y=15	X=47	Y=39	X=22	Y=20	X=43	Y=44	X=33	Y=14	X=48	Y=42
49	19	46	23	24	34	64	46	45	20	63	40
48	21	74	38	49	28	61	47	35	30	57	38
35	27	72	33	46	35	55	40	39	32	56	54
53	35	67	50	52	42	54	54	36	34	78	56

SUMMARY:

	Adjusted Means	Covariate Means	
A=1	28.479	52.0	MSBX = 63.333
A=2	40.331	47.0	SSEX = 5700
A=3	36.190	49.0	MSEY = 64.640

CONTRASTS:

LINEAR: $$t_{DB} = \frac{-28.479 + 0 + 36.190}{\sqrt{64.45\ [1 + 63.333/5700]\ [2/10]}}$$

$$= 2.13, \text{ df} = 26 \quad \text{prob} > 0.05$$

QUADRATIC: $t_{DB} = \dfrac{(28.479 - 2(40.331) + 36.190)}{\sqrt{64.64\ [1 + 63.333/5700]\ [6/10]}}$

= 2.55, df = 26 prob < 0.05, Bonferroni Critical Value is 2.379

PROTECTED LSD TESTS:

A = 1 vs A = 2: $t = \dfrac{28.479 - 40.331}{\sqrt{64.64\ [2/10 + (52 - 47)^2/5700]}}$

= -3.26, DF = 26, prob = 0.003

A = 1 vs A = 3: $t = \dfrac{28.479 - 36.190}{\sqrt{64.64\ [2/10 + (52 - 49)^2/5700]}}$

= -2.14, df = 26, prob = 0.042

A = 2 vs A = 3: $t = \dfrac{40.331 - 36.190}{\sqrt{64.64\ [2/10 + (52 - 49)^2/5700]}}$

= 1.15, df = 26, prob = 0.26

THEREFORE:

A = 2	3	1
40.331	36.190	28.479

NOTE: The `CONTRAST` statement in `PROC GLM` produced F-values equal to the square of the above t-values, although the probability levels were substantially different. The output of the `LSMEANS` statement with the `PDIFF` option were identical to the protected LSD tests above.

Reference:

Huitema, Bradley, E., 1980, *The Analysis of Covariance and Alternatives*, John Wiley and Sons, Toronto.

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