BIOMETRICS INFORMATION

(You're 95% likely to need this information)

PAMPHLET NO. # 12 DATE: November 9, 1988

SUBJECT: Determining Polynomial Contrast Coefficients

Orthogonal polynomial contrasts are often appropriate in analysis of variance situations for treatments which are quantitative. If the spacings between the quantitative levels (x) are equal, then the contrast coefficients can be found in many texts such as, Table A.17 in Snedecor and Cochran, Table 15.11 in Steel and Torrie or Table C.11 in Wetherill. When the levels are unequally spaced the contrast coefficients must be derived from first principles. This pamphlet will describe how to determine the linear and quadratic contrast coefficients for this situation.

STEP 1: Determine the linear contrast coefficients c_{1i} by:

i) subtracting the mean of the levels from each level

and ii) multiplying or dividing by a constant, k, to obtain integer values.

STEP 2: Determine the quadratic coefficients c_{2i} by:

$$c_{2i} = c_{1i}^2 + bc_{1i} + d$$

where $b = -C/B$ and $d = -B/n$ $B = \sum c_{1i}^2$ = sum of squares

$C = \sum c_{1i}^3$ = sum of cubes and n = number of levels.

NOTE: The coefficients c_{2i} may be divided or multiplied by any constant which reduces the size of the coefficients or makes them integers.

EXAMPLE 1: $x = 1, 2, 3, 4$ $\bar{x} = (1+2+3+4)/4 = 2.5$

STEP 1: i) Subtract mean: -1.5, -0.5, 0.5, 1.5

ii) Multiply by k=2: -3, -1, 1, 3 = c_{1i}

STEP 2: i) Calculate sums: $B = -3^2 + -1^2 + 1^2 + 3^2 = 20$

$C = -3^3 + -1^3 + 1^3 + 3^3 = 0$

ii) Calculate equation parameters: $b = 0$ and $d = -20/4 = -5$

iii) Calculate coefficients: $c_{2i} = c_{1i}^2 - 5$ = 4 -4 -4 4

iv) Divide by k2: c_{2i} = = 1 -1 -1 1

FINAL SET:

x =	1	2	3	4	
linear =	-3	-1	1	3	= c_{1i}
quadratic =	1	-1	-1	1	= c_{2i}

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EXAMPLE 2: $x = 10, 20, 50$ $\bar{x} = (10+20+50)/3 = 80/3$

STEP 1: i) Subtract mean: -50/3, -20/3, 70/3

ii) Multiply by 3/10: -5, -2, 7

STEP 2: i) Calculate sums: $B = 5^2 + -2^2 + 7^2 = 78$
$C = -5^3 + -2^3 + 7^3 = 210$

ii) Calculate equation parameters: $b = -210/78 = -105/39$
and $d = -78/3 = -26$

iii) Calculate coefficients: $c_{2i} = c_{1i}^2 - (105/39)c_{1i} - 26$

i.e. c_{2i} = 12.46 -16.62 4.15

iv) Multiply by 39/81: c_{2i} = 3 -4 1

FINAL SET: x = 10 20 50
linear = -5 -2 7 = c_{1i}
quadratic = 3 -4 1 = c_{2i}

EXAMPLE 3: $x = 4, 12, 25$ $\bar{x} = (4+12+25)/3 = 41/3$

STEP 1: i) Subtract mean: -29/3, -5/3, 34/3

ii) Multiply by 3: -29, -5, 34

STEP 2: i) Calculate sums: $B = -29^2 + -5^2 + 34^2 = 2022$
$C = -29^3 + -5^3 + 34^3 = 14790$

ii) Calculate equation parameters: $b = -14790/2022 = -2465/337$
and $d = -2022/3 = -674$

iii) Calculate coefficients: $c_{2i} = c_{1i}^2 - (2465/337)c_{1i} - 674$

i.e. c_{2i} = 379.12 -612.43 233.31

iv) Multiply by 337/9828: c_{2i} = 13 -21 8

FINAL SET: x = 4 12 25
linear = -29 -5 34 = c_{1i}
quadratic = 13 -21 8 = c_{2i}

CHECKING CALCULATIONS: If you have done your calculations correctly then the following should be true: i) $\sum c_{1i} = 0$ ii) $\sum c_{2i} = 0$ iii) $\sum c_{1i}\, c_{2i} = 0$.

HINT: Remember that $(-1)^3 = -1$!

References:

Snedecor, G.W. and W.C. Cochran. 1967. *Statistical Methods*, Sixth Ed., The Iowa State University Press, Ames, Iowa.

Steel, R.G.D. and J.H. Torrie. 1980. *Principles and Procedures of Statistics: A Biometrical Approach.* 2nd edition. McGraw-Hill Inc., New York, New York.

Wetherill, G.B. 1981. *Intermediate Statistical Methods*. Chapman and Hall, New York, New York.

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