

# Foundations of Mathematics

## 11 and 12 (2008)



# FOUNDATIONS OF MATHEMATICS

## GRADE 11

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

<b>Measurement</b>	<b>General Outcome:</b> Develop spatial sense and proportional reasoning.
<b>Specific Outcomes</b>	<b>Achievement Indicators</b>
<i>It is expected that students will:</i>	<i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
A1. Solve problems that involve the application of rates. [CN, PS, R]	1.1 Interpret rates in a given context, such as the arts, commerce, the environment, medicine or recreation. 1.2 Solve a rate problem that requires the isolation of a variable. 1.3 Determine and compare rates and unit rates. 1.4 Make and justify a decision, using rates. 1.5 Represent a given rate pictorially. 1.6 Draw a graph to represent a rate. 1.7 Explain, using examples, the relationship between the slope of a graph and a rate. 1.8 Describe a context for a given rate or unit rate. 1.9 Identify and explain factors that influence a rate in a given context. 1.10 Solve a contextual problem that involves rates or unit rates.

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<b>Measurement</b> (continued)	<b>General Outcome:</b> Develop spatial sense and proportional reasoning.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
A2. Solve problems that involve scale diagrams, using proportional reasoning. [CN, PS, R, V]	<p>2.1 Explain, using examples, how scale diagrams are used to model a 2-D shape or a 3-D object.</p> <p>2.2 Determine, using proportional reasoning, the scale factor, given one dimension of a 2-D shape or a 3-D object and its representation.</p> <p>2.3 Determine, using proportional reasoning, an unknown dimension of a 2-D shape or a 3-D object, given a scale diagram or a model.</p> <p>2.4 Draw, with or without technology, a scale diagram of a given 2-D shape according to a specified scale factor (enlargement or reduction).</p> <p>2.5 Solve a contextual problem that involves scale diagrams.</p>
A3. Demonstrate an understanding of the relationships among scale factors, areas, surface areas and volumes of similar 2-D shapes and 3-D objects. [C, CN, PS, R, V]	<p>3.1 Determine the area of a 2-D shape, given the scale diagram, and justify the reasonableness of the result.</p> <p>3.2 Determine the surface area and volume of a 3-D object, given the scale diagram, and justify the reasonableness of the result.</p> <p>3.3 Explain, using examples, the effect of a change in the scale factor on the area of a 2-D shape.</p> <p>3.4 Explain, using examples, the effect of a change in the scale factor on the surface area of a 3-D object.</p> <p>3.5 Explain, using examples, the effect of a change in the scale factor on the volume of a 3-D object.</p> <p>3.6 Explain, using examples, the relationships among scale factor, area of a 2-D shape, surface area of a 3-D object and volume of a 3-D object.</p> <p>3.7 Solve a spatial problem that requires the manipulation of formulas.</p> <p>3.8 Solve a contextual problem that involves the relationships among scale factors, areas and volumes.</p>

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<b>Geometry</b>	<b>General Outcome:</b> Develop spatial sense.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
B1. Derive proofs that involve the properties of angles and triangles. [CN, R, V]	<i>(It is intended that deductive reasoning be limited to direct proof.)</i> 1.1 Generalize, using inductive reasoning, the relationships between pairs of angles formed by transversals and parallel lines, with or without technology. 1.2 Prove, using deductive reasoning, properties of angles formed by transversals and parallel lines, including the sum of the angles in a triangle. 1.3 Generalize, using inductive reasoning, a rule for the relationship between the sum of the interior angles and the number of sides ( $n$ ) in a polygon, with or without technology. 1.4 Identify and correct errors in a given proof of a property involving angles. 1.5 Verify, with examples, that if lines are not parallel the angle properties do not apply.
B2. Solve problems that involve the properties of angles and triangles. [CN, PS, V]	2.1 Determine the measures of angles in a diagram that involves parallel lines, angles and triangles, and justify the reasoning. 2.2 Identify and correct errors in a given solution to a problem that involves the measures of angles. 2.3 Solve a contextual problem that involves angles or triangles. 2.4 Construct parallel lines, using only a compass or a protractor, and explain the strategy used. 2.5 Determine if lines are parallel, given the measure of an angle at each intersection formed by the lines and a transversal.
B3. Solve problems that involve the cosine law and the sine law, including the ambiguous case. [CN, PS, R]	3.1 Draw a diagram to represent a problem that involves the cosine law or sine law. 3.2 Explain the steps in a given proof of the sine law or cosine law. 3.3 Solve a problem involving the cosine law that requires the manipulation of a formula. 3.4 Explain, concretely, pictorially or symbolically, whether zero, one or two triangles exist, given two sides and a non-included angle. 3.5 Solve a problem involving the sine law that requires the manipulation of a formula. 3.6 Solve a contextual problem that involves the cosine law or the sine law.

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<b>Logical Reasoning</b>	<b>General Outcome:</b> Develop logical reasoning.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
C1. Analyze and prove conjectures, using inductive and deductive reasoning, to solve problems. [C, CN, PS, R]	<p>1.1 Make conjectures by observing patterns and identifying properties, and justify the reasoning.</p> <p>1.2 Explain why inductive reasoning may lead to a false conjecture.</p> <p>1.3 Compare, using examples, inductive and deductive reasoning.</p> <p>1.4 Provide and explain a counterexample to disprove a given conjecture.</p> <p>1.5 Prove algebraic and number relationships, such as divisibility rules, number properties, mental mathematics strategies or algebraic number tricks.</p> <p>1.6 Prove a conjecture, using deductive reasoning (not limited to two column proofs).</p> <p>1.7 Determine if a given argument is valid, and justify the reasoning.</p> <p>1.8 Identify errors in a given proof; e.g., a proof that ends with <math>2 = 1</math>.</p> <p>1.9 Solve a contextual problem involving inductive or deductive reasoning.</p>
C2. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [CN, PS, R, V]	<p><i>(It is intended that this outcome be integrated throughout the course by using sliding, rotation, construction, deconstruction and similar puzzles and games.)</i></p> <p>2.1 Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g.,</p> <ul style="list-style-type: none"> <li>• guess and check</li> <li>• look for a pattern</li> <li>• make a systematic list</li> <li>• draw or model</li> <li>• eliminate possibilities</li> <li>• simplify the original problem</li> <li>• work backward</li> <li>• develop alternative approaches.</li> </ul> <p>2.2 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.</p> <p>2.3 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.</p>

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<b>Statistics</b>	<b>General Outcome:</b> Develop statistical reasoning.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
D1. Demonstrate an understanding of normal distribution, including: <ul style="list-style-type: none"> <li>• standard deviation</li> <li>• <math>z</math>-scores.</li> </ul> [CN, PS, T, V]	1.1 Explain, using examples, the meaning of standard deviation. 1.2 Calculate, using technology, the population standard deviation of a data set. 1.3 Explain, using examples, the properties of a normal curve, including the mean, median, mode, standard deviation, symmetry and area under the curve. 1.4 Determine if a data set approximates a normal distribution, and explain the reasoning. 1.5 Compare the properties of two or more normally distributed data sets. 1.6 Explain, using examples that represent multiple perspectives, the application of standard deviation for making decisions in situations such as warranties, insurance or opinion polls. 1.7 Solve a contextual problem that involves the interpretation of standard deviation. 1.8 Determine, with or without technology, and explain the $z$ -score for a given value in a normally distributed data set. 1.9 Solve a contextual problem that involves normal distribution.

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<b>Statistics</b> (continued)	<b>General Outcome:</b> Develop statistical reasoning.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
D2. Interpret statistical data, using: <ul style="list-style-type: none"> <li>• confidence intervals</li> <li>• confidence levels</li> <li>• margin of error.</li> </ul> [C, CN, R]	<i>(It is intended that the focus of this outcome be on interpretation of data rather than on statistical calculations.)</i> <p>2.1 Explain, using examples, how confidence levels, margin of error and confidence intervals may vary depending on the size of the random sample.</p> <p>2.2 Explain, using examples, the significance of a confidence interval, margin of error or confidence level.</p> <p>2.3 Make inferences about a population from sample data, using given confidence intervals, and explain the reasoning.</p> <p>2.4 Provide examples from print or electronic media in which confidence intervals and confidence levels are used to support a particular position.</p> <p>2.5 Interpret and explain confidence intervals and margin of error, using examples found in print or electronic media.</p> <p>2.6 Support a position by analyzing statistical data presented in the media.</p>

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<b>Relations and Functions</b>	<b>General Outcome:</b> Develop algebraic and graphical reasoning through the study of relations.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
E1. Model and solve problems that involve systems of linear inequalities in two variables. [CN, PS, T, V]	<p>1.1 Model a problem, using a system of linear inequalities in two variables.</p> <p>1.2 Graph the boundary line between two half planes for each inequality in a system of linear inequalities, and justify the choice of solid or broken lines.</p> <p>1.3 Determine and explain the solution region that satisfies a linear inequality, using a test point when given a boundary line.</p> <p>1.4 Determine, graphically, the solution region for a system of linear inequalities, and verify the solution.</p> <p>1.5 Explain, using examples, the significance of the shaded region in the graphical solution of a system of linear inequalities.</p> <p>1.6 Solve an optimization problem, using linear programming.</p>

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<b>Relations and Functions (continued)</b>	<b>General Outcome:</b> Develop algebraic and graphical reasoning through the study of relations.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
E2. Demonstrate an understanding of the characteristics of quadratic functions, including: <ul style="list-style-type: none"> <li>• vertex</li> <li>• intercepts</li> <li>• domain and range</li> <li>• axis of symmetry.</li> </ul> [CN, PS, T, V]	<p><i>(It is intended that completion of the square not be required.)</i></p> <ol style="list-style-type: none"> <li>2.1 Determine, with or without technology, the intercepts of the graph of a quadratic function.</li> <li>2.2 Determine, by factoring, the roots of a quadratic equation, and verify by substitution.</li> <li>2.3 Determine, using the quadratic formula, the roots of a quadratic equation.</li> <li>2.4 Explain the relationships among the roots of an equation, the zeros of the corresponding function, and the <math>x</math>-intercepts of the graph of the function.</li> <li>2.5 Explain, using examples, why the graph of a quadratic function may have zero, one or two <math>x</math>-intercepts.</li> <li>2.6 Express a quadratic equation in factored form, using the zeros of a corresponding function or the <math>x</math>-intercepts of its graph.</li> <li>2.7 Determine, with or without technology, the coordinates of the vertex of the graph of a quadratic function.</li> <li>2.8 Determine the equation of the axis of symmetry of the graph of a quadratic function, given the <math>x</math>-intercepts of the graph.</li> <li>2.9 Determine the coordinates of the vertex of the graph of a quadratic function, given the equation of the function and the axis of symmetry, and determine if the <math>y</math>-coordinate of the vertex is a maximum or a minimum.</li> <li>2.10 Determine the domain and range of a quadratic function.</li> <li>2.11 Sketch the graph of a quadratic function.</li> <li>2.12 Solve a contextual problem that involves the characteristics of a quadratic function.</li> </ol>

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<b>Mathematics Research Project</b>	<b>General Outcome:</b> Develop an appreciation of the role of mathematics in society.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
F1. Research and give a presentation on a historical event or an area of interest that involves mathematics. [C, CN, ME, PS, R, T, V]	<p>1.1 Collect primary or secondary data (statistical or informational) related to the topic.</p> <p>1.2 Assess the accuracy, reliability and relevance of the primary or secondary data collected by:</p> <ul style="list-style-type: none"> <li>• identifying examples of bias and points of view</li> <li>• identifying and describing the data collection methods</li> <li>• determining if the data is relevant</li> <li>• determining if the data is consistent with information obtained from other sources on the same topic.</li> </ul> <p>1.3 Interpret data, using statistical methods if applicable.</p> <p>1.4 Identify controversial issues, if any, and present multiple sides of the issues with supporting data.</p> <p>1.5 Organize and present the research project, with or without technology.</p>



# FOUNDATIONS OF MATHEMATICS

## GRADE 12

[C] Communication	[PS] Problem Solving
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<b>Financial Mathematics</b>	<b>General Outcome:</b> Develop number sense in financial applications.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
A1. Solve problems that involve compound interest in financial decision making. [C, CN, PS, T, V]	<p>1.1 Explain the advantages and disadvantages of compound interest and simple interest.</p> <p>1.2 Identify situations that involve compound interest.</p> <p>1.3 Graph and compare, in a given situation, the total interest paid or earned for different compounding periods.</p> <p>1.4 Determine, given the principal, interest rate and number of compounding periods, the total interest of a loan.</p> <p>1.5 Graph and describe the effects of changing the value of one of the variables in a situation that involves compound interest.</p> <p>1.6 Determine, using technology, the total cost of a loan under a variety of conditions; e.g., different amortization periods, interest rates, compounding periods and terms.</p> <p>1.7 Compare and explain, using technology, different credit options that involve compound interest, including bank and store credit cards and special promotions.</p> <p>1.8 Solve a contextual problem that involves compound interest.</p>

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<b>Financial Mathematics</b> (continued)	<b>General Outcome:</b> Develop number sense in financial applications.
<b>Specific Outcomes</b>	<b>Achievement Indicators</b>
<i>It is expected that students will:</i>	<i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
A2. Analyze costs and benefits of renting, leasing and buying. [CN, PS, R, T]	2.1 Identify and describe examples of assets that appreciate or depreciate. 2.2 Compare, using examples, renting, leasing and buying. 2.3 Justify, for a specific set of circumstances, if renting, buying or leasing would be advantageous. 2.4 Solve a problem involving renting, leasing or buying that requires the manipulation of a formula. 2.5 Solve, using technology, a contextual problem that involves cost-and-benefit analysis.
A3. Analyze an investment portfolio in terms of: <ul style="list-style-type: none"> <li>• interest rate</li> <li>• rate of return</li> <li>• total return.</li> </ul> [ME, PS, R, T]	3.1 Determine and compare the strengths and weaknesses of two or more portfolios. 3.2 Determine, using technology, the total value of an investment when there are regular contributions to the principal. 3.3 Graph and compare the total value of an investment with and without regular contributions. 3.4 Apply the Rule of 72 to solve investment problems, and explain the limitations of the rule. 3.5 Determine, using technology, possible investment strategies to achieve a financial goal. 3.6 Explain the advantages and disadvantages of long-term and short-term investment options. 3.7 Explain, using examples, why smaller investments over a longer term may be better than larger investments over a shorter term. 3.8 Solve an investment problem.

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<b>Logical Reasoning</b>	<b>General Outcome:</b> Develop logical reasoning.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
B1. Analyze puzzles and games that involve numerical and logical reasoning, using problem-solving strategies. [CN, ME, PS, R]	<i>(It is intended that this outcome be integrated throughout the course by using games and puzzles such as chess, Sudoku, Nim, logic puzzles, magic squares, Kakuro and cribbage.)</i> 1.1 Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g., <ul style="list-style-type: none"> <li>• guess and check</li> <li>• look for a pattern</li> <li>• make a systematic list</li> <li>• draw or model</li> <li>• eliminate possibilities</li> <li>• simplify the original problem</li> <li>• work backward</li> <li>• develop alternative approaches.</li> </ul> 1.2 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game. 1.3 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

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<b>Logical Reasoning</b> (continued)	<b>General Outcome:</b> Develop logical reasoning.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
B2. Solve problems that involve the application of set theory. [CN, PS, R, V]	<p>2.1 Provide examples of the empty set, disjoint sets, subsets and universal sets in context, and explain the reasoning.</p> <p>2.2 Organize information such as collected data and number properties, using graphic organizers, and explain the reasoning.</p> <p>2.3 Explain what a specified region in a Venn diagram represents, using connecting words (and, or, not) or set notation.</p> <p>2.4 Determine the elements in the complement, the intersection or the union of two sets.</p> <p>2.5 Explain how set theory is used in applications such as Internet searches, database queries, data analysis, games and puzzles.</p> <p>2.6 Identify and correct errors in a given solution to a problem that involves sets.</p> <p>2.7 Solve a contextual problem that involves sets, and record the solution, using set notation.</p>
B3. Solve problems that involve conditional statements. [C, CN, PS, R]	<p>3.1 Analyze an “if-then” statement, make a conclusion, and explain the reasoning.</p> <p>3.2 Make and justify a decision, using “what if?” questions, in contexts such as probability, finance, sports, games or puzzles, with or without technology.</p> <p>3.3 Determine the converse, inverse and contrapositive of an “if-then” statement; determine its veracity; and, if it is false, provide a counterexample.</p> <p>3.4 Demonstrate, using examples, that the veracity of any statement does not imply the veracity of its converse or inverse.</p> <p>3.5 Demonstrate, using examples, that the veracity of any statement does imply the veracity of its contrapositive.</p> <p>3.6 Identify and describe contexts in which a biconditional statement can be justified.</p> <p>3.7 Analyze and summarize, using a graphic organizer such as a truth table or Venn diagram, the possible results of given logical arguments that involve biconditional, converse, inverse or contrapositive statements.</p>

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<b>Probability</b>	<b>General Outcome:</b> Develop critical thinking skills related to uncertainty.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
C1. Interpret and assess the validity of odds and probability statements. [C, CN, ME]	<p>1.1 Provide examples of statements of probability and odds found in fields such as media, biology, sports, medicine, sociology and psychology.</p> <p>1.2 Explain, using examples, the relationship between odds (part-part) and probability (part-whole).</p> <p>1.3 Express odds as a probability and vice versa.</p> <p>1.4 Determine the probability of, or the odds for and against, an outcome in a situation.</p> <p>1.5 Explain, using examples, how decisions may be based on probability or odds and on subjective judgments.</p> <p>1.6 Solve a contextual problem that involves odds or probability.</p>
C2. Solve problems that involve the probability of mutually exclusive and non–mutually exclusive events. [CN, PS, R, V]	<p>2.1 Classify events as mutually exclusive or non–mutually exclusive, and explain the reasoning.</p> <p>2.2 Determine if two events are complementary, and explain the reasoning.</p> <p>2.3 Represent, using set notation or graphic organizers, mutually exclusive (including complementary) and non–mutually exclusive events.</p> <p>2.4 Solve a contextual problem that involves the probability of mutually exclusive or non–mutually exclusive events.</p> <p>2.5 Solve a contextual problem that involves the probability of complementary events.</p> <p>2.6 Create and solve a problem that involves mutually exclusive or non–mutually exclusive events.</p>

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<b>Probability (continued)</b>	<b>General Outcome:</b> Develop critical thinking skills related to uncertainty.
<b>Specific Outcomes</b>	<b>Achievement Indicators</b>
<i>It is expected that students will:</i>	<i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
C3. Solve problems that involve the probability of two events. [CN, PS, R]	3.1 Compare, using examples, dependent and independent events. 3.2 Determine the probability of an event, given the occurrence of a previous event. 3.3 Determine the probability of two dependent or two independent events. 3.4 Create and solve a contextual problem that involves determining the probability of dependent or independent events.
C4. Solve problems that involve the fundamental counting principle. [PS, R, V]	4.1 Represent and solve counting problems, using a graphic organizer. 4.2 Generalize the fundamental counting principle, using inductive reasoning. 4.3 Identify and explain assumptions made in solving a counting problem. 4.4 Solve a contextual counting problem, using the fundamental counting principle, and explain the reasoning.
C5. Solve problems that involve permutations. [ME, PS, R, T, V]	<i>(It is intended that circular permutations not be included.)</i> 5.1 Represent the number of arrangements of $n$ elements taken $n$ at a time, using factorial notation. 5.2 Determine, with or without technology, the value of a factorial. 5.3 Simplify a numeric or algebraic fraction containing factorials in both the numerator and denominator. 5.4 Solve an equation that involves factorials. 5.5 Determine the number of permutations of $n$ elements taken $r$ at a time. 5.6 Determine the number of permutations of $n$ elements taken $n$ at a time where some elements are not distinct. 5.7 Explain, using examples, the effect on the total number of permutations of $n$ elements when two or more elements are identical. 5.8 Generalize strategies for determining the number of permutations of $n$ elements taken $r$ at a time. 5.9 Solve a contextual problem that involves probability and permutations.

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<b>Probability</b> (continued)	<b>General Outcome:</b> Develop critical thinking skills related to uncertainty.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
C6. Solve problems that involve combinations. [ME, PS, R, T, V]	6.1 Explain, using examples, why order is or is not important when solving problems that involve permutations or combinations. 6.2 Determine the number of combinations of $n$ elements taken $r$ at a time. 6.3 Generalize strategies for determining the number of combinations of $n$ elements taken $r$ at a time. 6.4 Solve a contextual problem that involves combinations and probability.

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<b>Relations and Functions</b>	<b>General Outcome:</b> Develop algebraic and graphical reasoning through the study of relations.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
D1. Represent data, using polynomial functions (of degree $\leq 3$ ), to solve problems. [C, CN, PS, T, V]	1.1 Describe, orally and in written form, the characteristics of polynomial functions by analyzing their graphs. 1.2 Describe, orally and in written form, the characteristics of polynomial functions by analyzing their equations. 1.3 Match equations in a given set to their corresponding graphs. 1.4 Graph data and determine the polynomial function that best approximates the data. 1.5 Interpret the graph of a polynomial function that models a situation, and explain the reasoning. 1.6 Solve, using technology, a contextual problem that involves data that is best represented by graphs of polynomial functions, and explain the reasoning.
D2. Represent data, using exponential and logarithmic functions, to solve problems. [C, CN, PS, T, V]	2.1 Describe, orally and in written form, the characteristics of exponential or logarithmic functions by analyzing their graphs. 2.2 Describe, orally and in written form, the characteristics of exponential or logarithmic functions by analyzing their equations. 2.3 Match equations in a given set to their corresponding graphs. 2.4 Graph data and determine the exponential or logarithmic function that best approximates the data. 2.5 Interpret the graph of an exponential or logarithmic function that models a situation, and explain the reasoning. 2.6 Solve, using technology, a contextual problem that involves data that is best represented by graphs of exponential or logarithmic functions, and explain the reasoning.

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	[V] Visualization

<b>Relations and Functions</b> (continued)	<b>General Outcome:</b> Develop algebraic and graphical reasoning through the study of relations.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
D3. Represent data, using sinusoidal functions, to solve problems. [C, CN, PS, T, V]	<p>3.1 Describe, orally and in written form, the characteristics of sinusoidal functions by analyzing their graphs.</p> <p>3.2 Describe, orally and in written form, the characteristics of sinusoidal functions by analyzing their equations.</p> <p>3.3 Match equations in a given set to their corresponding graphs.</p> <p>3.4 Graph data and determine the sinusoidal function that best approximates the data.</p> <p>3.5 Interpret the graph of a sinusoidal function that models a situation, and explain the reasoning.</p> <p>3.6 Solve, using technology, a contextual problem that involves data that is best represented by graphs of sinusoidal functions, and explain the reasoning.</p>

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

<b>Mathematics Research Project</b>	<b>General Outcome:</b> Develop an appreciation of the role of mathematics in society.
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following set of indicators <b>may</b> be used to determine whether students have met the corresponding specific outcome.</i>
E1. Research and give a presentation on a current event or an area of interest that involves mathematics. [C, CN, ME, PS, R, T, V]	1.1 Collect primary or secondary data (statistical or informational) related to the topic. 1.2 Assess the accuracy, reliability and relevance of the primary or secondary data collected by: <ul style="list-style-type: none"> <li>• identifying examples of bias and points of view</li> <li>• identifying and describing the data collection methods</li> <li>• determining if the data is relevant</li> <li>• determining if the data is consistent with information obtained from other sources on the same topic.</li> </ul> 1.3 Interpret data, using statistical methods if applicable. 1.4 Identify controversial issues, if any, and present multiple sides of the issues with supporting data. 1.5 Organize and present the research project, with or without technology.