Mathematics Grade 5

Integrated Resource Package 2007

British Columbia
The Best Place on Earth

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This Integrated Resource Package (IRP) provides basic information teachers will require in order to implement Mathematics K to 7. Once fully implemented, this document will supersede Mathematics K to 7 (1995).

> The information contained in this document is also available on the Internet at www.bced.gov.bc.ca/irp/irp.htm

The following paragraphs provide brief descriptions of the components of the IRP.

Introduction

The Introduction provides general information about Mathematics K to 7, including special features and requirements.

Included in this section are

- a rationale for teaching Mathematics K to 7 in BC schools
- goals for Mathematics K to 7
- descriptions of the curriculum organizers – groupings for prescribed learning outcomes that share a common focus
- a suggested timeframe for each grade
- a graphic overview of the curriculum content from K to 7
- additional information that sets the context for teaching Mathematics K to 7

Considerations for Program delivery

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners.

Prescribed Learning Outcomes

This section contains the *prescribed learning outcomes.* Prescribed learning outcomes are the legally required content standards for the provincial education system. They define the required attitudes, skills, and knowledge for each subject. The learning outcomes are statements of what students are expected to know and be able to do by the end of the grade.

The prescribed learning outcomes for the Mathematics K to 7 IRP are based on the Learning Outcomes contained within the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework (CCF) for K to 9 Mathematics available at www.wncp.ca.

Student Achievement

This section of the IRP contains information about classroom assessment and measuring student achievement, including sets of specific achievement indicators for each prescribed learning outcome. Achievement indicators are statements that describe what students should be able to do in order to demonstrate that they fully meet the expectations set out by the prescribed learning outcomes. Achievement indicators are not mandatory; they are provided to assist teachers in assessing how well their students achieve the prescribed learning outcomes.

The achievement indicators for the Mathematics K to 7 IRP are based on the achievement indicators contained within the WNCP Common Curriculum Framework for K to 9 Mathematics.

> The WNCP CCF for K to 9 Mathematics is available online at www.wncp.ca

Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of prescribed learning outcomes.

Classroom Assessment Model

This section contains a series of classroom units that address the learning outcomes. The units have been developed by BC teachers, and are provided to support classroom assessment. These units are suggestions only – teachers may use or modify the units to assist them as they plan for the implementation of this curriculum.

Each unit includes the prescribed learning outcomes and suggested achievement indicators, a suggested timeframe, a sequence of suggested assessment activities, and sample assessment instruments.

Learning Resources

This section contains general information on learning resources, providing a link to titles, descriptions, and ordering information for the recommended learning resources in the Mathematics K to 7 Grade Collections. [*Note: Grade Collections for Mathematics K to 7 will be updated as new resources matching the IRP are authorized.*]

Glossary

The glossary section provides a link to an online glossary that contains definitions for selected terms used in this Integrated Resource Package

INTRODUCTION

This Integrated Resource Package (IRP) sets out the provincially prescribed curriculum for Mathematics K to 7. The development of this IRP has been guided by the principles of learning:

- Learning requires the active participation of the student.
- People learn in a variety of ways and at different rates.
- Learning is both an individual and a group process.

In addition to these three principles, this document recognizes that British Columbia's schools include young people of varied backgrounds, interests, abilities, and needs. Wherever appropriate for this curriculum, ways to meet these needs and to ensure equity and access for all learners have been integrated as much as possible into the learning outcomes and achievement indicators.

The Mathematics K to 7 IRP is based on the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework (CCF) for Kindergarten to Grade 9 Mathematics (May 2006). A complete list of references used to inform the revisions of the WNCP CCF for K to 9 Mathematics as well as this IRP can be found at the end of this section of the IRP.

Mathematics K to 7, in draft form, was available for public review and response from September to November, 2006. Input from educators, students, parents, and other educational partners informed the development of this document.

Rationale

The aim of Mathematics K to 7 is to provide students with the opportunity to further their knowledge, skills, and attitudes related to mathematics.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Numeracy can be defined as the combination of mathematical knowledge, problem solving and communication skills required by all persons to function successfully within our technological world. Numeracy is more than knowing about numbers and number operations. (British Columbia Association of Mathematics Teachers 1998)

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of a variety of manipulatives and pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics. Information gathered from these discussions can be used for formative assessment to guide instruction.

As facilitators of learning educators are encouraged to highlight mathematics concepts as they occur within the K to 7 school environment and within home environments. Mathematics concepts are present within every school's subjects and drawing students' attention to these concepts as they occur can help to provide the "teachable moment."

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary. Positive learning experiences build self-confidence and develop attitudes that value learning mathematics.

Aboriginal perspective

Aboriginal students in British Columbia come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings including urban, rural, and isolated communities. Teachers need to understand the diversity of cultures and experiences of students.

Aboriginal students come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is also vital that teachers understand and respond to non-verbal cues so that student learning and mathematical understanding are optimized. Depending on their learning styles, students may look for connections in learning and learn best when mathematics is contextualized and not taught as discrete components.

A variety of teaching and assessment strategies is required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students. *The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks 1993).*

Affective Domain

Bloom's taxonomy of learning behaviours identified three domains of educational activities, affective (growth in feelings or emotional areas – attitude), cognitive (mental skills – knowledge), and psychomotor (manual or physical skills – skills). The affective domain involves the way in which we perceive and respond to things emotionally, such as feelings, values, appreciation, enthusiasms, motivations, and attitudes.

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help students develop and maintain positive attitudes and self-confidence. Research has shown that students who are more engaged with school and with mathematics are far more likely to be successful in school and in learning mathematics. (Nardi & Steward 2003). Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

Substantial progress has been made in research in the last decade that has examined the importance and use of the affective domain as part of the learning process. In addition there has been a parallel increase in specific research involving the affective domain and its' relationship to the learning of mathematics which has provided powerful evidence of the importance of this area to the learning of mathematics (McLeod 1988, 1992 & 1994; Hannula 2002 & 2006; Malmivuori 2001 & 2006). Teachers, students, and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Students who are feeling more comfortable with a subject, demonstrate more confidence and have the opportunity for greater academic achievement (Denton & McKinney 2004; Hannula 2006; Smith et al. 1998). Educators can include opportunities for active and co-operative learning in their mathematics lessons which has been shown in research to promote greater conceptual understanding, more positive attitudes and subsequently improved academic achievement from students (Denton & McKinney 2004). By allowing the sharing and discussion of answers and strategies used in mathematics, educators are providing rich opportunities for students mathematical development. Educators can foster greater conceptual understanding in students by having students practice certain topics and concepts in mathematics in a meaningful and engaging manner.

It is important for educators, students, and parents to recognize the relationship between the affective and cognitive domains and attempt to nurture those aspects of the affective domain that contribute to positive attitudes and success in learning.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that are integral to the nature of mathematics, including change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty. These components are woven throughout this curriculum.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, p. 184).

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (The Primary Program 2000, p. 146).

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms.

Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics.

These skills contribute to students' interaction with and understanding of their environment.

Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions, and justify their reasoning when solving routine and non-routine problems.

Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Goals for Mathematics K to 7

Mathematics K to 7 represents the first formal steps that students make towards becoming life-long learners of mathematics.

Goals for Mathematics K to 7

The Mathematics K-7 curriculum is meant to start students toward achieving the main goals of mathematics education:

- using mathematics confidently to solve problems
- using mathematics to better understand the world around us
- communicating and reasoning mathematically
- appreciating and valuing mathematics
- making connections between mathematics and its applications
- committing themselves to lifelong learning
- becoming mathematically literate and using mathematics to participate in, and contribute to, society

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- be able to use mathematics to make and justify decisions about the world around us
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

Curriculum Organizers

A curriculum organizer consists of a set of prescribed learning outcomes that share a common focus. The prescribed learning outcomes for Mathematics K to 7 progress in age-appropriate ways, and are grouped under the following curriculum organizers and suborganizers:

Curriculum Organizers and Suborganizers

Mathematics K-7

Number

Patterns and Relations
- Patterns
- Variables and Equations

Shape and Space
- Measurement
- 3-D Objects and 2-D Shapes
- Transformations

Statistics and Probability
- Data Analysis
- Chance and Uncertainty

These curriculum organizers reflect the main areas of mathematics that students are expected to address. The ordering of organizers, suborganizers, and outcomes in the Mathematics K to 7 curriculum does not imply an order of instruction. The order in which various topics are addressed is left to the professional judgment of teachers. Mathematics teachers are encouraged to integrate topics throughout the curriculum and within other subject areas to emphasize the connections between mathematics concepts.

Number

Students develop their concept of the number system and relationships between numbers. Concrete, pictorial and symbolic representations are used to help students develop their number sense. Computational fluency, the ability to connect understanding of the concepts with accurate, efficient and flexible computation strategies for multiple purposes, is stressed throughout the number organizer with an emphasis on the development of personal strategies, mental mathematics and estimation strategies.

The Number organizer does not contain any suborganizers.

Patterns and Relations

Students develop their ability to recognize, extend, create, and use numerical and non- numerical patterns to better understand the world around them as well as the world of mathematics. This organizer provides opportunities for students to look for relationships in the environment and to describe the relationships. These relationships should be examined in multiple sensory forms.

The Patterns and Relations organizer includes the following suborganizers:
- Patterns
- Variables and Equations

Shape and Space

Students develop their understanding of objects and shapes in the environment around them. This includes recognition of attributes that can be measured, measurement of these attributes, description of these attributes, the identification and use of referents, and positional change of 3-D objects and 2-D shapes on the environment and on the Cartesian plane.

The Shape and Space organizer includes the following suborganizers:
- Measurement
- 3-D Objects and 2-D Shapes
- Transformations

Statistics and Probability

Students collect, interpret and present data sets in relevant contexts to make decisions. The development of the concepts involving probability is also presented as a means to make decisions. The Shape and Space organizer includes the following suborganizers:
- Data Analysis
- Chance and Uncertainty

Key Concepts: Overview of Mathematics K to 7 Topics

	Kindergarten	Grade 1	Grade 2	Grade 3
Number	• number sequence to 10 • familiar number arrangements up to 5 objects • one-to-one correspondence • numbers in-depth to 10	• skip counting starting at 0 to 100 • arrangements up to 10 objects • numbers in-depth to 20 • addition & subtraction to 20 • mental math strategies to 18	• skip counting at starting points other than 0 to 100 • numbers in-depth to 100 • even, odd & ordinal numbers • addition & subtraction to 100 • mental math strategies to 18	• skip counting at starting points other than 0 to 1000 • numbers in-depth to 1000 • addition & subtraction to 1000 • mental math strategies for 2-digit numerals • multiplication up to 5 × 5 • representation of fractions
Patterns & Relations *Patterns*	• repeating patterns of two or three elements	• repeating patterns of two to four elements • representation of pattern	• repeating patterns of three to five elements • increasing patterns	• increasing patterns • decreasing patterns
Patterns & Relations *Variables & Equations*		• equalities & inequalities • symbol for equality	• equality & inequality • symbols for equality & inequality	• one-step addition and subtraction equations
Shape & Space *Measurement*	• direct comparison for length, mass & volume	• process of measurement using comparison	• days, weeks, months, & years • non-standard units of measure for length, height distance around, mass (weight)	• non-standard & standard units of time • measurements of length (cm, m) & mass (g, kg) • perimeter of regular & irregular shapes
Shape & Space *3-D Objects & 2-D Shapes*	• single attribute of 3-D objects	• one attribute of 3-D objects & 2-D shapes • composite 2-D shapes & 3-D objects • 2-D shapes in the environment	• two attributes of 3-D objects & 2-D shapes • cubes, spheres, cones, cylinders, pyramids • triangles, squares, rectangles, circles • 2-D shapes in the environment	• faces, edges & vertices of 3-D objects • triangles, quadrilaterals, pentagons, hexagons, octagons
Shape & Space *Transformations*				
Statistics & Probability *Data Analysis*			• data about self and others • concrete graphs and pictographs	• first-hand data • bar graphs
Statistics & Probability *Chance & Uncertainty*				

Grade 4	Grade 5	Grade 6	Grade 7
• numbers in-depth to 10 000 • addition & subtraction to 10 000 • multiplication & division of numbers • fractions less than or equal to one • decimals to hundredths	• numbers in-depth to 1 000 000 • estimation strategies for calculations & problem solving • mental mathematics strategies for multiplication facts to 81 & corresponding division facts • mental mathematics for multiplication • multiplication for 2-digit by 2-digit & division for 3-digit by 1-digit • decimal & fraction comparison • addition & subtraction of decimals	• numbers in-depth greater than 1 000 000 & smaller than one thousandth • factors & multiples • improper fractions & mixed numbers • ratio & whole number percent • integers • multiplication & division of decimals • order of operations excluding exponents	• divisibility rules • addition, subtraction, multiplication, & division of numbers • percents from 1% to 100% • decimal & fraction relationships for repeating & terminating decimals • addition & subtraction of positive fractions & mixed numbers • addition & subtraction of integers
• patterns in tables & charts	• prediction using a pattern rule	• patterns & relationships in graphs & tables including tables of value	• table of values & graphs of linear relations
• symbols to represent unknowns • one-step equations	• single-variable, one-step equations with whole number coefficients & solutions	• letter variable representation of number relationships • preservation of equality	• preservation of equality • expressions & equations • one-step linear equations
• digital clocks, analog clocks, & calendar dates • area of regular & irregular 2-D shapes	• perimeter & area of rectangles • length, volume, & capacity	• perimeter & area of rectangles • length, volume, & capacity	• properties of circles • area of triangles, parallelograms, & circles
• rectangular & triangular prisms	• parallel, intersecting, perpendicular, vertical & horizontal edges & faces • rectangles, squares, trapezoids, parallelograms & rhombuses	• types of triangles • regular & irregular polygons	• geometric constructions
• line symmetry	• 2-D shape single transformation	• combinations of transformations • single transformation in the first quadrant of the Cartesian plane	• four quadrants of the Cartesian plane • transformations in the four quadrants of the Cartesian plane
• many-to-one correspondence including bar graphs & pictographs	• first-hand & second-hand data • double bar graphs	• line graphs • methods of data collection • graph data	• central tendency, outliers & range • circle graphs
	• likelihood of a single outcome	• experimental & theoretical probability	• ratios, fractions, & percents to express probabilities • two independent events • tree diagrams for two independent events

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The following seven mathematical processes should be integrated within Mathematics K to 7.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students need to be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine 1991, p. 5).

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhances flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

Even more important than performing computational procedures or using calculators is the greater facility that students need – more than ever before – with estimation and mental mathematics (NCTM May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein 2001).

Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating.

Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you...?" or "How could you...?" the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K to 3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong 1993, p. 10). The use of visualization in the study of mathematics provides students with the opportunity to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to decide when to measure, when to estimate and to know several estimation strategies (Shaw & Cliatt 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Suggested Timeframe

Provincial curricula are developed in accordance with the amount of instructional time recommended by the Ministry of Education for each subject area. For Mathematics K to 7, the Ministry of Education recommends a time allotment of 20% (approximately 95 hours in Kindergarten and 185 hours in Grades 1 to 7) of the total instructional time for each school year. In the primary years, teachers determine the time allotments for each required area of study and may choose to combine various curricula to enable students to integrate ideas and see the application of mathematics concepts across curricula.

The Mathematics K to 7 IRP for grades 1 to 7 is based on approximately 170 hours of instructional time to allow flexibility to address local needs. For Kindergarten, this estimate is approximately 75 hours. Based on these recommendations, teachers should be spending about 2 to 2.5 hours each week on Mathematics in Kindergarten and 4.5 to 5 hours of instructional time each week on Mathematics grades 1 to 7.

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The following references have been used to inform the revisions of the BC Mathematics K to 7 IRP as well as the WNCP CCF for K-9 Mathematics upon which the Prescribed Learning Outcomes and Achievement Indicators are based.

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Considerations for Program Delivery

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners. Included in this section is information about

- alternative delivery policy
- inclusion, equity, and accessibility for all learners
- working with the Aboriginal community
- information and communications technology
- copyright and responsibility
- fostering the development of positive attitudes
- instructional focus
- applying mathematics

Alternative Delivery Policy

The Alternative Delivery policy does not apply to the Mathematics K to 7 curriculum.

The Alternative Delivery policy outlines how students, and their parents or guardians, in consultation with their local school authority, may choose means other than instruction by a teacher within the regular classroom setting for addressing prescribed learning outcomes contained in the Health curriculum organizer of the following curriculum documents:

- Health and Career Education K to 7, and Personal Planning K to 7 Personal Development curriculum organizer (until September 2008)
- Health and Career Education 8 and 9
- Planning 10

The policy recognizes the family as the primary educator in the development of children's attitudes, standards, and values, but the policy still requires that all prescribed learning outcomes be addressed and assessed in the agreed-upon alternative manner of delivery.

It is important to note the significance of the term "alternative delivery" as it relates to the Alternative Delivery policy. The policy does not permit schools to omit addressing or assessing any of the prescribed learning outcomes within the health and career education curriculum. Neither does it allow students to be excused from meeting any learning outcomes related to health. It is expected that students who arrange for alternative delivery will address the health-related learning outcomes and will be able to demonstrate their understanding of these learning outcomes.

For more information about policy relating to alternative delivery, refer to www.bced.gov.bc.ca/policy/

Inclusion, Equity, and Accessibility for All Learners

British Columbia's schools include young people of varied backgrounds, interests, and abilities. The Kindergarten to Grade 12 school system focuses on meeting the needs of all students. When selecting specific topics, activities, and resources to support the implementation of Mathematics K to 7, teachers are encouraged to ensure that these choices support inclusion, equity, and accessibility for all students. In particular, teachers should ensure that classroom instruction, assessment, and resources reflect sensitivity to diversity and incorporate positive role portrayals, relevant issues, and themes such as inclusion, respect, and acceptance.

Government policy supports the principles of integration and inclusion of students who have English as a second language and of students with special needs. Most of the prescribed learning outcomes and suggested achievement indicators in this IRP can be met by all students, including those with special needs and/or ESL needs. Some strategies may require adaptations to ensure that those with special and/or ESL needs can successfully achieve the learning outcomes. Where necessary, modifications can be made to the prescribed learning outcomes for students with Individual Education Plans.

For more information about resources and support for students with special needs, refer to www.bced.gov.bc.ca/specialed/

For more information about resources and support for ESL students, refer to www.bced.gov.bc.ca/esl/

Working with the Aboriginal Community

The Ministry of Education is dedicated to ensuring that the cultures and contributions of Aboriginal peoples in BC are reflected in all provincial curricula. To address these topics in the classroom in a way that is accurate and that respectfully reflects Aboriginal concepts of teaching and learning, teachers are strongly encouraged to seek the advice and support of local Aboriginal communities. Aboriginal communities are diverse in terms of language, culture, and available resources, and each community will have its own unique protocol to gain support for integration of local knowledge and expertise. To begin discussion of possible instructional and assessment activities, teachers should first contact Aboriginal education co-ordinators, teachers, support workers, and counsellors in their district who will be able to facilitate the identification of local resources and contacts such as Elders, chiefs, tribal or band councils, Aboriginal cultural centres, Aboriginal Friendship Centres, and Métis or Inuit organizations.

In addition, teachers may wish to consult the various Ministry of Education publications available, including the "Planning Your Program" section of the resource, *Shared Learnings* (2006). This resource was developed to help all teachers provide students with knowledge of, and opportunities to share experiences with, Aboriginal peoples in BC.

> For more information about these documents, consult the Aboriginal Education web site: www.bced.gov.bc.ca/abed/welcome.htm

Information and Communications Technology

The study of information and communications technology is increasingly important in our society. Students need to be able to acquire and analyze information, to reason and communicate, to make informed decisions, and to understand and use information and communications technology for a variety of purposes. Development of these skills is important for students in their education, their future careers, and their everyday lives.

Literacy in the area of information and communications technology can be defined as the ability to obtain and share knowledge through investigation, study, instruction, or transmission of information by means of media technology. Becoming literate in this area involves finding, gathering, assessing, and communicating information using electronic means, as well as developing the knowledge and skills to use and solve problems effectively with the technology. Literacy also involves a critical examination and understanding of the ethical and social issues related to the use of information and communications technology. Mathematics K to 7 provides opportunities for students to develop literacy in relation to information and communications technology sources, and to reflect critically on the role of these technologies in society.

Copyright and Responsibility

Copyright is the legal protection of literary, dramatic, artistic, and musical works; sound recordings; performances; and communications signals. Copyright provides creators with the legal right to be paid for their work and the right to say how their work is to be used. There are some exceptions in the law (i.e., specific things permitted) for schools but these are very limited, such as copying for private study or research. The copyright law determines how resources can be used in the classroom and by students at home

In order to respect copyright it is necessary to understand the law. It is unlawful to do the following, unless permission has been given by a copyright owner:

- photocopy copyrighted material to avoid purchasing the original resource for any reason
- photocopy or perform copyrighted material beyond a very small part – in some cases the copyright law considers it "fair" to copy whole works, such as an article in a journal or a photograph, for purposes of research and private study, criticism, and review
- show recorded television or radio programs to students in the classroom unless these are cleared for copyright for educational use (there are exceptions such as for news and news commentary taped within one year of broadcast that by law have record-keeping requirements – see the web site at the end of this section for more details)
- photocopy print music, workbooks, instructional materials, instruction manuals, teacher guides, and commercially available tests and examinations

- show video recordings at schools that are not cleared for public performance
- perform music or do performances of copyrighted material for entertainment (i.e., for purposes other than a specific educational objective)
- copy work from the Internet without an express message that the work can be copied

Permission from or on behalf of the copyright owner must be given in writing. Permission may also be given to copy or use all or some portion of copyrighted work through a licence or agreement. Many creators, publishers, and producers have formed groups or "collectives" to negotiate royalty payments and copying conditions for educational institutions. It is important to know what licences are in place and how these affect the activities schools are involved in. Some licences may also require royalty payments that are determined by the quantity of photocopying or the length of performances. In these cases, it is important to assess the educational value and merits of copying or performing certain works to protect the school's financial exposure (i.e., only copy or use that portion that is absolutely necessary to meet an educational objective).

It is important for education professionals, parents, and students to respect the value of original thinking and the importance of not plagiarizing the work of others. The works of others should not be used without their permission.

For more information about copyright, refer to www.cmec.ca/copyright/indexe.stm

Fostering the Development of Positive Attitudes in Mathematics

A positive attitude toward mathematics is often a result of a learning environment in the classroom that encourages students' own mathematical thinking and contributions to classroom activities and discussions. Teachers should provide a variety of instructional approaches used in the classroom in order to reach a variety of learning styles and dispositions. These include experiences that encourage students to

- enjoy and value mathematics
- develop mathematical habits of mind
- explore
- take risks
- exhibit curiosity
- make and correct errors
- persevere
- experience mathematics in non-threatening, engaging ways
- understand and appreciate the role of mathematics in human affairs

These learning opportunities enable students to gain confidence in their abilities to solve complex problems.

The assessment of attitudes is indirect, and based on inferences drawn from students' behaviour. We can see what students do and hear what they say, and from these observations make inferences and draw conclusions about their attitudes.

It is important for teachers to consider their role in developing a positive attitude in mathematics. Teachers and parents are role models from which students begin to develop their disposition toward mathematics. Teachers need to model these attitudes in order to help students develop them (Burns 2000). In this manner teachers need to "present themselves as problem solvers, as active learners who are seekers, willing to plunge into new situations, not always knowing the answer or what the outcome will be" (p. 29).

Instructional Focus

The Mathematics K to 7 courses are arranged into a number of organizers with mathematical processes integrated throughout. Students learn in different ways and at different rates. As in other subject areas, it is essential when teaching mathematics, that concepts are introduced to students in a variety of ways. Students should hear explanations, watch demonstrations, draw to represent their thinking, engage in experiences with concrete materials and be encouraged to visualize and discuss their understanding of concepts. Most students need a range of concrete or representational experiences with mathematics concepts before they develop symbolic or abstract understanding. The development of conceptual understanding should be emphasized throughout the curriculum as a means to develop students to become mathematical problem solvers.

Teaching through Problem Solving

Problem solving should be an integral part of all mathematics classrooms. Teachers are encouraged to weave problem solving throughout all curriculum organizers in the K to 7 mathematics curriculum on a regular basis. Problem solving provides a way of helping students learn mathematics.

Hiebert et al. (1996) encourage teachers to make mathematics problematic. A problem can be defined as any task or activity for which the students have not memorized a method or rule, nor is there an assumption by the students that there is only one correct way to solve the problem (Hiebert et al. 1997). Van de Walle (2006) notes that "a problem for learning mathematics also has these features:

- The problem must begin where the students are.
- The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn.
- The problem must require justifications and explanations for answers and methods. (p. 11)

Why teach through problem solving?

- The math makes more sense. When using real world math problems, students are able to make the connections between what math is and how they can apply it.
- Problems are more motivating when they are challenging. Although some students are anxious when they are not directed by the teacher, most enjoy a challenge they can be successful in solving.
- Problem solving builds confidence. It maximizes the potential for understanding as each child makes his own sense out of the problem and allows for individual strategies.
- Problem solving builds perseverance. Because an answer is not instantaneous, many children think they are unable to do the math. Through the experience of problem solving they learn to apply themselves for longer periods of time and not give up.
- Problems can provide practice with concepts and skills. Good problems enable students to learn and apply the concepts in a meaningful way and an opportunity to practice the skills.
- Problem solving provides students with insight into the world of mathematics. Mathematicians struggle to find solutions to many problems and often need to go down more than one path to arrive at a solution. This is a creative process that is difficult to understand if one has never had to struggle.
- Problem solving provides the teacher with insight into a student's mathematical thinking. As students choose strategies and solve problems, the teacher has evidence of their thinking and can inform instruction based on this.
- Students need to practice problem solving. If we are expecting students to confront new situations involving mathematics, they need practice to become independent problem solvers (Small 2005).

Polya (1957) characterized a general method which can be used to solve problems, and to describe how problem-solving should be taught and learned. He advocated for the following steps in solving a mathematical problem:

- Understand the problem – What is unknown? What is known? Is enough information provided to determine the solution? Can a figure or model be used to represent the situation?
- Make a plan – Is there a similar problem that has been solved before? Can the problem be restated so it makes more sense?
- Carry out the plan – Have all of the steps been completed correctly?
- Look back – Do the results look correct? Is there another way to solve the problem that would verify the results?

While a number of variations of the problem solving model proposed by Polya (Van de Walle 2006; Small 2006; Burns 2000) they all have similar characteristics. The incorporation of a wide variety of strategies to solve problems is essential to developing students' ability to be flexible problem solvers.

The Mathematics K to 7 (1995) IRP provides a number of useful strategies that students can use to increase their flexibility in solving problems. These include:

- look for a pattern
- construct a table
- make an organized list
- act it out
- draw a picture
- use objects
- guess and check
- work backward
- write an equation
- solve a simpler (or similar) problem
- make a model (BC Ministry of Education 1995)

During problem-solving experiences, students are encouraged to solve problems using ways that make sense to them. As students share different ways of solving problems they can learn strategies from each other. Teachers are encouraged to facilitate this process to in an open and non-threatening environment. I this manner, students can develop a repertoire of strategies from which to draw upon when mathematical problems are presented to them.

Problem solving requires a shift in student attitudes and how teachers model these attitudes in the classroom. In order to be successful, students must develop, and teachers model, the following characteristics:

- interest in finding solutions to problems
- confidence to try various strategies
- willingness to take risks
- ability to accept frustration when not knowing
- understanding the difference between not knowing the answer and not having found it yet (Burns 2000)

Problems are not just simple computations embedded in a story nor are they contrived, that is, they do not exist outside the math classroom. Students will be engaged if the problems relate to their lives; their culture, interests, families, current events. They are tasks that are rich and open-ended so there is more than one way of arriving at a solution, or multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill or understanding. The students should not know the answer immediately. Problem solving takes time and effort on the part of the student and the teacher. Teaching thought problem solving is one of the ways that teachers can bring increased depth to the Mathematics K to 7 curriculum.

Instruction should provide an emphasis on mental mathematics and estimation to check the reasonableness of paper and pencil exercises, and the solutions to problems which are determined through the use of technology, including calculators and computers. (It is assumed that all students have regular access to appropriate technology such calculators, or computers with graphing software and standard spreadsheet programs.) Concepts should be introduced using manipulatives, and gradually developed from the concrete to the pictorial to the symbolic.

Applying Mathematics

For students to view mathematics as relevant and useful, they must see how it can be applied in a variety of contexts. Mathematics helps students understand and interpret their world and solve problems that occur in their daily lives both within and outside of the school context.

Teachers are encouraged to incorporate, and make explicit, mathematics concepts which naturally occur across the subject areas. Possible situations where cross curricular integration may occur in K to 7 include the following:

Fine Arts

- pattern, line, and form
- fractions in rhythm and metre
- spatial awareness in dance, drama, and visual arts
- geometric shapes in visual arts, drama, and dance
- symmetry and unison
- transformations
- perspective and proportion in visual arts
- measuring and proportional reasoning for mixing and applying materials in visual arts

Health and Career Education

- creating schedules
- interpreting statistical data
- collecting, organizing, and interpreting data charts, graphs, diagrams, and tables
- using mathematics to develop a logical argument to support a position on a topic or issue

Language Arts

- reading literature with a mathematics theme
- creating a picture book or writing a story with mathematical content
- listening to stories to decode mathematical contexts
- examine the plot of a story from a mathematical perspective
- create graphic organizers provide an explanation, proof, or justification for an argument
- role-play or oral presentations of problems and solutions
- creating word walls, personal dictionaries, or glossaries of mathematics terms
- examine the roots of mathematical terms

Physical Education

- examining the benefits of various physical activity (e.g. burning calories)
- examining patterns in physical movement
- measuring distances
- estimate distances and other quantise using referents
- reading and recording dates and time

Science

- discussing the magnitude of numbers
- classifying and sorting objects
- examining patterns to make a hypothesis
- measuring quantities
- use of referents for measurement
- units and conversions between units
- reading and writing quantities in multiple formats (e.g., numerals, words)
- collecting, organizing and interpreting data charts, graphs, diagrams, and tables
- creating a logical argument to support a hypothesis
- mental mathematics for calculations

Social Studies

- discussing the magnitude of numbers and building referents for numbers
- using concepts of area, perimeter, and distances when mapping
- graphing using the Cartesian plane
- using circle concepts to explain latitude and longitude, time zones, great circle routes
- interpreting statistical data
- collecting, organizing, and interpreting data charts, graphs, diagrams, and tables
- reading and recording dates and time
- examining the history of mathematics in context of world events
- using mathematics to develop a logical argument to support a position on a topic or issue

Students can also be encouraged to identify and examine the mathematics around them. In this way, students will come to see that mathematics is present outside of the classroom. There are many aspects of students' daily lives where they may encounter mathematic such as

- making purchases
- reading bus schedules
- reading sports statistics
- interpreting newspaper and media sources
- following a recipe
- estimating time to complete tasks
- estimating quantities
- creating patterns when doodling

Making these connections explicit for students helps to solidify the importance of mathematics.

PRESCRIBED LEARNING OUTCOMES

Prescribed learning outcomes are content standards for the provincial education system; they are the prescribed curriculum. Clearly stated and expressed in measurable and observable terms, learning outcomes set out the required attitudes, skills, and knowledge – what students are expected to know and be able to do – by the end of the subject and grade.

Schools have the responsibility to ensure that all prescribed learning outcomes in this curriculum are met; however, schools have flexibility in determining how delivery of the curriculum can best take place.

It is expected that student achievement will vary in relation to the learning outcomes. Evaluation, reporting, and student placement with respect to these outcomes are dependent on the professional judgment and experience of teachers, guided by provincial policy.

Prescribed learning outcomes for Mathematics K to 7 are presented by grade and by curriculum organizer and suborganizer, and are coded alphanumerically for ease of reference; however, this arrangement is not intended to imply a required instructional sequence.

Wording of Prescribed Learning Outcomes

All learning outcomes complete the stem, "It is expected that students will …."

When used in a prescribed learning outcome, the word "including" indicates that any ensuing item **must be addressed**. Lists of items introduced by the word "including" represent a set of minimum requirements associated with the general requirement set out by the outcome. The lists are not necessarily exhaustive, however, and teachers may choose to address additional items that also fall under the general requirement set out by the outcome.

Domains of Learning

Prescribed learning outcomes in BC curricula identify required learning in relation to one or more of the three domains of learning: cognitive, psychomotor, and affective. The following definitions of the three domains are based on Bloom's taxonomy.

The **cognitive domain** deals with the recall or recognition of knowledge and the development of intellectual abilities. The cognitive domain can be further specified as including three cognitive levels: knowledge, understanding and application, and higher mental processes. These levels are determined by the verb used in the learning outcome, and illustrate how student learning develops over time.

- Knowledge includes those behaviours that emphasize the recognition or recall of ideas, material, or phenomena.
- Understanding and application represents a comprehension of the literal message contained in a communication, and the ability to apply an appropriate theory, principle, idea, or method to a new situation.
- Higher mental processes include analysis, synthesis, and evaluation. The higher mental processes level subsumes both the knowledge and the understanding and application levels.

The **affective domain** concerns attitudes, beliefs, and the spectrum of values and value systems.

The **psychomotor domain** includes those aspects of learning associated with movement and skill demonstration, and integrates the cognitive and affective consequences with physical performances.

Domains of learning and cognitive levels also form the basis of the Assessment Overview Tables provided for each grade in the Classroom Assessment Model. In addition, domains of learning and, particularly, cognitive levels, inform the design and development of the Grades 4 and 7 Foundation Skills Assessment (FSA).

Prescribed Learning Outcomes

Grade 5

GRADE 5

It is expected that students will:

NUMBER

A1 represent and describe whole numbers to 1 000 000 [C, CN, V, T]

A2 use estimation strategies including
- front-end rounding
- compensation
- compatible numbers

in problem-solving contexts [C, CN, ME, PS, R, V]

A3 apply mental mathematics strategies and number properties, such as
- skip counting from a known fact
- using doubling or halving
- using patterns in the 9s facts
- using repeated doubling or halving

to determine answers for basic multiplication facts to 81 and related division facts [C, CN, ME, R, V]

A4 apply mental mathematics strategies for multiplication, such as
- annexing then adding zero
- halving and doubling
- using the distributive property [C, ME, R]

A5 demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems [C, CN, PS, V]

A6 Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems [C, CN, PS]

A7 demonstrate an understanding of fractions by using concrete and pictorial representations to
- create sets of equivalent fractions
- compare fractions with like and unlike denominators [C, CN, PS, R, V]

A8 describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially, and symbolically [C, CN, R, V]

A9 relate decimals to fractions (to thousandths) [CN, R, V]

A10 compare and order decimals (to thousandths) by using
- benchmarks
- place value
- equivalent decimals [CN, R, V]

A11 demonstrate an understanding of addition and subtraction of decimals (limited to thousandths) [C, CN, PS, R, V]

PATTERNS AND RELATIONS

Patterns

B1 determine the pattern rule to make predictions about subsequent elements [C, CN, PS, R, V]

Variables and Equations

B2 solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions [C, CN, PS, R]

GRADE 5

SHAPE AND SPACE

Measurement

C1 design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions [C, CN, PS, R, V]

C2 demonstrate an understanding of measuring length (mm) by
- selecting and justifying referents for the unit mm
- modelling and describing the relationship between mm and cm units, and between mm and m units

[C, CN, ME, PS, R, V]

C3 demonstrate an understanding of volume by
- selecting and justifying referents for cm^3 or m^3 units
- estimating volume by using referents for cm^3 or m^3
- measuring and recording volume (cm^3 or m^3)
- constructing rectangular prisms for a given volume [C, CN, ME, PS, R, V]

C4 demonstrate an understanding of capacity by
- describing the relationship between mL and L
- selecting and justifying referents for mL or L units
- estimating capacity by using referents for mL or L
- measuring and recording capacity (mL or L) [C, CN, ME, PS, R, V]

3-D Objects and 2-D Shapes

C5 describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are
- parallel
- intersecting
- perpendicular
- vertical
- horizontal [C, CN, R, T, V]

C6 identify and sort quadrilaterals, including
- rectangles
- squares
- trapezoids
- parallelograms
- rhombuses

according to their attributes [C, R, V]

Transformations

C7 perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image [C, CN, T, V]

C8 identify a single transformation, including a translation, rotation, and reflection of 2-D shapes [C, T, V]

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

Grade 5

Statistics and Probability

Data Analysis

D1 differentiate between first-hand and second-hand data [C, R, T, V]

D2 construct and interpret double bar graphs to draw conclusions [C, PS, R, T, V]

Chance and Uncertainty

D3 describe the likelihood of a single outcome occurring using words such as
- impossible
- possible
- certain [C, CN, PS, R]

D4 compare the likelihood of two possible outcomes occurring using words such as
- less likely
- equally likely
- more likely [C, CN, PS, R]

Student Achievement

This section of the IRP contains information about classroom assessment and student achievement, including specific achievement indicators that may be used to assess student performance in relation to each prescribed learning outcome. Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of prescribed learning outcomes.

Classroom Assessment and Evaluation

Assessment is the systematic gathering of information about what students know, are able to do, and are working toward. Assessment evidence can be collected using a wide variety of methods, such as

- observation
- student self-assessments and peer assessments
- quizzes and tests (written, oral, practical)
- samples of student work
- projects and presentations
- oral and written reports
- journals and learning logs
- performance reviews
- portfolio assessments

Assessment of student achievement is based on the information collected through assessment activities. Teachers use their insight, knowledge about learning, and experience with students, along with the specific criteria they establish, to make judgments about student performance in relation to prescribed learning outcomes.

Three major types of assessment can be used in conjunction with each other to support student achievement.

- Assessment for learning is assessment for purposes of greater learning achievement.
- Assessment as learning is assessment as a process of developing and supporting students' active participation in their own learning.
- Assessment of learning is assessment for purposes of providing evidence of achievement for reporting.

Assessment for Learning

Classroom assessment for learning provides ways to engage and encourage students to become involved in their own day-to-day assessment – to acquire the skills of thoughtful self-assessment and to promote their own achievement.

This type of assessment serves to answer the following questions:

- What do students need to learn to be successful?
- What does the evidence of this learning look like?

Assessment for learning is criterion-referenced, in which a student's achievement is compared to established criteria rather than to the performance of other students. Criteria are based on prescribed learning outcomes, as well as on suggested achievement indicators or other learning expectations.

Students benefit most when assessment feedback is provided on a regular, ongoing basis. When assessment is seen as an opportunity to promote learning rather than as a final judgment, it shows students their strengths and suggests how they can develop further. Students can use this information to redirect their efforts, make plans, communicate with others (e.g., peers, teachers, parents) about their growth, and set future learning goals.

Assessment for learning also provides an opportunity for teachers to review what their students are learning and what areas need further attention. This information can be used to inform teaching and create a direct link between assessment and instruction. Using assessment as a way of obtaining feedback on instruction supports student achievement by informing teacher planning and classroom practice.

Assessment as Learning

Assessment as learning actively involves students in their own learning processes. With support and guidance from their teacher, students take responsibility for their own learning, constructing meaning for themselves. Through a process of continuous self-assessment, students develop the ability to take stock of what they have already learned, determine what they have not yet learned, and decide how they can best improve their own achievement.

Although assessment as learning is student-driven, teachers can play a key role in facilitating how this assessment takes place. By providing regular opportunities for reflection and self-assessment, teachers can help students develop, practise, and become comfortable with critical analysis of their own learning.

Assessment of Learning

Assessment of learning can be addressed through summative assessment, including large-scale assessments and teacher assessments. These summative assessments can occur at the end of the year or at periodic stages in the instructional process.

Large-scale assessments, such as Foundation Skills Assessment (FSA) and Graduation Program exams, gather information on student performance throughout the province and provide information

for the development and revision of curriculum. These assessments are used to make judgments about students' achievement in relation to provincial and national standards.

Assessment of learning is also used to inform formal reporting of student achievement.

For Ministry of Education reporting policy, refer to www.bced.gov.bc.ca/policy/policies/student_reporting.htm

Assessment *for* Learning	Assessment *as* Learning	Assessment *of* Learning
Formative assessment *ongoing in the classroom* • teacher assessment, student self-assessment, and/or student peer assessment • criterion-referenced criteria based on prescribed learning outcomes identified in the provincial curriculum, reflecting performance in relation to a specific learning task • involves both teacher and student in a process of continual reflection and review about progress • teachers adjust their plans and engage in corrective teaching in response to formative assessment	Formative assessment *ongoing in the classroom* • self-assessment • provides students with information on their own achievement and prompts them to consider how they can continue to improve their learning • student-determined criteria based on previous learning and personal learning goals • students use assessment information to make adaptations to their learning process and to develop new understandings	Summative assessment *occurs at end of year or at key stages* • teacher assessment • may be either criterion-referenced (based on prescribed learning outcomes) or norm-referenced (comparing student achievement to that of others) • information on student performance can be shared with parents/guardians, school and district staff, and other education professionals (e.g., for the purposes of curriculum development) • used to make judgments about students' performance in relation to provincial standards

For more information about assessment for, as, and of learning, refer to the following resource developed by the Western and Northern Canadian Protocol (WNCP): *Rethinking Assessment with Purpose in Mind.*

This resource is available online at www.wncp.ca

In addition, the BC Performance Standards describe levels of achievement in key areas of learning (reading, writing, numeracy, social responsibility, and information and communications technology integration) relevant to all subject areas. Teachers may wish to use the Performance Standards as resources to support ongoing formative assessment in mathematics.

BC Performance Standards are available at www.bced.gov.bc.ca/perf_stands/

Criterion-Referenced Assessment and Evaluation

In criterion-referenced evaluation, a student's performance is compared to established criteria rather than to the performance of other students. Evaluation in relation to prescribed curriculum requires that criteria be established based on the learning outcomes.

Criteria are the basis for evaluating student progress. They identify, in specific terms, the critical aspects of a performance or a product that indicate how well the student is meeting the prescribed learning outcomes. For example, weighted criteria, rating scales, or scoring guides (reference sets) are ways that student performance can be evaluated using criteria.

Wherever possible, students should be involved in setting the assessment criteria. This helps students develop an understanding of what high-quality work or performance looks like.

Criterion-referenced assessment and evaluation may involve these steps:

Step 1	Identify the prescribed learning outcomes and suggested achievement indicators (as articulated in this IRP) that will be used as the basis for assessment.
Step 2	Establish criteria. When appropriate, involve students in establishing criteria.
Step 3	Plan learning activities that will help students gain the attitudes, skills, or knowledge outlined in the criteria.
Step 4	Prior to the learning activity, inform students of the criteria against which their work will be evaluated.
Step 5	Provide examples of the desired levels of performance.
Step 6	Conduct the learning activities.
Step 7	Use appropriate assessment instruments (e.g., rating scale, checklist, scoring guide) and methods (e.g., observation, collection, self-assessment) based on the particular assignment and student.
Step 8	Review the assessment data and evaluate each student's level of performance or quality of work in relation to criteria.
Step 9	Where appropriate, provide feedback and/or a letter grade to indicate how well the criteria are met.
Step 10	Communicate the results of the assessment and evaluation to students and parents/guardians.

Key Elements

Key elements provide an overview of content in each curriculum organizer. They can be used to determine the expected depth and breadth of the prescribed learning outcomes.

Note that some topics appear at multiple grade levels in order to emphasize their importance and to allow for developmental learning.

Achievement Indicators

To support the assessment of provincially prescribed curricula, this IRP includes sets of achievement indicators in relation to each learning outcome.

Achievement indicators, taken together as a set, define the specific level of attitudes demonstrated, skills applied, or knowledge acquired by the student in relation to a corresponding prescribed learning outcome. They describe what evidence to look for to determine whether or not the student has fully met the intent of the learning outcome. Since each achievement indicator defines only one aspect of the corresponding learning outcome, the entire set of achievement indicators should be considered when determining whether students have fully met the learning outcome.

In some cases, achievement indicators may also include suggestions as to the type of task that would provide evidence of having met the learning outcome (e.g., a constructed response such as a list, comparison, or analysis; a product created and presented such as a report, poster, letter, or model; a particular skill demonstrated such as map making or critical thinking).

Achievement indicators support the principles of assessment for learning, assessment as learning, and assessment of learning. They provide teachers and parents with tools that can be used to reflect on what students are learning, as well as provide students with a means of self-assessment and ways of defining how they can improve their own achievement.

Achievement indicators are not mandatory; they are suggestions only, provided to assist in the assessment of how well students achieve the prescribed learning outcomes.

The following pages contain the suggested achievement indicators corresponding to each prescribed learning outcome for the Mathematics K to 7 curriculum. The achievement indicators are arranged by curriculum organizer for each grade; however, this order is not intended to imply a required sequence of instruction and assessment.

Student Achievement

Grade 5

KEY ELEMENTS: GRADE 5

MATHEMATICAL PROCESS (INTEGRATED)

The following mathematical processes have been integrated within the prescribed learning outcomes and achievement indicators for the grade: communication, connections, mental mathematics and estimation, problem solving, reasoning, technology, and visualization.

NUMBER – develop number sense

- whole numbers to 1 000 000
- estimation strategies for calculations and problem solving
- mental mathematics strategies for multiplication facts to 81 and corresponding division facts
- mental mathematics for multiplication
- multiplication for 2-digit by 2-digit and division for 3-digit by 1-digit
- decimal and fraction comparison
- addition and subtraction of decimals to thousandths

PATTERNS AND RELATIONS – use patterns to describe the world and solve problems

Patterns

- prediction using a pattern rule

Variables and Equations

- single-variable, one-step equations with whole number coefficients and solutions

SHAPE AND SPACE – use direct and indirect measurement to solve problems

Measurement

- perimeter and area of rectangles
- length, volume and capacity

3-D Objects and 2-D Shapes

- parallel, intersecting, perpendicular, vertical and horizontal edges and faces
- quadrilaterals including rectangles, squares, trapezoids, parallelograms and rhombuses

Transformations

- 2-D shape single transformation

STATISTICS AND PROBABILITY – collect, display and analyze data to solve problems

Data Analysis

- first-hand and second-hand data
- double bar graphs

Chance and Uncertainty

- likelihood of a single outcome

NUMBER

General Outcome: Develop number sense.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
A1 represent and describe whole numbers to 1 000 000 [C, CN, V, T]	❑ write a given numeral using proper spacing without commas (e.g., 934 567) ❑ describe the pattern of adjacent place positions moving from right to left ❑ describe the meaning of each digit in a given numeral ❑ provide examples of large numbers used in print or electronic media ❑ express a given numeral in expanded notation (e.g., 45 321 = (4 × 10 000) + (5 × 1000) + (3 × 100) + (2 × 10) + (1 × 1) or 40 000 + 5000 + 300 + 20 + 1) ❑ write the numeral represented by a given expanded notation
A2 use estimation strategies including - front-end rounding - compensation - compatible numbers in problem-solving contexts [C, CN, ME, PS, R, V]	❑ provide a context for when estimation is used to - make predictions - check reasonableness of an answer - determine approximate answers ❑ describe contexts in which overestimating is important ❑ determine the approximate solution to a given problem not requiring an exact answer ❑ estimate a sum or product using compatible numbers ❑ estimate the solution to a given problem using compensation and explain the reason for compensation ❑ select and use an estimation strategy for a given problem ❑ apply front-end rounding to estimate - sums (e.g., 253 + 615 is more than 200 + 600 = 800) - differences (e.g., 974 – 250 is close to 900 – 200 = 700) - products (e.g., the product of 23 × 24 is greater than 20 × 20 (400) and less than 25 × 25 (625)) - quotients (e.g., the quotient of 831 ÷ 4 is greater than 800 ÷ 4 (200))

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

Prescribed Learning Outcomes	Suggested Achievement Indicators
A3 apply mental mathematics strategies and number properties, such as - skip counting from a known fact - using doubling or halving - using patterns in the 9s facts - using repeated doubling or halving to determine answers for basic multiplication facts to 81 and related division facts [C, CN, ME, R, V]	❑ describe the mental mathematics strategy used to determine a given basic fact, such as - skip count up by one or two groups from a known fact (e.g., if 5 × 7 = 35, then 6 × 7 is equal to 35 + 7 and 7 × 7 is equal to 35 + 7 + 7) - skip count down by one or two groups from a known fact (e.g., if 8 × 8 = 64, then 7 × 8 is equal to 64 – 8 and 6 × 8 is equal to 64 – 8 – 8) - doubling (e.g., for 8 × 3 think 4 × 3 = 12, and 8 × 3 = 12 + 12) - patterns when multiplying by 9 (e.g., for 9 × 6, think 10 × 6 = 60, and 60 – 6 = 54; for 7 × 9, think 7 × 10 = 70, and 70 – 7 = 63) - repeated doubling (e.g., if 2 × 6 is equal to 12, then 4 × 6 is equal to 24 and 8 × 6 is equal to 48) - repeated halving (e.g., for 60 ÷ 4, think 60 ÷ 2 = 30 and 30 ÷ 2 = 15) ❑ explain why multiplying by zero produces a product of zero ❑ explain why division by zero is not possible or undefined (e.g., 8 ÷ 0) ❑ recall multiplication facts to 81and related division facts
A4 apply mental mathematics strategies for multiplication, such as - annexing then adding zero - halving and doubling - using the distributive property [C, ME, R]	❑ determine the products when one factor is a multiple of 10, 100, or 1000 by annexing zero or adding zeros (e.g., for 3 × 200 think 3 × 2 and then add two zeros) ❑ apply halving and doubling when determining a given product (e.g., 32 × 5 is the same as 16 × 10) ❑ apply the distributive property to determine a given product involving multiplying factors that are close to multiples of 10 (e.g., 98 × 7 = (100 × 7) – (2 × 7))
A5 demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems [C, CN, PS, V]	❑ illustrate partial products in expanded notation for both factors (e.g., for 36 × 42, determine the partial products for (30 + 6) × (40 + 2)) ❑ represent both 2-digit factors in expanded notation to illustrate the distributive property (e.g., to determine the partial products of 36 × 42, (30 + 6) × (40 + 2) = 30 × 40 + 30 × 2 + 6 × 40 + 6 × 2 = 1200 + 60 + 240 + 12 = 1512) ❑ model the steps for multiplying 2-digit factors using an array and base ten blocks, and record the process symbolically ❑ describe a solution procedure for determining the product of two given 2-digit factors using a pictorial representation, such as an area model ❑ solve a given multiplication problem in context using personal strategies and record the process

Prescribed Learning Outcomes	Suggested Achievement Indicators
A6 Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems [C, CN, PS]	❑ model the division process as equal sharing using base ten blocks and record it symbolically ❑ explain that the interpretation of a remainder depends on the context ❑ ignore the remainder (e.g., making teams of 4 from 22 people) ❑ round up the quotient (e.g., the number of five passenger cars required to transport 13 people) ❑ express remainders as fractions (e.g., five apples shared by two people) ❑ express remainders as decimals (e.g., measurement and money) ❑ solve a given division problem in context using personal strategies, and record the process
A7 demonstrate an understanding of fractions by using concrete and pictorial representations to - create sets of equivalent fractions - compare fractions with like and unlike denominators [C, CN, PS, R, V]	❑ create a set of equivalent fractions and explain why there are many equivalent fractions for any given fraction using concrete materials ❑ model and explain that equivalent fractions represent the same quantity ❑ determine if two given fractions are equivalent using concrete materials or pictorial representations ❑ formulate and verify a rule for developing a set of equivalent fractions
A8 describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially, and symbolically [C, CN, R, V]	❑ identify equivalent fractions for a given fraction ❑ compare two given fractions with unlike denominators by creating equivalent fractions ❑ position a given set of fractions with like and unlike denominators on a number line and explain strategies used to determine the order ❑ write the decimal for a given concrete or pictorial representation of part of a set, part of a region, or part of a unit of measure ❑ represent a given decimal using concrete materials or a pictorial representation ❑ represent an equivalent tenth, hundredth, or thousandth for a given decimal using a grid ❑ express a given tenth as an equivalent hundredth and thousandth ❑ express a given hundredth as an equivalent thousandth ❑ describe the value of each digit in a given decimal
A9 relate decimals to fractions (to thousandths) [CN, R, V]	❑ write a given decimal in fractional form ❑ write a given fraction with a denominator of 10, 100, or 1000 as a decimal ❑ express a given pictorial or concrete representation as a fraction or decimal (e.g., 250 shaded squares on a thousandth grid can be expressed as 0.250 or $^{25}/_{1000}$)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

Prescribed Learning Outcomes	Suggested Achievement Indicators
A10 compare and order decimals (to thousandths) by using - benchmarks - place value - equivalent decimals [CN, R, V]	❑ order a given set of decimals by placing them on a number line that contains benchmarks, 0.0, 0.5, 1.0 ❑ order a given set of decimals including only tenths using place value ❑ order a given set of decimals including only hundredths using place value ❑ order a given set of decimals including only thousandths using place value ❑ explain what is the same and what is different about 0.2, 0.20, and 0.200 ❑ order a given set of decimals including tenths, hundredths, and thousandths using equivalent decimals
A11 demonstrate an understanding of addition and subtraction of decimals (limited to thousandths) [C, CN, PS, R, V]	❑ place the decimal point in a sum or difference using front-end estimation (e.g., for 6.3 + 0.25 + 306.158, think 6 + 306, so the sum is greater than 312) ❑ correct errors of decimal point placements in sums and differences without using paper and pencil ❑ explain why keeping track of place value positions is important when adding and subtracting decimals ❑ predict sums and differences of decimals using estimation strategies ❑ solve a given problem that involves addition and subtraction of decimals, limited to thousandths

PATTERNS AND RELATIONS (PATTERNS)

General Outcome: Use patterns to describe the world and solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
B1 determine the pattern rule to make predictions about subsequent elements [C, CN, PS, R, V]	❑ extend a given pattern with and without concrete materials, and explain how each element differs from the proceeding one ❑ describe, orally or in writing, a given pattern using mathematical language, such as one more, one less, five more ❑ write a mathematical expression to represent a given pattern, such as $r + 1$, $r - 1$, $r + 5$ ❑ describe the relationship in a given table or chart using a mathematical expression ❑ determine and explain why a given number is or is not the next element in a pattern ❑ predict subsequent elements in a given pattern ❑ solve a given problem by using a pattern rule to determine subsequent elements ❑ represent a given pattern visually to verify predictions

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

General Outcome: Represent algebraic expressions in multiple ways.

Prescribed Learning Outcomes	Suggested Achievement Indicators
	The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.
It is expected that students will:	*Students who have fully met the prescribed learning outcome are able to:*
B2 solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions [C, CN, PS, R]	❑ express a given problem in context as an equation where the unknown is represented by a letter variable ❑ solve a given single-variable equation with the unknown in any of the terms (e.g., $n + 2 = 5$, $4 + a = 7$, $6 = r - 2$, $10 = 2c$ ❑ create a problem in context for a given equation

SHAPE AND SPACE (MEASUREMENT)

General Outcome: Use direct or indirect measurement to solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
C1 design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions [C, CN, PS, R, V]	❑ construct or draw two or more rectangles for a given perimeter in a problem-solving context ❑ construct or draw two or more rectangles for a given area in a problem-solving context ❑ illustrate that for any given perimeter, the square or shape closest to a square will result in the greatest area ❑ illustrate that for any given perimeter, the rectangle with the smallest possible width will result in the least area ❑ provide a real-life context for when it is important to consider the relationship between area and perimeter
C2 demonstrate an understanding of measuring length (mm) by - selecting and justifying referents for the unit mm - modelling and describing the relationship between mm and cm units, and between mm and m units [C, CN, ME, PS, R, V]	❑ provide a referent for one millimetre and explain the choice ❑ provide a referent for one centimetre and explain the choice ❑ provide a referent for one metre and explain the choice ❑ show that 10 millimetres is equivalent to 1 centimetre using concrete materials (e.g., ruler) ❑ show that 1000 millimetres is equivalent to 1 metre using concrete materials (e.g., metre stick) ❑ provide examples of when millimetres are used as the unit of measure
C3 demonstrate an understanding of volume by - selecting and justifying referents for cm^3 or m^3 units - estimating volume by using referents for cm^3 or m^3 - measuring and recording volume (cm^3 or m^3) - constructing rectangular prisms for a given volume [C, CN, ME, PS, R, V]	❑ identify the cube as the most efficient unit for measuring volume and explain why ❑ provide a referent for a cubic centimetre and explain the choice ❑ provide a referent for a cubic metre and explain the choice ❑ determine which standard cubic unit is represented by a given referent ❑ estimate the volume of a given 3-D object using personal referents ❑ determine the volume of a given 3-D object using manipulatives and explain the strategy ❑ construct a rectangular prism for a given volume ❑ explain that many rectangular prisms are possible for a given volume by constructing more than one rectangular prism for the same given volume

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

Prescribed Learning Outcomes	Suggested Achievement Indicators
C4 demonstrate an understanding of capacity by - describing the relationship between mL and L - selecting and justifying referents for mL or L units - estimating capacity by using referents for mL or L - measuring and recording capacity (mL or L) [C, CN, ME, PS, R, V]	❑ demonstrate that 1000 millilitres is equivalent to 1 litre by filling a 1 litre container using a combination of smaller containers ❑ provide a referent for a litre and explain the choice ❑ provide a referent for a millilitre and explain the choice ❑ determine which capacity unit is represented by a given referent ❑ estimate the capacity of a given container using personal referents ❑ determine the capacity of a given container using materials that take the shape of the inside of the container (e.g., a liquid, rice, sand, beads) and explain the strategy

SHAPE AND SPACE (3-D OBJECTS AND 2-D SHAPES)

General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
C5 describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are - parallel - intersecting - perpendicular - vertical - horizontal [C, CN, R, T, V]	❑ identify parallel, intersecting, perpendicular, vertical, and horizontal edges and faces on 3-D objects ❑ identify parallel, intersecting, perpendicular, vertical, and horizontal sides on 2-D shapes ❑ provide examples from the environment that show parallel, intersecting, perpendicular, vertical, and horizontal line segments ❑ find examples of edges, faces, and sides that are parallel, intersecting, perpendicular, vertical, and horizontal in print and electronic media such as newspapers, magazines, and the internet ❑ draw 2-D shapes or 3-D objects that have edges, faces and sides that are parallel, intersecting, perpendicular, vertical, or horizontal ❑ describe the faces and edges of a given 3-D object using terms, such as parallel, intersecting, perpendicular, vertical, or horizontal ❑ describe the sides of a given 2-D shape using terms, such as parallel, intersecting, perpendicular, vertical, or horizontal
C6 identify and sort quadrilaterals, including - rectangles - squares - trapezoids - parallelograms - rhombuses according to their attributes [C, R, V]	❑ identify and describe the characteristics of a pre-sorted set of quadrilaterals ❑ sort a given set of quadrilaterals and explain the sorting rule ❑ sort a given set of quadrilaterals according to the lengths of the sides ❑ sort a given set of quadrilaterals according to whether or not opposite sides are parallel

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SHAPE AND SPACE (TRANSFORMATIONS)

General Outcome: Describe and analyze position and motion of objects and shapes.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
C7 perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image [C, CN, T, V]	❑ translate a given 2-D shape horizontally, vertically or diagonally, and describe the position and orientation of the image ❑ rotate a given 2-D shape about a point, and describe the position and orientation of the image ❑ reflect a given 2-D shape in a line of reflection, and describe the position and orientation of the image ❑ perform a transformation of a given 2-D shape by following instructions ❑ draw a 2-D shape, translate the shape, and record the translation by describing the direction and magnitude of the movement ❑ draw a 2-D shape, rotate the shape, and describe the direction of the turn (clockwise or counterclockwise), the fraction of the turn, and the point of rotation ❑ draw a 2-D shape, reflect the shape, and identify the line of reflection and the distance of the image from the line of reflection ❑ predict the result of a single transformation of a 2-D shape and verify the prediction
C8 identify a single transformation, including a translation, rotation, and reflection of 2-D shapes [C, T, V]	❑ provide an example of a translation, a rotation and a reflection ❑ identify a given single transformation as a translation, rotation, or reflection ❑ describe a given rotation by the direction of the turn (clockwise or counterclockwise)

STATISTICS AND PROBABILITY (DATA ANALYSIS)

General Outcome: Collect, display and analyze data to solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
D1 differentiate between first-hand and second-hand data [C, R, T, V]	❑ explain the difference between first-hand and second-hand data ❑ formulate a question that can best be answered using first-hand data and explain why ❑ formulate a question that can best be answered using second-hand data and explain why ❑ find examples of second-hand data in print and electronic media, such as newspapers, magazines, and the internet
D2 construct and interpret double bar graphs to draw conclusions [C, PS, R, T, V]	❑ determine the attributes (title, axes, intervals, and legend) of double bar graphs by comparing a given set of double bar graphs ❑ represent a given set of data by creating a double bar graph, label the title and axes, and create a legend without the use of technology ❑ draw conclusions from a given double bar graph to answer questions ❑ provide examples of double bar graphs used in a variety of print and electronic media, such as newspapers, magazines, and the internet ❑ solve a given problem by constructing and interpreting a double bar graph

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

STATISTICS AND PROBABILITY (CHANCE AND UNCERTAINTY)

General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
D3 describe the likelihood of a single outcome occurring using words such as - impossible - possible - certain [C, CN, PS, R]	❑ provide examples of events that are impossible, possible, or certain from personal contexts ❑ classify the likelihood of a single outcome occurring in a probability experiment as impossible, possible, or certain ❑ design and conduct a probability experiment in which the likelihood of a single outcome occurring is impossible, possible, or certain ❑ conduct a given probability experiment a number of times, record the outcomes, and explain the results
D4 compare the likelihood of two possible outcomes occurring using words such as - less likely - equally likely - more likely [C, CN, PS, R]	❑ identify outcomes from a given probability experiment which are less likely, equally likely, or more likely to occur than other outcomes ❑ design and conduct a probability experiment in which one outcome is less likely to occur than the other outcome ❑ design and conduct a probability experiment in which one outcome is equally as likely to occur as the other outcome ❑ design and conduct a probability experiment in which one outcome is more likely to occur than the other outcome

Classroom Assessment Model

The Classroom Assessment Model outlines a series of assessment units for Mathematics K to 7.

These units have been structured by grade level and theme. Collectively the units address all of the prescribed learning outcomes for each grade, and provide one suggested means of organizing, ordering, and delivering the required content. This organization is not intended to prescribe a linear means of delivery. Teachers are encouraged to reorder the learning outcomes and to modify, organize, and expand on the units to meet the needs of their students, to respond to local requirements, and to incorporate relevant recommended learning resources as applicable. (See the Learning Resources section later in this IRP for information about the recommended learning resources for Mathematics K to 7). In addition, teachers are encouraged to consider ways to adapt assessment strategies from one grade to another.

Considerations for Instruction and Assessment in Mathematics K to 7

It is highly recommended that parents and guardians be kept informed about all aspects of Mathematics K to 7. Suggested strategies for involving parents and guardians are found in the Introduction to this IRP.

Teachers are responsible for setting a positive classroom climate in which students feel comfortable learning about and discussing topics in Mathematics K to 7. Guidelines that may help educators establish a positive climate that is open to free inquiry and respectful of various points of view can be found in the section on Establishing a Positive Classroom Climate in the Introduction to this IRP.

Teachers may also wish to consider the following:

- Involve students in establishing guidelines for group discussion and presentations. Guidelines might include using appropriate listening and speaking skills, respecting students who are reluctant to share personal information in group settings, and agreeing to maintain confidentiality if sharing of personal information occurs.
- Promote critical thinking and open-mindedness, and refrain from taking sides on one point of view.
- Develop and discuss procedures associated with recording and using personal information that may be collected as part of students' work for the purposes of instruction and/or assessment (e.g., why the information is being collected, what the information will be used for, where the information will be kept; who can access it – students, administrators, parents; how safely it will be kept).
- Ensure students are aware that if they disclose personal information that indicates they are at risk for harm, then that information cannot be kept confidential. For more information, see the section on Confidentiality in the Introduction to this IRP.

Classroom Assessment and Evaluation

Teachers should consider using a variety of assessment instruments and techniques to assess students' abilities to meet the prescribed learning outcomes. Tools and techniques for assessment in Mathematics K to 7 can include

- teacher assessment tools such as observation checklists, rating scales, and scoring guides
- self-assessment tools such as checklists, rating scales, and scoring guides
- peer assessment tools such as checklists, rating scales, and scoring guides
- journals or learning logs
- video (to record and critique student demonstration or performance)
- written tests, oral tests (true/false, multiple choice, short answer)
- questionnaires, worksheets
- portfolios
- student-teacher conferences

Assessment in Mathematics K to 7 can also occur while students are engaged in, and based on the product of, activities such as

- class and group discussions
- interviews and questioning
- sharing strategies
- object manipulation
- models and constructions
- charts, graphs, diagrams
- games
- experiments
- artwork, songs/stories, dramas
- centres/stations
- demonstrations and presentations
- performance tasks
- projects

For more information about student assessment, refer to the section on Student Achievement, as well as to the Assessment Overview Tables in each grade of the Classroom Assessment Model.

Information and Communications Technology

The Mathematics K to 7 curriculum requires students to be able to use and analyse the most current information to make informed decisions on a range of topics. This information is often found on the Internet as well as in other information and communications technology resources. When organizing for instruction and assessment, teachers should consider how students will best be able to access the relevant technology, and ensure that students are aware of school district policies on safe and responsible Internet and computer use.

Contents of the Model

Assessment Overview Tables

The Assessment Overview Tables provide teachers with suggestions and guidelines for assessment of each grade of the curriculum. These tables identify the domains of learning and cognitive levels of the learning outcomes, along with a listing of suggested assessment activities and a suggested weight for grading for each curriculum organizer.

Overview

Each grade includes an overview of the assessment units:

- Learning at Previous Grades, indicating any relevant learning based on prescribed learning outcomes from earlier grades of the same subject area. It is assumed that students will have already acquired this learning; if they have not, additional introductory instruction may need to take place before undertaking the suggested assessment outlined in the unit. Note that some topics appear at multiple grade levels in order to emphasize their importance and to allow for reinforcement and developmental learning.
- Curriculum Correlation – a table that shows which curriculum organizers and suborganizers are addressed by each unit in this grade of the Classroom Assessment Model.

Prescribed Learning Outcomes

Each unit begins with a listing of the prescribed learning outcomes that are addressed by that unit. Collectively, the units address all the learning outcomes for that grade; some outcomes may appear in more than one unit. The units may not address all of the achievement indicators for each of the outcomes.

Suggested Assessment Activities

Assessment activities have been included for each set of prescribed learning outcomes and corresponding achievement indicators. Each assessment activity consists of two parts:

- Planning for Assessment – outlining the background information to explain the classroom context, opportunities for students to gain and practise learning, and suggestions for preparing the students for assessment
- Assessment Strategies – describing the assessment task, the method of gathering assessment information, and the assessment criteria as defined by the learning outcomes and achievement indicators.

A wide variety of activities have been included to address a variety of learning and teaching styles. The assessment activities describe a variety of tools and methods for gathering evidence of student performance. These assessment activities are also referenced in the Assessment Overview Tables, found at the beginning of each grade in the Model.

These strategies are suggestions only, designed to provide guidance for teachers in planning instruction and assessment to meet the prescribed learning outcomes.

Assessment Instruments

Sample assessment instruments have been included at the end of each grade where applicable, and are provided to help teachers determine the extent to which students are meeting the prescribed learning outcomes. These instruments contain criteria specifically keyed to one or more of the suggested assessment activities contained in the units. Ongoing formative assessment will be required throughout the year to guide instruction and provide evidence that students have met the breadth and depth of the prescribed learning outcomes.

Classroom Assessment Model

Grade 5

Grade 5: Assessment Overview Table

The purpose of this table is to provide teachers with suggestions and guidelines for formative and summative classroom-based assessment and grading of Grade 5 Mathematics.

Curriculum Organizers	Suggested Assessment Activities		Suggested Weight for Grading	Number of Outcomes	Number of Outcomes by Domain*		
					K	U&A	HMP
Number	• observations • interviews • journals/logs • games • models • questioning	• portfolios • presentations • projects • self-assessment • peer assessment	45-55%	11	4	5	2
Patterns and Relations	• observations • interviews • portfolios • journals/logs • questioning	• demonstrations • projects • peer assessment • self-assessment	5-15%	2	0	1	1
Shape and Space	• observations • interviews • journals/logs • games • questioning	• portfolios • demonstrations • projects • self-assessment • peer assessment	25-35%	8	3	5	0
Statistics and Probability	• observations • interviews • journals/logs • games	• questioning • self-assessment • peer assessment • portfolios	5-15%	4	0	3	1
		Totals	**100%**	**25**	**7**	**14**	**4**

* The following abbreviations are used to represent the three cognitive levels within the cognitive domain: K = Knowledge; U&A = Understanding and Application; HMP = Higher Mental Processes.

GRADE 5

OVERVIEW

Learning at Previous Grades

- whole numbers to 10 000
- addition with answers to 10 000 and corresponding subtraction
- multiplication by 0 and 1 and division by 1
- mental mathematics strategies for multiplication facts to 9 × 9 and corresponding division facts
- multiplication of 2- or 3- digit by 1-digit
- division of 2-digit divisor by 1-digit dividend
- fractions less than or equal to one
- decimal representation to hundredths and relation to fractions
- addition and subtraction of decimals to hundredths
- pattern relationships in tables and charts
- symbols to represent unknowns
- one-step equations
- digital, analog and 24 hour clocks and calendar dates
- area of regular and irregular 2-D shapes
- rectangular and triangular prisms
- line symmetry
- many-to-one correspondence including bar graphs and pictographs

Curriculum Correlation

The following table shows which curriculum organizers and suborganizers are addressed by each unit in this grade of the Classroom Assessment Model. Note that some curriculum organizers/suborganizers are addressed in more than one unit.

	Party Planning	Amusement Park Games	Volume/ Capacity	Quadrilaterals	2-D, 3-D	Number Balance Scale	Math in Art	Find the Errors
Number	X	X						X
Patterns and Relations *Patterns*	X							
Variables and Equations						X		
Space and Shape *Measurement*	X		X					
3-D Objects and 2-D Shapes		X		X	X		X	
Transformations		X						
Statistics and Probability *Data Analysis*	X							
Chance and Uncertainty		X						

Party Planning

Prescribed Learning Outcomes

It is expected that students will:

A1 represent and describe whole numbers to 1 000 000 [C, CN, V, T]

A2 use estimation strategies including
- front-end rounding
- compensation
- compatible numbers

in problem-solving contexts [C, CN, ME, PS, R, V]

A3 apply mental mathematics strategies and number properties, such as
- skip counting from a known fact
- using doubling or halving
- using patterns in the 9s facts
- using repeated doubling or halving

to determine answers for basic multiplication facts to 81 and related division facts [C, CN, ME, R, V]

A4 apply mental mathematics strategies for multiplication, such as
- annexing then adding zero
- halving and doubling
- using the distributive property [C, ME, R]

A5 demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems [C, CN, PS, V]

A6 Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems [C, CN, PS]

B1 determine the pattern rule to make predictions about subsequent elements [C, CN, PS, R, V]

C4 demonstrate an understanding of capacity by
- describing the relationship between mL and L
- selecting and justifying referents for mL or L units
- estimating capacity by using referents for mL or L
- measuring and recording capacity (mL or L) [C, CN, ME, PS, R, V]

D1 differentiate between first-hand and second-hand data [C, R, T, V]

D2 construct and interpret double bar graphs to draw conclusions [C, PS, R, T, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Have students work in groups to create and submit a proposal for a class party. The proposal is to include - menu choices - entertainment choices - a budget - a justification for cost per students and total cost of the party As a class, select the best proposal and justify their decisions. In many Aboriginal communities feasts are held. Students may be able to assist in planning for the feast.	• Prior to the start of the project, discuss criteria for assessing the presentation. Ask students to consider how they will know what is the best proposal? What does "best" mean? Suggested criteria could include - data are accurately represented by graphs, charts and tables - conclusions drawn from the data are logical - arguments are supported by the data For more detailed suggestions as to possible criteria, see the **Party Planning** rubric provided at the end of this grade. Have students include their final proposals in their math portfolios, adding comments such as the following: - This is an example of ____. - I want you to notice that ____. - I think I did a good job on it because ____. - Something new I learned by doing this activity was ____.
• Have students formulate a question and survey their classmates in order to determine menu and entertainment choices. Proposals should include evidence that this menu would appeal to other students in the class. (e.g., a survey of student preferences would provide evidence that potato chips are preferred over popcorn). Have students create graphs, tables or charts to represent their findings. Explain that they will use this evidence to support their arguments.	• Assess students' ability to gather data, using interview questions such as the following: - Why did you choose this question? - Who are you going to survey and why did you select these people? - What are you noticing? - When interviewing students look for evidence that the questions they formulated can be answered by surveying the class and will lead to information about the class' food preference. Look for evidence to verify that - graphs are legible - graphs accurately represent the data - intervals are clearly identifiable - labels are visible

Planning for Assessment	Assessment Strategies
• Prior to calculating costs, verify that students are able to identify equivalent capacities (e.g., have students explain how many 250 ml sized individual servings are contained in a 2L bottle).	• Observe students' ability to demonstrate an understanding of capacity by having them - demonstrate that 1 000 mL is equivalent to 1 L by filling a 1 L container using a combination of smaller containers, or by describing the relationship between mL and L - identify an appropriate referent when estimating capacity of a container (e.g., I think there are about 6 pop cans in a 2 L bottle)
• Have students estimate the quantity and cost of various food and beverage items. Proposals will outline unit costs of food items as well as the total cost for quantities required.	• To assess students' abilities to calculate costs: - observe students' use of personal procedures, base 10 blocks, manipulatives, or pictorial representations when applying multiplication and division in determining cost of menu items - observe what strategies students use when applying mental math strategies for multiplication (e.g., annexing zero; halving or doubling or using the distributive property) to determine food costs - verify that students can use division strategies when calculating unit costs (students may demonstrate this using manipulatives, an algorithm or verbal explanation) - verify that students can describe the meaning of each digit in a given numeral when talking about large quantities - verify students' abilities to round to the nearest dollar and to explain why their estimate is reasonable - verify that students use estimation strategies (e.g., front-end rounding, compensations, compatible numbers) when calculating costs - verify that students can predict patterns when projecting unit costs to determine costs for whole class

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Provide opportunities for students to present their proposals to a committee of peers (the class) and justify their arguments using results from the data they collected. As well, students can submit a written report that includes charts, tables, graphs, discussion, rationale, etc.	• When reviewing written reports or listening to presentations, look for evidence - that students are able to formulate a question that can be answered by gathering first-hand data and provide justification for using first-hand data as opposed to second-hand data (e.g., second-hand data will not show us what our class wants) - of students' ability to create a double bar graph, interpret the data and draw conclusions - of students' ability to represent, extend and explain a pattern (e.g., unit cost, cost for 2 units, 3 units) - of students' ability to justify their argument using the data collected Use questions or discussion prompts such as the following to interview students after their oral presentations: - Explain why you selected these menu items. (What questions did you ask in your survey?) - Explain how you gathered evidence for your suggestions and what conclusions you drew from your data. - Which food items were good bargains? Why do you think they are good value for your money? - Explain any decisions you had to make about choosing one food item over another when calculating food costs. - Explain how you used multiplication or division strategies (including mental math strategies) to help figure out food costs.
• After oral presentations of all the proposals, each student will then select the best, least expensive proposal and explain why he or she thinks this is the "best" (most desirable, most economic). If necessary, prompt for an explanation: After listening to other proposals have you changed your mind about which is the most desirable and least expensive proposal? If so, what convinced you of this?	• To assess student selections, look for evidence of - students' abilities to justify their choices using criteria specified (most desirable/least expensive) - students' references to the data, charts, tables, and graphs to explain or justify their choices

Amusement Park Games

Prescribed Learning Outcomes

It is expected that students will:

A7 demonstrate an understanding of fractions by using concrete and pictorial representations to
- create sets of equivalent fractions
- compare fractions with like and unlike denominators [C, CN, PS, R, V]

A8 describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically [C, CN, R, V]

A9 relate decimals to fractions (to thousandths) [CN, R, V]

A10 compare and order decimals (to thousandths) by using
- benchmarks
- place value
- equivalent decimals [CN, R, V]

C1 design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions [C, CN, PS, R, V]

C2 demonstrate an understanding of measuring length (mm) by
- selecting and justifying referents for the unit mm
- modelling and describing the relationship between mm and cm units, and between mm and m units [C, CN, ME, PS, R, V]

C7 perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image [C, CN, T, V]

C8 identify a single transformation, including a translation, rotation, and reflection of 2-D shapes [C, T, V]

D3 describe the likelihood of a single outcome occurring using words such as
- impossible
- possible
- certain [C, CN, PS, R]

D4 compare the likelihood of two possible outcomes occurring using words such as
- less likely
- equally likely
- more likely [C, CN, PS, R]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• In this unit, students will design part of an amusement park by creating 3 games of chance, a booth from which the games are played, and decorations for the area surrounding the booth.	• Have students include their final products (games, designs for booth and tileway) in their math portfolios, adding comments describing their work and what it represents.

Planning for Assessment	Assessment Strategies
• Explain to students that they will be creating games of chance for an amusement park. They will design 3 different games of chance: one that favours the vendor, another that favours the customer, and a third that offers an equally likely chance for either vendor or customer to win.	• Work with students to establish criteria for assessing this task. Suggested criteria to consider include the following: - games created reflect conditions specified by the project (e.g., vendor has a more than likely chance to win, customer has a more likely chance to win, and both customer and vendor have an equally likely chance to win) - student is able to use the language of probability (likely, equally likely, etc.) to describe his or her game While students are working on the project, circulate and initiate conversation, using prompts such as the following: - explain how this game favours the vendor - explain how this game favours the customer - explain how this game gives both the customer and the vendor an equal opportunity of winning While student are working on the project, circulate and observe, looking for evidence of how students - use the language of probability to describe their games - explain outcomes of chance events - compare the likelihood of 2 possible outcomes demonstrate how different outcomes can occur when the games are played repeatedly Have students reflect on their learning in their math journals. Encourage them to consider ways that they were good mathematicians (e.g., persevered, asked good questions, took risks and tried new ideas, talked about math ideas with a buddy; used mathematical language such as *likely, unlikely, equally likely*). When assessing math journals, look for evidence that students are able to reflect on their learning and identify ideas that are confusing, difficult, or new learning.

Planning for Assessment	Assessment Strategies
• Have students design a booth for their game. The booth should fit onto a platform that has an area which can comfortably fit 3 players and take up the smallest area possible on the fairgrounds. Have students specify the dimensions of the booth and platform using appropriate units of measurement.	• Discuss criteria for assessment. Criteria will ideally include the extent to which students identify a referent and explain the reason why they chose this referent for the measurements they used. Circulate while students are working on the task and look for evidence that students are - designing a booth to specs - using appropriate units of measurement Circulate as students work on task and use the following prompts to invite conversation, discussions: - Explain why you chose to design your model using those dimensions. - Explain why you chose to measure your booth using that unit of measurement. - If you had to measure your booth using only mm, what would the dimensions be? - If you had to measure your booth using only cm, what would the dimensions be? - Give a referent for the booth using millimetres, centimetres, or metres (e.g., this booth is as tall as …, as wide as…). Listen for students' ability to - justify their choice of a referent - explain how the referent was used to make the estimate - describe the relationship between millimetres, centimetres, and metres Have students reflect on their learning in their math journals. When assessing math journals, look for evidence that students are able to reflect on their learning and identify ideas that are confusing, difficult, or new learning.
• Have students design a tiled pathway around the perimeter of the amusement park using a tile pattern that is transformed by reflection, rotation or translation. Offer students the choice of drawing the tile by hand or using technology. Ask students to - describe the transformed pattern using the vocabulary translation, rotation, and reflection - describe what happens to a shape when it is moved (translated, rotated, reflected)	• When reviewing students' work, look for evidence that - the shapes are indeed transformed - the reflections, rotations, and translations preserve the integrity of the shape and are correct transformations As students describe their designs, listen for appropriate use of vocabulary, including *rotation*, *translation*, and *reflection*. For example: - students should recognize that when you slide an image it remains the same size and shape - students should be able to identify the line of reflection between a 2-D shape and its reflected image - given a rotation, students identify the fraction of the turn (¼, ½, ¾)

PLANNING FOR ASSESSMENT

- Have students work in pairs to create a gameboard and play a racing game. The gameboard is to contain seven number lines (lanes) that are variously divided into halves, thirds, quarters, fifths, sixths, eighths, and tenths (see the Gameboard Samples sheet included at the end of this grade). Have students label the intervals using proper fractions.

  Once the boards are complete, give each pair of students a set of cards such as the following to play the game:

$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{2}{10}$
$\frac{1}{5}$	$\frac{2}{5}$	$\frac{4}{10}$	$\frac{6}{10}$	$\frac{3}{5}$
$\frac{4}{6}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{6}{8}$	$\frac{10}{10}$

  The game is a race is for a player to be the first to get all of the markers to the finish line. On their turn, students will move along the race lanes according to distance indicated on a chosen card. Encourage them to check their opponent's moves and to challenge those they feel are inappropriate (even when these moves are in their favour). Encourage a spirit of collaboration, so that students help each other make the best moves possible.

  In another version of this game, students draw a second gameboard using decimals and fractions with denominators of 10, 100, and 1000 (again, see the Gameboard Samples sheet included at the end of this grade). They play the game as described previously, using cards such as the following:

$\frac{200}{100}$	0.4	0.50	0.800	$\frac{70}{100}$
$\frac{3}{10}$	$\frac{60}{100}$	0.20	$\frac{30}{100}$	$\frac{500}{100}$
0.10	0.2	$\frac{5}{10}$	0.400	$\frac{600}{1000}$
0.100	$\frac{100}{100}$	$\frac{4}{10}$	0.3	0.90

ASSESSMENT STRATEGIES

- As students are creating the board or playing the game, observe to look for evidence that they
  - are dividing and labelling the intervals accurately
  - understand equivalent fractions and are able to compare fractions with like and unlike denominators
  - are able to relate proper fractions to their decimal equivalents (e.g., 5 = $\frac{5}{10}$, 0.50 = $\frac{50}{100}$, 0.500 = $\frac{50}{1000}$)
  - are having conversations with their peers involving explanations and justifications of moves they are making
  - are challenging each other when they perceive a wrong move

  To enrich your assessment, circulate while students are playing the games, and have them
  - explain how they know they are moving the correct distance as specified on the card (e.g., Why are you moving $\frac{250}{1000}$ of the way when the card reads 0.25 or 0.250?)
  - given a decimal, show the equivalent fraction by moving their marker on the gameboard and explaining their action
  - given a tenth, express an equivalent in hundredths and thousandths
  - explain the value of each digit in a given decimal

Volume/Capacity

Prescribed Learning Outcomes

It is expected that students will:

C3 demonstrate an understanding of volume by
- selecting and justifying referents for cm^3 or m^3 units
- estimating volume by using referents for cm^3 or m^3
- measuring and recording volume (cm^3 or m^3)
- constructing rectangular prisms for a given volume [C, CN, ME, PS, R, V]

C4 demonstrate an understanding of capacity by
- describing the relationship between mL and L
- selecting and justifying referents for mL or L units
- estimating capacity by using referents for mL or L
- measuring and recording capacity (mL or L) [C, CN, ME, PS, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Give students containers of different sizes and shapes. Have them order these from largest capacity/volume to smallest capacity/volume. Have them provide examples from real life contexts that represent these quantities (e.g., 250 ml contains a bit less than the average pop can; the smallest Dienes block has a volume of 1 cm^3).	• As students are ordering the containers, ask them to provide a referent for each of the different containers. (e.g., 250 ml contains a bit less than the average pop can. The smallest Dienes block has a volume of 1 cm^3) Have them explain why they chose this referent. Ask them to give the approximate capacity/volume in L and then give an equivalent in mL. Have students use their journals to illustrate and explain how they know their ordering is correct. An exemplary journal entry is one that includes an illustration and detailed explanation justifying the ordering using referents. Students should be able to explain how they know they are right. A minimally meet expectations journal entry includes an illustration and some explanation, but no/little justification for the ordering or use of referents.
• Have students predict how many small containers (50 ml, 100 ml, 125 ml, 250 ml, 500 ml) are needed to fill a one litre container. Ask them to justify their predictions. Have students think of as many possible combinations to fill a one litre containers, using the following containers: 50 ml, 100 ml, 125 ml, 250 ml, 500 ml. Have them write or draw these in their math journals.	• When observing students performing this task look for evidence that predictions are reasonable, and that students are able to identify appropriate combinations. When reviewing math journal entries look for evidence that students are able to - give a range of combinations - identify appropriate combinations

Quadrilaterals

Prescribed Learning Outcomes

It is expected that students will:

C6 identify and sort quadrilaterals, including
- rectangles
- squares
- trapezoids
- parallelograms
- rhombuses

according to their attributes [C, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Have students work in partners to play a classifying game: - partner A classifies a given set of quadrilaterals - partner B attempts to guess the classification rule, selects another quadrilateral that would fit into that set, and then explains why that piece belongs to the set - on the next turn, partners switch roles (partner B becomes the rule maker and partner A guesses) Have student write instructions for how to play the game and include examples of possible scenarios.	• While students are playing the game, circulate and look for evidence that they are able to correctly sort and identify quadrilaterals according to their attributes. When reviewing students' game instructions, look for evidence that they are able to explain the game and attributes of quadrilaterals as they describe their examples.

2-D, 3-D

Prescribed Learning Outcomes

It is expected that students will:

C5 describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are
- –parallel
- –intersecting
- –perpendicular
- –vertical
- –horizontal [C, CN, R, T, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Have students create a glossary (in comic strip/ dictionary/short story format) to define these words: parallel, intersecting, perpendicular, vertical, and horizontal.	• Work with students to establish criteria for a good glossary. Some suggested criteria could include - defining the terms clearly using mathematical language - giving examples from the real world - including pertinent and accurate illustrations Have students include their glossary in their math portfolios.

Planning for Assessment	Assessment Strategies
• Have students pair up to play a shape-describing game. Partner A draws a 2-D shape that Partner B cannot see and describes it to Partner B using appropriate vocabulary (parallel, intersecting, perpendicular, horizontal, vertical lines). Partner B responds by drawing a shape with these criteria, and may ask questions as he or she attempts to draw. When Partner B is finished, Partner B will verify the shape with Partner A. Partners switch roles.	• As students play the shape-describing game, notice whether or not a student is able to use the vocabulary to describe a shape's attributes. Have students conduct self-assessment in pairs. Partners reflect on how they did on the task, with reference to questions such as the following: - How well did you work together as a team? - How did you deal with disagreements? - What did you enjoy about this collaborative activity? - What were the challenges faced? - What would you do differently next time? - How can working with a partner help you with your learning? Have students comment on the activity and on the self-assessment process in their math journals. Suggested sentence prompts for math journals include the following: - Today, I ____. - Something surprising was ____. - I noticed that ____. - Something challenging was ____. - Something my partner and I did well ____. - Next time I would ____. • Have students share their math journal entry with the class. When reviewing student journal entries notice the extent to which students - used the vocabulary to explain the task. - are able to reflect on their learning, identify new ideas, areas of confusion or difficulty, surprises, misconceptions in their prior knowledge, etc. - are able to describe how well they are working with partners

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Have students pair up. Partner A chooses a geometric object from a set of geometric solids. He or she then describes its attributes using vocabulary such as parallel faces, perpendicular sides, horizontal, vertical, etc without naming the object to Partner B, who cannot see the object. Partner B tries to identify the object. Partners switch roles.	• As students play the game, notice whether or not a student is able to use the vocabulary to describe a shape's attributes Ask partners to reflect on how they did on the task, with reference to questions such as the following: - How well did they work together as a team? - How did they deal with disagreements? - What did they enjoy about this collaborative activity? - What were the challenges faced? - What would they do differently next time? - How can working with a partner help you with your learning? Have students use their math journals to record their reflections. When reviewing student journal entries, notice - how students used the vocabulary to explain the task. - to what extent students are able to reflect on their learning, identify new ideas, areas of confusion or difficulty, surprises, misconceptions in their prior knowledge, etc. - to what extent students are able to describe how well they are working with partners

Number Balance Scale

Prescribed Learning Outcomes

It is expected that students will:

B2 solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions [C, CN, PS, R]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Have students draw a balance scale. In pairs, have Partner A think of a number and write the number on one side of the scale. On the other side of the scale, Partner B creates a single variable equation to match that number. For ex. $12 = n + 4$. Have students take turns and switch roles. Have students create examples for each of the operations of multiplication, addition, subtraction and division.	• Circulate and observe, looking for evidence of students' ability to match numbers with single variable equations. Have students include their working sheet in their math portfolios.
• Have students, given an equation, describe a context that represents that equation (e.g., given $2n = 10$, students might write "John is 10 and he is twice as old as Sherry. How old is Sherry?"). Given a word problem, have students write an equation.	• Circulate and observe students' ability to - create an appropriate context for a given equation and vice versa - represent a situation using an equation Have students include this work in their math journals. Look for evidence that students are able to create a word problem for a given equation and explain their answer to their particular problem.

Math in Art

Prescribed Learning Outcomes

It is expected that students will:

C5 describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are
- parallel
- intersecting
- perpendicular
- vertical
- horizontal [C, CN, R, T, V]

C6 identify and sort quadrilaterals, including
- rectangles
- squares
- trapezoids
- parallelograms
- rhombuses

according to their attributes [C, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Show students prints of paintings by Robert Davidson, Emily Carr, Pablo Picasso, Georges Braques, or other artists. Conduct a whole class discussion where you model how to describe these paintings using vocabulary such as parallel, perpendicular, intersecting, vertical, horizontal, trapezoids, parallelograms, rectangles, squares, and rhombuses. Then have students describe what they see. Students may then select a print and, using the vocabulary, describe in writing what they see.	• When listening to student discussions and/or when reviewing their writing or drawings look for evidence that students are able to - use the vocabulary correctly - identify quadrilaterals - can explain the difference between different quadrilaterals
• Using specified criteria, have students draw a scene such as landscape. Sample instructions to students: Draw a landscape that includes parallel, perpendicular, intersecting, vertical, and horizontal lines, trapezoids, parallelograms, rectangles, squares, and rhombuses.	• Have students include their art work in their math portfolios along with annotations describing their work and what they have learned.

Find The Errors

Prescribed Learning Outcomes

It is expected that students will:

A1 represent and describe whole numbers to 1 000 000 [C, CN, V, T]
A5 demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems [C, CN, PS, V]
A6 Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems [C, CN, PS]
A11 demonstrate an understanding of addition and subtraction of decimals (limited to thousandths) [C, CN, PS, R, V]

PLANNING FOR ASSESSMENT

- Have students correct a fictitious student's worksheet, supplying written explanations about why a given answer is incorrect. The fictitious worksheet might contain entries such as the following:

  (a)
  $$\begin{array}{r} 56 \\ \times 15 \\ \hline 280 \\ +56 \\ \hline \mathbf{336} \end{array}$$

  (b)
  $$\begin{array}{r} 1.560 \\ 1.23 \\ .18 \\ \hline \mathbf{17.01} \end{array}$$

  (c) $250 \div 5 = \mathbf{50}$

  $$\begin{array}{r} 58.4 \\ -1.45 \\ \hline \mathbf{56.95} \end{array}$$

  (d) **4 517 063** = (4 × 1 000 000) + (5 × 100 000) + (1 × 1000) + (7 × 1000) + (6 × 10) + (3 × 1)

  (e) 60 000 + 7000 + 500 + 2 = **67 502**

- Have students make up their own version using the sample provided as a template. Students then exchange sheets with a partner and repeat the exercise.

ASSESSMENT STRATEGIES

- When reviewing a student's work, look for evidence of students' understanding of and abilities to
  - multiplication of 2 digit by 2 digit problems
  - division (3-digit by 1-digit) with and without remainders
  - explain why keeping track of place value positions is important when adding and subtracting decimals.
  - solve a problem that involves addition and subtraction of decimals, limited to thousandths.
  - represent and describe whole numbers to 1 000 000 by expressing a given numeral in expanded notation; by describing the meaning of each digit in a given numeral, by writing a given numeral represented by expanded notation.

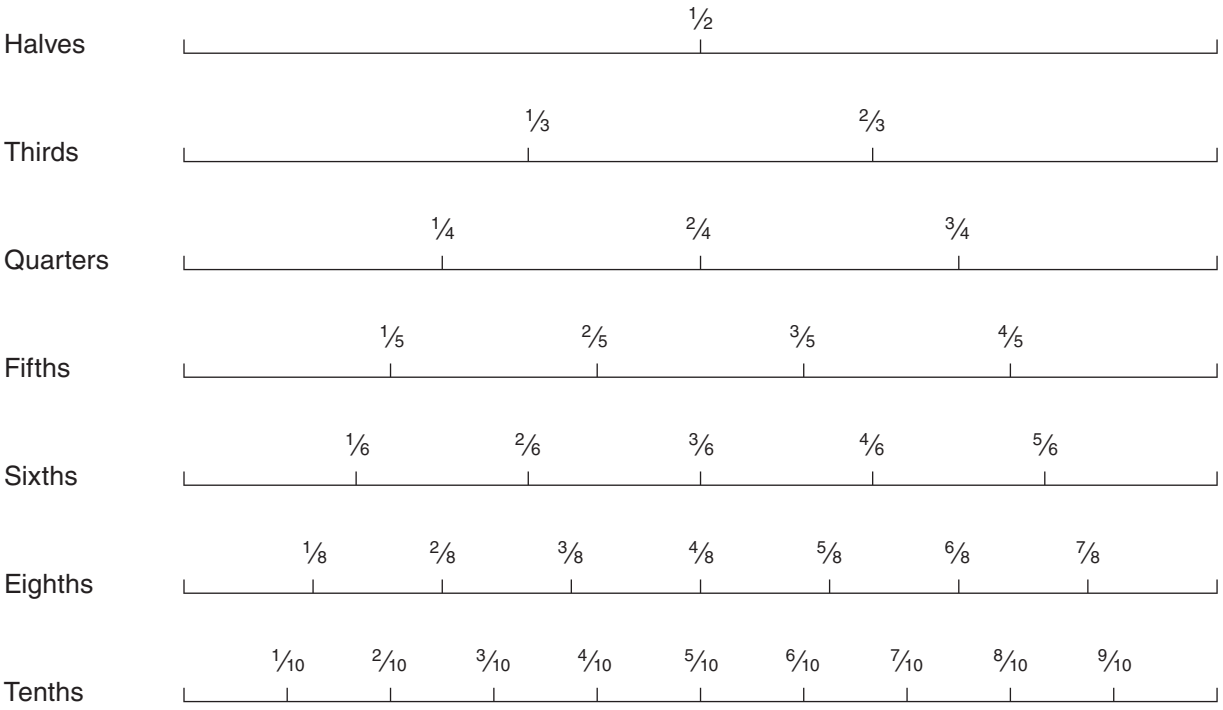
  Have students include this working sheet in their math portfolios.

Party Planning

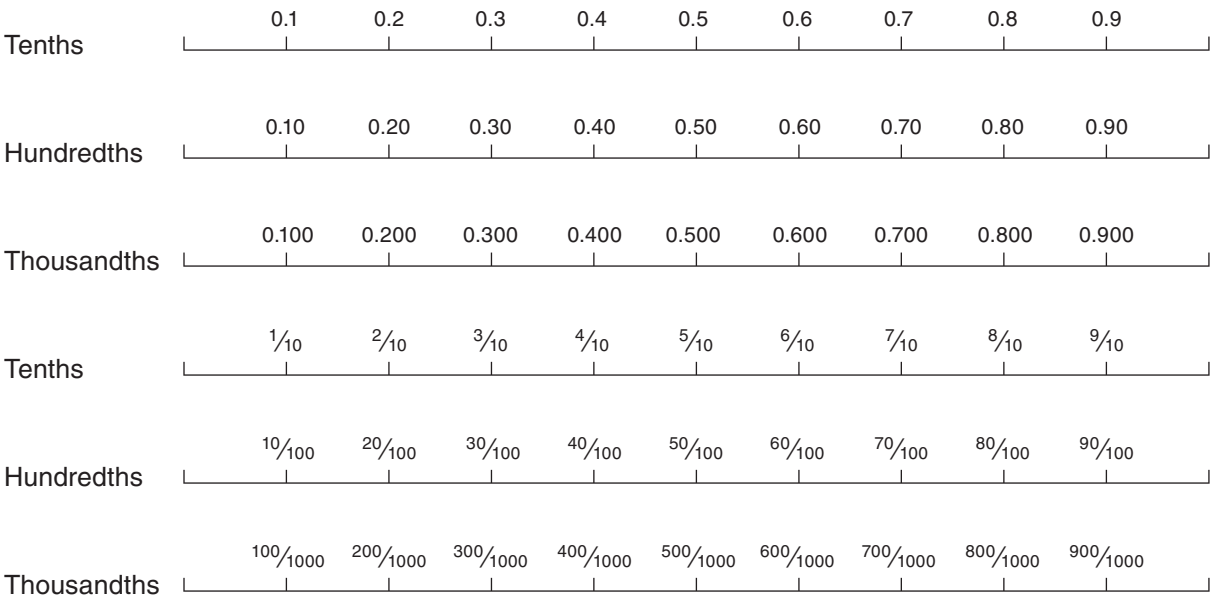
Does not yet meet expectations	Minimally meets expectations	Fully meets expectations	Exceeds expectations
Understanding concepts of data and statistics			
• unable to formulate questions • did not survey students to determine menu and entertainment choices • demonstrates an inability to draw conclusions • conclusions are not supported by the data	• able to formulate some questions • uses some of the survey data for menu and entertainment choices • demonstrates limited ability to draw conclusions • conclusions are partially inconsistent with data	• able to formulate appropriate questions and is able to identify student preferred menus and entertainment • demonstrates the ability to draw conclusions • conclusions are consistent with and fully supported by the data	• able to formulate the most appropriate questions and incorporate student preferences in menus and entertainment choices • demonstrates an ability to draw conclusions, extend conclusions, and make logical predictions based on the data
Applications of mathematical procedures			
• is unable to determine cost per student • is unable to justify costs	• is able to calculate some costs per students • tries to justify cost per student	• calculates cost per students • is able to justify cost per student	• accurately calculates cost per students • justifies cost per student
Communication (Presentation – argument, graphs)			
• describes and explains few justifications in the proposal; advances weak argument • axis, titles, intervals, and data not accurately drawn	• describes and explains some justifications in the proposal; provides very little argument or justification • axis, titles, intervals, and data drawn with some errors	• presents most of their justifications in proposal; justifies argument • axis, titles , intervals, and data accurately drawn with a few errors	• accurately presents all justifications in the proposal; well supported argument • axis, titles, intervals, and data accurately drawn; no errors and/or omissions

GAMEBOARD SAMPLES

SAMPLE 1



SAMPLE 2



LEARNING RESOURCES

This section contains general information on learning resources, and provides a link to the titles, descriptions, and ordering information for the recommended learning resources in the Mathematics K to 7 Grade Collections.

What Are Recommended Learning Resources?

Recommended learning resources are resources that have undergone a provincial evaluation process using teacher evaluators and have Minister's Order granting them provincial recommended status. These resources may include print, video, software and CD-ROMs, games and manipulatives, and other multimedia formats. They are generally materials suitable for student use, but may also include information aimed primarily at teachers.

Information about the recommended resources is organized in the format of a Grade Collection. A Grade Collection can be regarded as a "starter set" of basic resources to deliver the curriculum. In many cases, the Grade Collection provides a choice of more than one resource to support curriculum organizers, enabling teachers to select resources that best suit different teaching and learning styles. Teachers may also wish to supplement Grade Collection resources with locally approved materials.

How Can Teachers Choose Learning Resources to Meet Their Classroom Needs?

Teachers must use either

- provincially recommended resources
  OR
- resources that have been evaluated through a local, board-approved process

Prior to selecting and purchasing new learning resources, an inventory of resources that are already available should be established through consultation with the school and district resource centres. The ministry also works with school districts to negotiate cost-effective access to various learning resources.

What Are the Criteria Used to Evaluate Learning Resources?

The Ministry of Education facilitates the evaluation of learning resources that support BC curricula, and that will be used by teachers and/or students for instructional and assessment purposes. Evaluation criteria focus on content, instructional design, technical considerations, and social considerations.

> Additional information concerning the review and selection of learning resources is available from the ministry publication, Evaluating, Selecting and Managing Learning Resources: A Guide (Revised 2002)
> www.bced.gov.bc.ca/irp/resdocs/esm_guide.pdf

What Funding is Available for Purchasing Learning Resources?

As part of the selection process, teachers should be aware of school and district funding policies and procedures to determine how much money is available for their needs. Funding for various purposes, including the purchase of learning resources, is provided to school districts. Learning resource selection should be viewed as an ongoing process that requires a determination of needs, as well as long-term planning to co-ordinate individual goals and local priorities.

What Kinds of Resources Are Found in a Grade Collection?

The Grade Collection charts list the recommended learning resources by media format, showing links to the curriculum organizers. Each chart is followed by an annotated bibliography. Teachers should check with suppliers for complete and up-to-date ordering information. Most suppliers maintain web sites that are easy to access.

Mathematics K to 7 Grade Collections

The Grade Collections for Mathematics K to 7 include newly recommended learning resources as well as relevant resources previously recommended for prior versions of the Mathematics K to 7 curriculum. The ministry updates the Grade Collections on a regular basis as new resources are developed and evaluated.

> Please check the following ministry web site for the most current list of recommended learning resources in the Grade Collections for each IRP: www.bced.gov.bc.ca/irp_resources/lr/resource/gradcoll.htm

GLOSSARY

The British Columbia Ministry of Education recognizes the limitation of a glossary available only in print format. An online glossary has been developed by Alberta Education to support the implementation of their revised Kindergarten to Grade 9 Program of Studies. This glossary is based on the WNCP CCF for K-9 Mathematics and therefore also supports the British Columbia Mathematics K to 7 IRP.

This online glossary provides additional supports for teachers indlucing definitions, diagrams, pictures, and interactive applets that cannot be provided through a conventional print glossary. As a result, the Ministry of Education encourages educational stakeholders to access the glossary through a link which is provided on the British Columbia Ministry of Education website.

To access the glossary, follow the links for curriculum support material from the mathematics IRP main page at www.bced.gov.bc.ca/irp/irp_math.htm