



This document represents an updating of the 2000 IRP. This updating has been undertaken for the purpose of

- clarifying the Prescribed Learning Outcomes
- introducing Suggested Achievement Indicators
- addressing content overload

Resources previously recommended for the 2000 version of the curriculum, where still valid, continue to support this updated IRP. (See the Learning Resources section in this IRP for additional information.)

PRINCIPLES OF MATHEMATICS 10 TO 12

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ACKNOWLEDGMENTS

Acknowledgments.....	III
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PREFACE

Preface	V
---------------	---

INTRODUCTION TO PRINCIPLES OF MATHEMATICS 10 TO 12

Rationale.....	3
Requirements and Graduation Credits.....	4
Graduation Program Examination.....	4
Goals for Principles of Mathematics 10 to 12.....	4
Curriculum Organizers.....	5
Mathematical Processes	6
Principles of Mathematics 10 to 12: At a Glance	9
Suggested Timeframe.....	10

CONSIDERATIONS FOR PROGRAM DELIVERY

Alternative Delivery Policy.....	13
Instructional Focus	13
Fostering the Development of Positive Attitudes	13
Applying Mathematics	14
Involving Parents and Guardians.....	14
Confidentiality.....	14
Inclusion, Equity, and Accessibility for All Learners	14
Working with the School and Community	15
Working with Aboriginal Community	15
Information and Communications Technology	15
Copyright and Responsibility	16

PRESCRIBED LEARNING OUTCOMES

Introduction.....	19
Wording of Prescribed Learning Outcomes.....	19
Domains of Learning.....	19
Principles of Mathematics 10 Prescribed Learning Outcomes.....	20
Principles of Mathematics 11 Prescribed Learning Outcomes.....	22
Principles of Mathematics 12 Prescribed Learning Outcomes.....	24

STUDENT ACHIEVEMENT

Introduction.....	29
Classroom Assessment and Evaluation.....	29
Key Elements.....	32
Achievement Indicators.....	32
Principles of Mathematics 10	
Key Elements	34
Suggested Achievement Indicators	35

TABLE OF CONTENTS

Principles of Mathematics 11	
Key Elements	46
Suggested Achievement Indicators	47
Principles of Mathematics 12	
Key Elements	56
Suggested Achievement Indicators	57

LEARNING RESOURCES

Introduction	67
Principles of Mathematics 10 to 12 Grade Collections	67

GLOSSARY

Glossary	71
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PRINCIPLES OF MATHEMATICS 10 TO 12 IRP REFINEMENT TEAM

Robert Sidley, School District No. 38 (Richmond)

Chris Van Bergyk, School District No. 23 (Central Okanagan)

Hold Fast Consultants Inc.

Preliminary Review

Preliminary Review

IRP writing and editing

This Integrated Resource Package (IRP) provides basic information teachers will require in order to implement Principles of Mathematics 10 to 12. This document supersedes the Principles of Mathematics 10 to 12 portion of the *Mathematics 10 to 12 Integrated Resource Package* (2000).

The information contained in this document is also available on the Internet at www.bced.gov.bc.ca/irp/irp.htm

The following paragraphs provide brief descriptions of the components of the IRP.

INTRODUCTION

The Introduction provides general information about Principles of Mathematics 10 to 12, including special features and requirements.

Included in this section are

- a rationale for teaching Principles of Mathematics 10 to 12 in BC schools
- the curriculum goals
- descriptions of the curriculum organizers – groupings for prescribed learning outcomes that share a common focus
- a suggested timeframe for each curriculum organizer
- a graphic overview of the curriculum content

CONSIDERATIONS FOR PROGRAM DELIVERY

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners.

PRESCRIBED LEARNING OUTCOMES

This section contains the *prescribed learning outcomes*, the legally required content standards for the provincial education system. The learning outcomes define the required knowledge, skills, and attitudes for each subject. They are statements of what students are expected to know and be able to do by the end of the course.

STUDENT ACHIEVEMENT

This section of the IRP contains information about classroom assessment and measuring student achievement, including sets of specific achievement indicators for each prescribed learning outcome. Achievement indicators are statements that describe what students should be able to do in order to demonstrate that they fully meet the expectations set out by the prescribed learning outcomes. Achievement indicators are not mandatory; they are provided to assist in the assessment of how well students achieve the prescribed learning outcomes.

Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of the prescribed learning outcomes.

LEARNING RESOURCES

This section contains general information on learning resources, and provides a link to titles, descriptions, and ordering information for the recommended learning resources in the Principles of Mathematics 10 to 12 Grade Collections.

GLOSSARY

The glossary defines selected terms used in this Integrated Resource Package.



INTRODUCTION

Principles of Mathematics 10 to 12

This Integrated Resource Package (IRP) sets out the provincially prescribed curriculum for Principles of Mathematics 10 to 12. The development of this IRP has been guided by the principles of learning:

- Learning requires the active participation of the student.
- People learn in a variety of ways and at different rates.
- Learning is both an individual and a group process.

In addition to these three principles, this document recognizes that British Columbia's schools include young people of varied backgrounds, interests, abilities, and needs. Wherever appropriate for this curriculum, ways to meet these needs and to ensure equity and access for all learners have been integrated as much as possible into the learning outcomes and achievement indicators.

The prescribed learning outcomes in the Principles of Mathematics 10 to 12 IRP are based on *The Common Curriculum Framework for K to 12 Mathematics* (Western and Northern Canadian Protocol for Collaboration in Basic Education, 1996). The Achievement Indicators were developed, in part, using the following documents:

- *Mathematics 10 to 12 Integrated Resource Package* (British Columbia Ministry of Education, 2000);
- *Principles of Mathematics 10 Provincial Examination Specifications* (British Columbia Ministry of Education, 2004);
- *Principles of Mathematics 12 Provincial Examination Specifications* (British Columbia Ministry of Education, 2004);
- *Outcomes with Assessment Standards for Pure Mathematics 10* (Alberta Learning, 2002);
- *Outcomes with Assessment Standards for Pure Mathematics 20* (Alberta Learning, 2002); and,
- *Pure Mathematics 30: Information Bulletin* (Alberta Learning, 2004)

This document represents an updating of the 2000 IRP. This updating has been undertaken for the purpose of

- clarifying the prescribed learning outcomes
- introducing suggested achievement indicators
- addressing content overload

Resources previously recommended for the 2000 version of the curriculum continue to support this updated IRP. (See the Learning Resources section later in this IRP for additional information.)

Principles of Mathematics 10 to 12, in draft form, was available for public review and response from November to December, 2005. Feedback from educators, students, parents, and other educational partners informed the development of this updated IRP.

RATIONALE

Mathematics is increasingly important in our technological society. Students today require the ability to reason and communicate, to solve problems, and to understand and use mathematics. Development of these skills helps students become numerate.

Numeracy can be defined as the combination of mathematical knowledge, problem solving and communication skills required by all persons to function successfully within our technological world. Numeracy is more than knowing about numbers and number operations. (British Columbia Association of Mathematics Teachers, 1998)

Becoming numerate involves developing the ability to explore, conjecture, reason logically, and use a variety of mathematical methods to solve problems. It also involves the development of self-confidence and the ability to use quantitative and spatial information in problem solving and decision making. As students develop their numeracy skills and concepts, they generally grow more confident and motivated in their mathematical explorations. This growth occurs as they learn to enjoy and value mathematics, to think analytically, and to understand and appreciate the role of mathematics in everyday life.

The provincial mathematics curriculum emphasizes the development of numeracy skills and concepts and their practical application in higher education and the workplace. The curriculum places emphasis on probability and statistics, reasoning and communication, measurement, and problem

solving. To ensure that students are prepared for the demands of both further education and the workplace, the graduate years of the mathematics curriculum (Grades 10 to 12) help students develop a more sophisticated sense of numeracy. At the same time, the curriculum investigates the creative and aesthetic aspects of mathematics by exploring the connections between mathematics, art, and design.

REQUIREMENTS AND GRADUATION CREDITS

Principles of Mathematics 10 and 11 *or* 12 are two of the courses available for students to satisfy the Graduation Program mathematics requirement. Principles of Mathematics 10, 11, and 12 are each designated as four-credit courses, and must be reported as such to the Ministry of Education for transcript purposes. Letter grades and percentages must be reported for these courses. It is not possible to obtain partial credit for these courses.

The course codes for Principles of Mathematics 10 to 12 are MA 10, MA 11, and MA 12. These courses are also available in French (Principes des mathématiques 10, Principes des mathématiques 11, Principes des mathématiques 12; course codes MTH 10, MTH 11, MTH 12).

GRADUATION PROGRAM EXAMINATION

Principles of Mathematics 10 has a Graduation Program examination, worth 20% of the final

course mark. Students are required to take this exam to receive credit for the course. Principles of Mathematics 12 has an optional Graduation Program examination, worth 40% of the final course mark for students who choose to write it. Although students are not required to take this exam to receive credit for the course, they should be advised that some post-secondary institutions require Grade 12 exams to meet entrance requirements, and that writing Grade 12 exams also provides opportunities for provincial scholarships.

For more information, refer to the Ministry of Education examinations web site:
www.bced.gov.bc.ca/exams/

GOALS FOR PRINCIPLES OF MATHEMATICS 10 TO 12

Students following the Principles of Mathematics pathway will develop their understanding of symbol manipulation and of generalizations of more sophisticated mathematical concepts. Although there is an increased focus in this pathway on the applications of mathematics, one of the primary purposes of Principles of Mathematics will be to develop the formalism students will need to continue on with the study of calculus.

Goals for Principles of Mathematics 10 to 12

The aim of Principles of Mathematics 10 to 12 is to develop the formalism students will need to continue on with the study of calculus.

Students will

- become numerate citizens with the ability to explore, conjecture, reason logically, and use a variety of mathematical methods to solve problems
- develop self-confidence and the ability to use quantitative and spatial information in problem solving and decision-making
- learn to enjoy and value mathematics, to think analytically, and to understand and appreciate the role of mathematics in everyday life
- be prepared for the demands of both further education and the workplace

CURRICULUM ORGANIZERS

A curriculum organizer consists of a set of prescribed learning outcomes that share a common focus. The prescribed learning outcomes for Principles of Mathematics 10 to 12 progress

in age-appropriate ways, and are grouped under the following curriculum organizers and suborganizers.

Note that the ordering of these organizers and suborganizers is not intended to imply an order of instruction.

Curriculum Organizers and Suborganizers			
PRINCIPLES OF MATHEMATICS			
Number	Patterns and Relations <ul style="list-style-type: none"> • Patterns • Variables and Equations • Relations and Functions 	Shape and Space <ul style="list-style-type: none"> • Measurement • 3-D Objects and 2-D Shapes • Transformations 	Statistics and Probability <ul style="list-style-type: none"> • Chance and Uncertainty

Number

In this organizer, students further develop their number sense as they develop an intuitive feeling about numbers and their multiple relationships. It is important that students continue to develop computational fluency and the ability to detect arithmetic errors.

The Number organizer describes the knowledge and skills that students need in order to understand and perform calculations proficiently, decide which arithmetic operations can be used to solve problems, and then solve the problems.

Patterns and Relations

Students need to recognize, extend, create and use patterns as a routine aspect of their lives. This organizer provides opportunities for students to examine the relationships among physical things, as well as the data used to describe those things. These relationships will be described visually, symbolically, orally, and in written form.

The Patterns and Relations organizer includes the following suborganizers:

- Patterns – use patterns to describe the world and solve problems

- Variables and Equations – represent algebraic expressions in multiple ways
- Relations and Functions – use algebraic and graphical models to generalize patterns, make predictions, and solve problems

Shape and Space

It is important that students look for and use similarity, congruence, patterns, transformations, and dilatations in the solution of a range of problems. This organizer provides opportunities for students to study geometric representations of algebraic relations and to develop and refine their reasoning skills.

The Shape and Space organizer includes the following suborganizers:

- Measurement – describe and compare everyday phenomena, using trigonometry
- 3-D Objects and 2-D Shapes – describe the characteristics of geometric objects and shapes and analyse the relationships among them
- Transformations – perform, analyse, and create transformations algebraically and graphically

Statistics and Probability

The language that students use to describe chance becomes more sophisticated and involves the vocabulary of probability theory.

The Statistics and Probability organizer includes the suborganizer Chance and Uncertainty – use experimental or theoretical probability to represent and solve problems involving uncertainty

MATHEMATICAL PROCESSES

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The following seven mathematical processes should be integrated within Principles of Mathematics 10 to 12.

Communication

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students need

to be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. “Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding... Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching” (Caine and Caine, 1991, p. 5).

Mental Mathematics and Estimation

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility.

Even more important than performing computational procedures or using calculators is the greater facility that students need – more than ever before – with estimation and mental mathematics (NCTM, May 2005).

Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001).

Mental mathematics “provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers” (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating.

Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type “How would you...?” or “How could you...?” the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies

for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

Reasoning

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels.

Visualization

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world” (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with the opportunity to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to decide when to measure, when to estimate and to know several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

PRINCIPLES OF MATHEMATICS 10 TO 12: AT A GLANCE

Principles of Mathematics 10	Principles of Mathematics 11	Principles of Mathematics 12
Number		
<ul style="list-style-type: none"> explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system use basic arithmetic operations on real numbers to solve problems use exact values, arithmetic operations, and algebraic operations on real numbers to solve problems 		
Patterns and Relations		
<p>Patterns</p> <ul style="list-style-type: none"> generate and analyse arithmetic number patterns <p>Variables and Equations</p> <ul style="list-style-type: none"> generalize operations on polynomials to include rational expressions <p>Relations and Functions</p> <ul style="list-style-type: none"> examine the nature of relations with an emphasis on functions represent data, using linear function models 	<p>Variables and Equations</p> <ul style="list-style-type: none"> represent and analyse situations that involve expressions, equations, and inequalities <p>Relations and Functions</p> <ul style="list-style-type: none"> represent and analyse quadratic, polynomial, and rational functions 	<p>Patterns</p> <ul style="list-style-type: none"> generate and analyse exponential patterns <p>Variables and Equations</p> <ul style="list-style-type: none"> solve exponential, logarithmic, and trigonometric equations and identities <p>Relations and Functions</p> <ul style="list-style-type: none"> represent and analyse exponential and logarithmic functions
Shape and Space		
<p>Measurement</p> <ul style="list-style-type: none"> solve problems involving triangles, including those found in 3-D and 2-D applications <p>3-D Objects and 2-D Shapes</p> <ul style="list-style-type: none"> solve coordinate geometry problems involving points, lines, and line segments 	<p>3-D Objects and 2-D Shapes</p> <ul style="list-style-type: none"> solve coordinate geometry problems involving points, lines, and line segments, and justify the solutions develop and apply the geometric properties of circles to solve problems 	<p>Transformations</p> <ul style="list-style-type: none"> perform, analyse, and create transformations of functions and relations that are described by equations or graphs
Statistics and Probability		
		<p>Chance and Uncertainty</p> <ul style="list-style-type: none"> solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations, combinations, and combining of simpler probabilities

SUGGESTED TIMEFRAME

Provincial curricula are developed in accordance with the amount of instructional time recommended by the Ministry of Education for each subject area. Teachers may choose to combine various curricula to enable students to integrate ideas and make meaningful connections.

In each of Grades 10, 11, and 12, a minimum of 100 hours of instructional time is recommended for the study of Principles of Mathematics. Although a

four-credit course is typically equivalent to 120 hours, this timeframe allows for flexibility to address local needs.

The following table shows the number of hours suggested to deliver the prescribed learning outcomes in each curriculum organizer.

These estimations are provided as suggestions only; when delivering the prescribed curriculum teachers should adjust the instructional time as necessary.

Suggested Timeframe for Principles of Mathematics 10 to 12

Curriculum Organizer (Suborganizer)	Suggested Timeframe	Curriculum Organizer (Suborganizer)	Suggested Timeframe	Curriculum Organizer (Suborganizer)	Suggested Timeframe
Principles of Mathematics 10		Principles of Mathematics 11		Principles of Mathematics 12	
Number	20 - 25 hrs				
Patterns and Relations		Patterns and Relations		Patterns and Relations	
(Patterns)	5 - 10 hrs	(Variables and Equations)	25 - 35 hrs	(Patterns)	5 - 10 hrs
(Variables and Equations)	20 - 25 hrs	(Relations and Functions)	35 - 40 hrs	(Variables and Equations)	20 - 25 hrs
(Relations and Functions)	15 - 20 hrs			(Relations and Functions)	25 - 30 hrs
Shape and Space		Shape and Space		Shape and Space	
(Measurement)	10 - 15 hrs	(3-D Objects and 2-D Shapes)	25 - 35 hrs	(Transformations)	15 - 20 hrs
(3-D Objects and 2-D Shapes)	15 - 20 hrs				
				Statistics and Probability	
				(Chance and Uncertainty)	20 - 25 hrs



CONSIDERATIONS FOR PROGRAM DELIVERY

Principles of Mathematics 10 to 12

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners. Included in this section is information about

- Alternative Delivery policy
- instructional focus
- fostering the development of positive attitudes
- applying mathematics
- involving parents and guardians
- confidentiality
- inclusion, equity, and accessibility for all learners
- working with the school and community
- working with the Aboriginal community
- information and communications technology
- copyright and responsibility

ALTERNATIVE DELIVERY POLICY

The Alternative Delivery policy does not apply to Principles of Mathematics 10 to 12.

The Alternative Delivery policy outlines how students, and their parents or guardians, in consultation with their local school authority, may choose means other than instruction by a teacher within the regular classroom setting for addressing prescribed learning outcomes contained in the Health curriculum organizer of the following curriculum documents:

- Health and Career Education K to 7, and Personal Planning K to 7 Personal Development curriculum organizer (until September 2008)
- Health and Career Education 8 and 9
- Planning 10

The policy recognizes the family as the primary educator in the development of children's attitudes, standards, and values, but the policy still requires that all prescribed learning outcomes be addressed and assessed in the agreed-upon alternative manner of delivery.

It is important to note the significance of the term "alternative delivery" as it relates to the Alternative Delivery policy. The policy does not permit schools to omit addressing or assessing any of the prescribed learning outcomes within the health and career

education curriculum. Neither does it allow students to be excused from meeting any learning outcomes related to health. It is expected that students who arrange for alternative delivery will address the health-related learning outcomes and will be able to demonstrate their understanding of these learning outcomes.

For more information about policy relating to alternative delivery, refer to www.bced.gov.bc.ca/policy/

INSTRUCTIONAL FOCUS

The Principles of Mathematics 10 to 12 courses are arranged into four organizers with problem solving integrated throughout. Decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations allows more time for concept development.

In addition to problem solving, other critical thinking processes – reasoning and making connections – are vital to increasing students' mathematical power and must be integrated throughout the program. A minimum of half the available time within all organizers should be dedicated to activities related to these processes.

Instruction should provide a balance between estimation and mental mathematics, paper and pencil exercises, and the appropriate use of technology, including calculators and computers. (It is assumed that all students have regular access to appropriate technology such as graphing calculators, or computers with graphing software and standard spreadsheet programs.) Concepts should be introduced using manipulatives, and gradually developed from the concrete to the pictorial to the symbolic.

FOSTERING THE DEVELOPMENT OF POSITIVE ATTITUDES

Students should be exposed to experiences that encourage them to enjoy and value mathematics, develop mathematical habits of mind, and understand and appreciate the role of mathematics in human affairs. They should be encouraged to

explore, take risks, exhibit curiosity, and make and correct errors, so that they gain confidence in their abilities to solve complex problems. The assessment of attitudes is indirect, and based on inferences drawn from students' behaviour. We can see what students do and hear what they say, and from these observations make inferences and draw conclusions about their attitudes.

APPLYING MATHEMATICS

For students to view mathematics as relevant and useful, they must see how it can be applied to a wide variety of real-world applications. Mathematics helps students understand and interpret their world and solve problems that occur in their daily lives.

INVOLVING PARENTS AND GUARDIANS

The family is essential in the development of students' attitudes and values. The school plays a supportive role by focussing on the prescribed learning outcomes in the Grades 10 to 12 Mathematics curriculum. Parents and guardians are encouraged to support, enrich, and extend the curriculum at home.

It is highly recommended that schools inform parents and guardians about the Principles of Mathematics curriculum. Teachers (along with school and district administrators) may choose to do so by

- informing parents/guardians and students, via a course outline at the beginning of the course, of the prescribed learning outcomes for the course
- responding to parent and guardian requests to discuss course unit plans, learning resources, etc.

CONFIDENTIALITY

The *Freedom of Information and Protection of Privacy Act* (FOIPPA) applies to students, to school districts, and to all curricula. Teachers, administrators, and district staff should consider the following:

- Be aware of district and school guidelines regarding the provisions of FOIPPA and how it applies to all subjects, including Principles of Mathematics 10 to 12.

- Do not use students' Personal Education Numbers (PEN) on any assignments that students wish to keep confidential.
- Ensure students are aware that if they disclose personal information that indicates they are at risk for harm, then that information cannot be kept confidential.
- Inform students of their rights under FOIPPA, especially the right to have access to their own personal information in their school records. Inform parents of their rights to access their children's school records.
- Minimize the type and amount of personal information collected, and ensure that it is used only for purposes that relate directly to the reason for which it is collected.
- Inform students that they will be the only ones recording personal information about themselves unless they, or their parents, have consented to teachers collecting that information from other people (including parents).
- Provide students and their parents with the reason(s) they are being asked to provide personal information in the context of the Principles of Mathematics 10 to 12 curriculum.
- Inform students and their parents that they can ask the school to correct or annotate any of the personal information held by the school, in accordance with Section 29 of FOIPPA.
- Ensure students are aware that their parents may have access to the schoolwork they create only insofar as it pertains to students' progress.
- Ensure that any information used in assessing students' progress is up-to-date, accurate, and complete.

For more information about confidentiality, refer to www.mser.gov.bc.ca/privacyaccess/

INCLUSION, EQUITY, AND ACCESSIBILITY FOR ALL LEARNERS

British Columbia's schools include young people of varied backgrounds, interests, and abilities. The Kindergarten to Grade 12 school system focusses on meeting the needs of all students. When selecting specific topics, activities, and resources to support the implementation of Principles of Mathematics 10

to 12, teachers are encouraged to ensure that these choices support inclusion, equity, and accessibility for all students. In particular, teachers should ensure that classroom instruction, assessment, and resources reflect sensitivity to diversity and incorporate positive role portrayals, relevant issues, and themes such as inclusion, respect, and acceptance.

Government policy supports the principles of integration and inclusion of students for whom English is a second language and of students with special needs. Most of the prescribed learning outcomes and suggested achievement indicators in this IRP can be met by all students, including those with special needs and/or ESL needs. Some strategies may require adaptations to ensure that those with special and/or ESL needs can successfully achieve the learning outcomes. Where necessary, modifications can be made to the prescribed learning outcomes for students with Individual Education Plans.

For more information about resources and support for students with special needs, refer to www.bced.gov.bc.ca/specialed/

For more information about resources and support for ESL students, refer to www.bced.gov.bc.ca/esl/

WORKING WITH THE SCHOOL AND COMMUNITY

This curriculum addresses a wide range of skills and understandings that students are developing in other areas of their lives. It is important to recognize that learning related to this curriculum extends beyond the Mathematics classroom.

Community organizations may also support the curriculum with locally developed learning resources, guest speakers, workshops, and field studies. Teachers may wish to draw on the expertise of these community organizations and members.

WORKING WITH THE ABORIGINAL COMMUNITY

The Ministry of Education is dedicated to ensuring that the cultures and contributions of Aboriginal peoples in BC are reflected in all provincial curricula.

To address these topics in the classroom in a way that is accurate and that respectfully reflects Aboriginal concepts of teaching and learning, teachers are strongly encouraged to seek the advice and support of local Aboriginal communities. Aboriginal communities are diverse in terms of language, culture, and available resources, and each community will have its own unique protocol to gain support for integration of local knowledge and expertise. To begin discussion of possible instructional and assessment activities, teachers should first contact Aboriginal education co-ordinators, teachers, support workers, and counsellors in their district who will be able to facilitate the identification of local resources and contacts such as elders, chiefs, tribal or band councils, Aboriginal cultural centres, Aboriginal Friendship Centres, and Métis or Inuit organizations.

In addition, teachers may wish to consult the various Ministry of Education publications available, including the “Planning Your Program” section of the resource, *Shared Learnings*. This resource was developed to help all teachers provide students with knowledge of, and opportunities to share experiences with, Aboriginal peoples in BC.

For more information about these documents, consult the Aboriginal Education web site: www.bced.gov.bc.ca/abed/welcome.htm

INFORMATION AND COMMUNICATIONS TECHNOLOGY

The study of information and communications technology is increasingly important in our society. Students need to be able to acquire and analyse information, to reason and communicate, to make informed decisions, and to understand and use information and communications technology for a variety of purposes. Development of these skills is important for students in their education, their future careers, and their everyday lives.

Literacy in the area of information and communications technology can be defined as the ability to obtain and share knowledge through investigation, study, instruction, or transmission

of information by means of media technology. Becoming literate in this area involves finding, gathering, assessing, and communicating information using electronic means, as well as developing the knowledge and skills to use and solve problems effectively with the technology. Literacy also involves a critical examination and understanding of the ethical and social issues related to the use of information and communications technology. When planning for instruction and assessment in Principles of Mathematics 10 to 12, teachers should provide opportunities for students to develop literacy in relation to information and communications technology sources, and to reflect critically on the role of these technologies in society.

COPYRIGHT AND RESPONSIBILITY

Copyright is the legal protection of literary, dramatic, artistic, and musical works; sound recordings; performances; and communications signals. Copyright provides creators with the legal right to be paid for their work and the right to say how their work is to be used. The law permits certain exceptions for schools (i.e., specific things permitted) but these are very limited, such as copying for private study or research. The copyright law determines how resources can be used in the classroom and by students at home.

In order to respect copyright it is necessary to understand the law. It is unlawful to do the following, unless permission has been given by a copyright owner:

- photocopy copyrighted material to avoid purchasing the original resource for any reason
- photocopy or perform copyrighted material beyond a very small part – in some cases the copyright law considers it “fair” to copy whole works, such as an article in a journal or a photograph, for purposes of research and private study, criticism, and review
- show recorded television or radio programs to students in the classroom unless these are cleared for copyright for educational use

(there are exceptions such as for news and news commentary taped within one year of broadcast that by law have record-keeping requirements – see the web site at the end of this section for more details)

- photocopy print music, workbooks, instructional materials, instruction manuals, teacher guides, and commercially available tests and examinations
- show video recordings at schools that are not cleared for public performance
- perform music or do performances of copyrighted material for entertainment (i.e., for purposes other than a specific educational objective)
- copy work from the Internet without an express message that the work can be copied.

Permission from or on behalf of the copyright owner must be given in writing. Permission may also be given to copy or use all or some portion of copyrighted work through a licence or agreement. Many creators, publishers, and producers have formed groups or “collectives” to negotiate royalty payments and copying conditions for educational institutions. It is important to know what licences are in place and how these affect the activities schools are involved in. Some licences may also require royalty payments that are determined by the quantity of photocopying or the length of performances. In these cases, it is important to assess the educational value and merits of copying or performing certain works to protect the school’s financial exposure (i.e., only copy or use that portion that is absolutely necessary to meet an educational objective).

It is important for education professionals, parents, and students to respect the value of original thinking and the importance of not plagiarizing the work of others. The works of others should not be used without their permission.

For more information about copyright, refer to <http://cmec.ca/copyright/indexe.stm>



PRESCRIBED LEARNING OUTCOMES

Principles of Mathematics 10 to 12

Prescribed learning outcomes are content standards for the provincial education system; they are the prescribed curriculum. Clearly stated and expressed in measurable and observable terms, learning outcomes set out the required knowledge, skills, and attitudes – what students are expected to know and be able to do – by the end of the specified course. All prescribed learning outcomes complete the stem, “It is expected that students will....”

Schools have the responsibility to ensure that all prescribed learning outcomes in this curriculum are met; however, schools have flexibility in determining how delivery of the curriculum can best take place.

It is expected that student achievement will vary in relation to the learning outcomes. Evaluation, reporting, and student placement with respect to these outcomes are dependent on the professional judgment and experience of teachers, guided by provincial policy.

Prescribed learning outcomes for Principles of Mathematics 10 to 12 are presented by curriculum organizer and suborganizer, and are coded alphanumerically for ease of reference; however, this arrangement is not intended to imply a required instructional sequence.

WORDING OF PRESCRIBED LEARNING OUTCOMES

When used in a prescribed learning outcome, the word “**including**” indicates that any ensuing item **must be addressed**. Lists of items introduced by the word “including” represent a set of minimum requirements associated with the general requirement set out by the outcome. The lists are not necessarily exhaustive, however, and teachers may choose to address additional items that also fall under the general requirement set out by the outcome.

Conversely, the abbreviation “**e.g.**” (for example) in a prescribed learning outcome indicates that the ensuing items are provided for illustrative purposes or clarification, and **are not required**. Presented in parentheses, the list of items introduced by “e.g.”

is neither exhaustive nor prescriptive, nor is it put forward in any special order of importance or priority. Teachers are free to substitute items of their own choosing that they feel best address the intent of the prescribed learning outcome.

DOMAINS OF LEARNING

Prescribed learning outcomes in BC curricula identify required learning in relation to one or more of the three domains of learning: cognitive, psychomotor, and affective. The following definitions of the three domains are based on Bloom’s taxonomy.

The **cognitive domain** deals with the recall or recognition of knowledge and the development of intellectual abilities. The cognitive domain can be further specified as including three cognitive levels: knowledge, understanding and application, and higher mental processes. These levels are determined by the verb used in the learning outcome, and illustrate how student learning develops over time.

- *Knowledge* includes those behaviours that emphasize the recognition or recall of ideas, material, or phenomena.
- *Understanding and application* represents a comprehension of the literal message contained in a communication, and the ability to apply an appropriate theory, principle, idea, or method to a new situation.
- *Higher mental processes* include analysis, synthesis, and evaluation. The higher mental processes level subsumes both the knowledge and the understanding and application levels.

The **affective domain** concerns attitudes, beliefs, and the spectrum of values and value systems.

The **psychomotor domain** includes those aspects of learning associated with movement and skill demonstration, and integrates the cognitive and affective consequences with physical performances.

Domains of learning and, particularly, cognitive levels, inform the design and development of the Graduation Program examinations for Principles of Mathematics 10 and 12.

Prescribed Learning Outcomes: Principles of Mathematics 10

It is expected that students will:

NUMBER

- A1 classify numbers as natural, whole, integer, rational, or irrational and describe contexts where they are used
- A2 describe how natural, whole, integer, rational, and irrational number sets are “nested” within the real number system
- A3 perform arithmetic operations on irrational numbers, using appropriate decimal approximations
- A4 perform operations on irrational numbers of monomial and binomial form, using exact values
- A5 explain and apply the exponent laws for powers of numbers, including

- $x^m \cdot x^n = x^{m+n}$
- $x^m \div x^n = x^{m-n}$
- $(x^m)^n = x^{mn}$
- $(xy)^m = x^m y^m$
- $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}, y \neq 0$
- $x^0 = 1, x \neq 0$
- $x^{-m} = \frac{1}{x^m}, x \neq 0$

- A6 explain and apply the exponent laws for powers of numbers and for variables with rational exponents

PATTERNS AND RELATIONS

Patterns

- B1 use expressions to represent general terms for arithmetic growth, and apply these expressions to solve problems
- B2 use expressions to represent sums for arithmetic growth, and apply these expressions to solve problems
- B3 relate arithmetic sequences to linear functions defined over the natural numbers

Variables and Equations

- B4 factor polynomial expressions of the form
 - $ax^2 + bx + c$
 - $a^2x^2 - b^2y^2$
- B5 find the product of polynomials (i.e., monomials, binomials, trinomials)
- B6 divide a polynomial (P or $P(x)$) by a binomial (D or $D(x)$) and express the result in the forms
 - $\frac{P}{D} = Q + \frac{R}{D}$
 - $P(x) = D(x)Q(x) + R$, where Q and $Q(x)$ denote the quotient and R denotes the remainder
- B7 determine equivalent forms of simple rational expressions with polynomial numerators, and denominators that are factorable monomials, binomials, or trinomials
- B8 determine the non-permissible values for the variable in rational expressions with polynomial numerators, and denominators that are factorable monomials, binomials, or trinomials
- B9 perform the operations of addition, subtraction, multiplication, and division on rational expressions with polynomial numerators, and denominators that are monomials, binomials, or trinomials
- B10 find and verify the solutions of rational equations that reduce to linear form

Organizer 'Patterns and Relations' continued on page 21

Prescribed Learning Outcomes: Principles of Mathematics 10

Organizer 'Patterns and Relations' continued from page 20

Relations and Functions

- B11 describe a linear function in terms of
 - ordered pairs
 - a rule, in word or equation form
 - a graph
- B12 use function notation to evaluate and represent linear functions
- B13 determine the following characteristics of the graph of a linear function, given its equation
 - x - and y -intercepts
 - slope
 - domain
 - range
- B14 sketch the graph of a linear function given its equation in the form
 - $ax + by + c = 0$ (general form)
 - $y = mx + b$ (slope-intercept form)
- B15 represent linear data, using linear function models
- B16 solve problems involving partial variation and arithmetic sequences as applications of linear functions

SHAPE AND SPACE

Measurement

- C1 solve 2-D and 3-D problems involving two right triangles
- C2 extend the concepts of sine and cosine for positive angles through to 180°
- C3 apply the sine and cosine laws to solve problems (excluding the ambiguous case)

3-D Objects and 2-D Shapes

- C4 solve problems involving distances between points in the coordinate plane
- C5 solve problems involving midpoints of line segments
- C6 solve problems involving rise, run, and slope of line segments
- C7 determine the equation of a line, given information that uniquely determines the line
- C8 solve problems involving slopes of
 - parallel lines
 - perpendicular lines

Prescribed Learning Outcomes: Principles of Mathematics 11

It is expected that students will:

PATTERNS AND RELATIONS

Variables and Equations

- A1 graph linear inequalities, in two variables
- A2 solve systems of linear equations, in two variables:
 - algebraically (elimination and substitution)
 - graphically
- A3 solve systems of linear equations, in three variables:
 - algebraically
 - with a graphing calculator
- A4 solve nonlinear equations
 - by factoring
 - graphically
 - with a graphing calculator
- A5 use the Remainder Theorem to evaluate polynomial expressions
- A6 use the Rational Zero Theorem and the Factor Theorem to determine factors of polynomials
- A7 determine the solution to a system of nonlinear equations, using a graphing calculator as appropriate

Relations and Functions

- A8 determine the characteristics of the graph of a quadratic function, including
 - vertex
 - domain and range
 - axis of symmetry
 - x - and y -intercepts
- A9 perform operations on functions and compositions of functions
- A10 determine the inverse of a function
- A11 connect algebraic and graphical transformations of quadratic functions, using completing the square as required
- A12 model real-world situations, using quadratic functions
- A13 solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using
 - factoring
 - the quadratic formula
 - graphing
- A14 determine the character of the real and non-real roots of a quadratic equation, using
 - the discriminant in the quadratic formula
 - graphing
- A15 describe, graph, and analyse polynomial and rational functions, using a graphing calculator as appropriate
- A16 formulate and apply strategies to solve absolute value equations, radical equations, rational equations, and inequalities

Prescribed Learning Outcomes: Principles of Mathematics 11

SHAPE AND SPACE

3-D Objects and 2-D Shapes

- B1 solve problems involving distances between points and lines
- B2 verify and prove assertions in plane geometry, using coordinate geometry
- B3 investigate the following geometric circle properties using computers with dynamic geometry software, and prove them using established concepts and theorems:
- the perpendicular bisector of a chord contains the centre of the circle
 - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
 - the inscribed angles subtended by the same arc are congruent
 - the angle inscribed in a semicircle is a right angle
 - the opposite angles of a cyclic quadrilateral are supplementary
 - a tangent to a circle is perpendicular to the radius at the point of tangency
 - the tangent segments to a circle, from any external point, are congruent
 - the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord
- B4 solve problems and justify the solution strategy using circle properties, including
- the perpendicular bisector of a chord contains the centre of the circle
 - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
 - the inscribed angles subtended by the same arc are congruent
 - the angle inscribed in a semicircle is a right angle
 - the opposite angles of a cyclic quadrilateral are supplementary
 - a tangent to a circle is perpendicular to the radius at the point of tangency
 - the tangent segments to a circle, from any external point, are congruent
 - the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord

Prescribed Learning Outcomes: Principles of Mathematics 12

It is expected that students will:

PATTERNS AND RELATIONS

Patterns

- A1 derive and apply expressions to represent general terms for geometric growth and to solve problems
- A2 derive and apply expressions to represent sums for geometric growth and to solve problems
- A3 estimate sums of expressions represented by infinite geometric processes where the common ratio, r , is $-1 < r < 1$

Variables and Equations

- A4 solve exponential equations having bases that are powers of one another
- A5 solve and verify exponential and logarithmic equations
- A6 solve and verify exponential and logarithmic identities
- A7 distinguish between degree and radian measure, and solve problems using both
- A8 determine the exact and the approximate values of trigonometric ratios for any multiples of 0° , 30° , 45° , 60° and 90° , and 0 rad, $\frac{\pi}{6}$ rad, $\frac{\pi}{4}$ rad, $\frac{\pi}{3}$ rad, $\frac{\pi}{2}$ rad
- A9 solve first and second degree trigonometric equations over a specified domain
 - algebraically
 - graphically
- A10 determine the general solutions to trigonometric equations where the domain is the set of real numbers
- A11 analyse trigonometric identities
 - graphically
 - algebraically for general cases
- A12 use sum, difference, and double angle identities for sine and cosine to verify and simplify trigonometric expressions

Relations and Functions

- A13 change functions from exponential form to logarithmic form and vice versa
- A14 model, graph, and apply exponential functions to solve problems
- A15 model, graph, and apply logarithmic functions to solve problems
- A16 describe the three primary trigonometric functions as circular functions with reference to the unit circle and an angle in standard position
- A17 sketch and analyse the graphs of sine, cosine, and tangent functions, for
 - amplitude, if defined
 - period
 - domain and range
 - asymptotes, if any
 - behaviour under transformations
- A18 use trigonometric functions to model and solve problems

Prescribed Learning Outcomes: Principles of Mathematics 12

SHAPE AND SPACE

Transformations

- B1 describe how vertical and horizontal translations of functions affect graphs and their related equations:
- $y = f(x - h)$
 - $y - k = f(x)$
- B2 describe how compressions and expansions of functions affect graphs and their related equations:
- $y = af(x)$
 - $y = f(kx)$
- B3 describe how reflections of functions in both axes and in the line $y = x$ affect graphs and their related equations:
- $y = f(-x)$
 - $y = -f(x)$
 - $y = f^{-1}(x)$
- B4 using the graph and/or the equation of $f(x)$, describe and sketch $\frac{1}{f(x)}$
- B5 using the graph and/or the equation of $f(x)$, describe and sketch $|f(x)|$
- B6 describe and perform single transformations and combinations of transformations on functions and relations

STATISTICS AND PROBABILITY

Chance and Uncertainty

- C1 use the fundamental counting principle to determine the number of different ways to perform multi-step operations
- C2 use factorial notation to determine different ways of arranging n distinct objects in a sequence
- C3 determine the number of permutations of n different objects taken r at a time, and use this to solve problems
- C4 determine the number of combinations of n different objects taken r at a time, and use this to solve problems
- C5 solve problems, using the binomial theorem where the exponent n belongs to the set of natural numbers
- C6 construct a sample space for up to three events
- C7 classify events as independent or dependent
- C8 solve problems, using the probabilities of mutually exclusive and complementary events
- C9 determine the conditional probability of two events
- C10 solve probability problems involving permutations, combinations, and conditional probability



STUDENT ACHIEVEMENT

Principles of Mathematics 10 to 12

This section of the IRP contains information about classroom assessment and student achievement, including specific achievement indicators to assist in the assessment of student achievement in relation to each prescribed learning outcome. Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of prescribed learning outcomes.

CLASSROOM ASSESSMENT AND EVALUATION

Assessment is the systematic gathering of information about what students know, are able to do, and are working toward. Assessment evidence can be collected using a wide variety of methods, such as

- observation
- student self-assessments and peer assessments
- quizzes and tests (written, oral, practical)
- samples of student work
- projects and presentation
- oral and written reports
- journals and learning logs
- performance reviews
- portfolio assessments

Assessment of student performance is based on the information collected through assessment activities. Teachers use their insight, knowledge about learning, and experience with students, along with the specific criteria they establish, to make judgments about student performance in relation to prescribed learning outcomes.

Three major types of assessment can be used in conjunction to support student achievement.

- Assessment **for** learning is assessment for purposes of greater learning achievement.
- Assessment **as** learning is assessment as a process of developing and supporting students' active participation in their own learning.
- Assessment **of** learning is assessment for purposes of providing evidence of achievement for reporting.

Assessment for Learning

Classroom assessment for learning provides ways to engage and encourage students to become involved in their own day-to-day assessment – to acquire the skills of thoughtful self-assessment and to promote their own achievement.

This type of assessment serves to answer the following questions:

- What do students need to learn to be successful?
- What does the evidence of this learning look like?

Assessment for learning is criterion-referenced, in which a student's achievement is compared to established criteria rather than to the performance of other students. Criteria are based on prescribed learning outcomes, as well as on suggested achievement indicators or other learning expectations.

Students benefit most when assessment feedback is provided on a regular, ongoing basis. When assessment is seen as an opportunity to promote learning rather than as a final judgment, it shows students their strengths and suggests how they can develop further. Students can use this information to redirect their efforts, make plans, communicate with others (e.g., peers, teachers, parents) about their growth, and set future learning goals.

Assessment for learning also provides an opportunity for teachers to review what their students are learning and what areas need further attention. This information can be used to inform teaching and create a direct link between assessment and instruction. Using assessment as a way of obtaining feedback on instruction supports student achievement by informing teacher planning and classroom practice.

Assessment as Learning

Assessment as learning actively involves students in their own learning processes. With support and guidance from their teacher, students take responsibility for their own learning, constructing meaning for themselves. Through a process of continuous self-assessment, students develop the ability to take stock of what they have already learned, determine what they have not yet learned, and decide how they can best improve their own achievement.

Although assessment as learning is student-driven, teachers can play a key role in facilitating how this assessment takes place. By providing regular opportunities for reflection and self-assessment, teachers can help students develop, practise, and become comfortable with critical analysis of their own learning.

Assessment of Learning

Assessment of learning can be addressed through summative assessment, including large-scale assessments and teacher assessments. These summative assessments can occur at the end of the year or at periodic stages in the instructional process.

Large-scale assessments, such as Foundation Skills Assessment (FSA) and Graduation Program exams, gather information on student performance throughout the province and provide information for the development and revision of curriculum. These assessments are used to make judgments about students' achievement in relation to provincial and national standards.

Assessment of learning is also used to inform formal reporting of student achievement.

For Ministry of Education reporting policy, refer to www.bced.gov.bc.ca/policy/policies/student_reporting.htm

Assessment for Learning	Assessment as Learning	Assessment of Learning
<p>Formative assessment is ongoing in the classroom</p> <ul style="list-style-type: none"> • teacher assessment, student self-assessment, and/or student peer assessment • criterion-referenced – criteria based on prescribed learning outcomes identified in the provincial curriculum, reflecting performance in relation to a specific learning task • involves both teacher and student in a process of continual reflection and review about progress • teachers adjust their plans and engage in corrective teaching in response to formative assessment 	<p>Formative assessment is ongoing in the classroom</p> <ul style="list-style-type: none"> • self-assessment • provides students with information on their own achievement and prompts them to consider how they can continue to improve their learning • student-determined criteria based on previous learning and personal learning goals • students use assessment information to make adaptations to their learning process and to develop new understandings 	<p>Summative assessment occurs at end of year or at key stages</p> <ul style="list-style-type: none"> • teacher assessment • may be either criterion-referenced (based on prescribed learning outcomes) or norm-referenced (comparing student achievement to that of others) • information on student performance can be shared with parents/guardians, school and district staff, and other education professionals (e.g., for the purposes of curriculum development) • used to make judgments about students' performance in relation to provincial standards

For more information about assessment for, as, and of learning, refer to the following resource developed by the Western and Northern Canadian Protocol (WNCP): *Rethinking Assessment with Purpose in Mind*.

This resource is available online at www.wncp.ca/

Criterion-Referenced Assessment and Evaluation

In criterion-referenced evaluation, a student's performance is compared to established criteria rather than to the performance of other students. Evaluation in relation to prescribed curriculum requires that criteria be established based on the learning outcomes.

Criteria are the basis for evaluating student progress. They identify, in specific terms, the critical aspects of a performance or a product that indicate how well the student is meeting the prescribed learning outcomes. For example, weighted criteria, rating scales, or scoring guides (reference sets) are ways that student performance can be evaluated using criteria.

Wherever possible, students should be involved in setting the assessment criteria. This helps students develop an understanding of what high-quality work or performance looks like.

Criterion-referenced assessment and evaluation may involve these steps:

- | | |
|----------------|--|
| Step 1 | Identify the prescribed learning outcomes and suggested achievement indicators (as articulated in this IRP) that will be used as the basis for assessment. |
| Step 2 | Establish criteria. When appropriate, involve students in establishing criteria. |
| Step 3 | Plan learning activities that will help students gain the knowledge, skills, and attitudes outlined in the criteria. |
| Step 4 | Prior to the learning activity, inform students of the criteria against which their work will be evaluated. |
| Step 5 | Provide examples of the desired levels of performance. |
| Step 6 | Conduct the learning activities. |
| Step 7 | Use appropriate assessment instruments (e.g., rating scale, checklist, scoring guide) and methods (e.g., observation, collection, self-assessment) based on the particular assignment and student. |
| Step 8 | Review the assessment data and evaluate each student's level of performance or quality of work in relation to criteria. |
| Step 9 | Where appropriate, provide feedback and/or a letter grade to indicate how well the criteria are met. |
| Step 10 | Communicate the results of the assessment and evaluation to students and parents/guardians. |

KEY ELEMENTS

Key elements provide an overview of content in each curriculum organizer. They can be used to determine the expected depth and breadth of the prescribed learning outcomes.

Note that some topics appear at multiple grade levels in order to emphasize their importance and to allow for developmental learning.

ACHIEVEMENT INDICATORS

To support the assessment of provincially prescribed curricula, this IRP includes sets of achievement indicators in relation to each learning outcome.

Achievement indicators, taken together as a set, define the specific level of knowledge acquired, skills applied, or attitudes demonstrated by the student in relation to a corresponding prescribed learning outcome. They describe what evidence to look for to determine whether or not a student has fully met the intent of the learning outcome. Since each achievement indicator defines only one aspect of the corresponding learning outcome, the entire set of achievement indicators should be considered when determining whether students have fully met the learning outcome.

In some cases, achievement indicators may also include suggestions as to the type of task that would provide evidence of having met the learning outcome (e.g., problem solving; a constructed response such as a list, comparison, analysis, or chart; a product created and presented such as a report, poster, or model; a particular skill demonstrated).

Achievement indicators support the principles of assessment *for* learning, assessment *as* learning, and assessment *of* learning. They provide teachers and parents with tools that can be used to reflect on what students are learning, as well as provide students with a means of self-assessment and ways of defining how they can improve their own achievement.

Achievement indicators are not mandatory; they are suggestions only, provided to assist in the assessment of how well students achieve the prescribed learning outcomes.

Achievement indicators may be useful to provincial examination development teams and inform the development of exam items. However, examination questions, item formats, exemplars, rubrics, or scoring guides will not necessarily be limited to the achievement indicators as outlined in the Integrated Resource Packages.

Specifications for provincial examinations are available online at www.bced.gov.bc.ca/exams/specs/

The following pages contain the suggested achievement indicators corresponding to each prescribed learning outcome for the Principles of Mathematics 10 to 12 curriculum. The achievement indicators are arranged by curriculum organizer and suborganizer for each grade; however, this order is not intended to imply a required sequence of instruction and assessment.



STUDENT ACHIEVEMENT

Principles of Mathematics 10

Key Elements: Principles of Mathematics 10

Mathematical Process (Integrated)

The following mathematical processes have been integrated within the prescribed learning outcomes and achievement indicators for the course: communication, problem solving, connections, mental mathematics and estimation, reasoning, technology, and visualization.

NUMBER

- number sets and their relationships, including natural, whole, integer, rational, and irrational
- basic arithmetic operations on irrational numbers using decimal approximations and exact values
- exponent laws, including numerical base, variable base, whole number exponent, and rational exponent

PATTERNS AND RELATIONS*Patterns*

- arithmetic sequences and series

Variables and Equations

- operations on polynomials, including factoring, product of polynomials, and division by a binomial
- non-permissible values in rational expressions
- operations on rational expressions
- solutions of rational equations that reduce to linear form

Relations and Functions

- nature of relations with an emphasis on linear functions
- linear data sets and associated graphs
- function notation
- characteristics of linear functions, including intercepts, slope, domain, and range
- general and slope-intercept form of linear equations
- direct variation, partial variation, and arithmetic sequences

SHAPE AND SPACE*Measurement*

- 2-D and 3-D applications of right triangles
- sine and cosine for angles between 0° and 180°
- sine and cosine laws (excluding the ambiguous case)

3-D Objects and 2-D Shapes

- distance between points in the coordinate plane using the Pythagorean Theorem or distance formula
- midpoint of line segments
- slope of lines and line segments
- equation of a line
- parallel and perpendicular lines, including slopes and equations of the lines

NUMBER

Students demonstrate an understanding of and proficiency with calculations, including making decisions concerning which arithmetic operation or operations to use to solve a problem and then solve the problem.

Prescribed Learning Outcomes	Suggested Achievement Indicators
<i>It is expected that students will:</i>	<p><i>The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.</i></p> <p><i>Students who have fully met the prescribed learning outcome are able to:</i></p>
A1 classify numbers as natural, whole, integer, rational, or irrational and describe contexts where they are used	<ul style="list-style-type: none"> <input type="checkbox"/> describe how the different number sets comprise the real number system <input type="checkbox"/> categorize a given set of numbers into their appropriate number sets
A2 describe how natural, whole, integer, rational, and irrational number sets are “nested” within the real number system	<ul style="list-style-type: none"> <input type="checkbox"/> explain how a given number can belong to more than one number set <input type="checkbox"/> use a graphic organizer to demonstrate the relationship between number systems
A3 perform arithmetic operations on irrational numbers, using appropriate decimal approximations	<ul style="list-style-type: none"> <input type="checkbox"/> use order of operations and a calculator to evaluate a given expression involving more than one radical symbol; examples: <ul style="list-style-type: none"> – $\sqrt{\sqrt{32}} \approx 2.38$ – $3\left(\sqrt[3]{85}\right) + 5\left(\sqrt{17}\right) \approx 33.8$
A4 perform operations on irrational numbers of monomial and binomial form, using exact values <i>Organizer ‘Number’ continued on page 36</i>	<ul style="list-style-type: none"> <input type="checkbox"/> write a given power with a positive rational exponent in equivalent radical form and vice versa <input type="checkbox"/> convert given mixed radicals to entire radicals and vice versa, including radicals with cube roots and variables <input type="checkbox"/> perform the operations of addition and subtraction with square roots and those cube roots involving easily recognized values <input type="checkbox"/> determine the product of <ul style="list-style-type: none"> – two given mixed or entire radicals – a given mixed radical and a binomial involving radicals – two given binomials involving entire radicals only <input type="checkbox"/> divide a given monomial radical by a monomial radical <input type="checkbox"/> determine the product of <ul style="list-style-type: none"> – two given binomials involving mixed or entire radicals – the square of a given mixed radical and a binomial involving radicals <input type="checkbox"/> divide a given binomial radical by a binomial radical <input type="checkbox"/> rationalize the denominator of a given expression involving division of a monomial by a binomial

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p>Organizer 'Number' continued from page 35</p> <p>A5 explain and apply the exponent laws for powers of numbers, including</p> <ul style="list-style-type: none"> - $x^m \cdot x^n = x^{m+n}$ - $x^m \div x^n = x^{m-n}$ - $(x^m)^n = x^{mn}$ - $(xy)^m = x^m y^m$ - $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}, y \neq 0$ - $x^0 = 1, x \neq 0$ - $x^{-m} = \frac{1}{x^m}, x \neq 0$ 	<ul style="list-style-type: none"> <input type="checkbox"/> demonstrate, using examples, the process of addition and subtraction of exponents <input type="checkbox"/> verify the exponent laws using examples <input type="checkbox"/> simplify a given expression using the exponent laws <input type="checkbox"/> identify the error(s) in a given simplification of an expression involving powers <input type="checkbox"/> determine the sum of two given powers (e.g., $5^2 + 5^3$) and record the process <input type="checkbox"/> determine the difference of two given powers (e.g., $4^3 - 4^2$) and record the process <input type="checkbox"/> use patterning to explain why $x^0 = 1$ when $x \neq 0$ <input type="checkbox"/> apply the exponent laws to evaluate a given expression
<p>A6 explain and apply the exponent laws for powers of numbers and for variables with rational exponents</p>	<ul style="list-style-type: none"> <input type="checkbox"/> manipulate a given expression containing more than one power with rational exponents (e.g., $\left(\frac{1}{27x^{-3}}\right)^{\frac{2}{3}} = \left(\frac{x^3}{27}\right)^{\frac{2}{3}} = \frac{x^2}{9}$) <input type="checkbox"/> convert a given expression containing more than one radical symbol with different indexes to powers with rational exponents (e.g., $\sqrt[4]{\sqrt{x^7}} = x^{\frac{7}{8}}$) <input type="checkbox"/> simplify a given expression and convert back to radical form (e.g., $\sqrt{\left(\frac{x^2}{y^4}\right)^{\frac{1}{4}}} = \frac{\sqrt[4]{x}}{\sqrt{y}}$)

PATTERNS AND RELATIONS

Students use algebraic and graphical models to generalize patterns, make predictions, and solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>It is expected that students will:</i></p>	<p><i>The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.</i></p> <p><i>Students who have fully met the prescribed learning outcome are able to:</i></p>
<p>Patterns</p> <p>B1 use expressions to represent general terms for arithmetic growth, and apply these expressions to solve problems</p>	<ul style="list-style-type: none"> <input type="checkbox"/> generate the next three terms, given four consecutive terms of an arithmetic sequence <input type="checkbox"/> solve given sequence problems where both the first term and the common difference are known <input type="checkbox"/> solve given sequence problems where either the first term or the common difference is unknown, but not both <input type="checkbox"/> solve given sequence problems where both the first term and the common difference are unknown <input type="checkbox"/> generate the next three terms of an arithmetic sequence when a problem is given
<p>B2 use expressions to represent sums for arithmetic growth, and apply these expressions to solve problems</p>	<ul style="list-style-type: none"> <input type="checkbox"/> solve a given series problem where both the first term and the common difference are known <input type="checkbox"/> solve a given series problem where either the first term or the common difference are unknown, but not both <input type="checkbox"/> solve a given series problem where neither the first term nor the common difference are known
<p>B3 relate arithmetic sequences to linear functions defined over the natural numbers</p>	<ul style="list-style-type: none"> <input type="checkbox"/> generate the graph of a linear equation that represents a given arithmetic sequence <input type="checkbox"/> generate an equation for a linear function from a given table, graph, or description given orally or in writing <input type="checkbox"/> describe the discrete nature of a given data set and explain the significance of the y-intercept as it relates to an arithmetic sequence (i.e., y-intercept occurs when $n = 0$ and the term $t_0 = (a - d)$)
<p>Variables and Equations</p> <p>B4 factor polynomial expressions of the form</p> <ul style="list-style-type: none"> – $ax^2 + bx + c$ – $a^2x^2 - b^2y^2$ <p><i>Suborganizer 'Variables and Equations' continued on page 38</i></p>	<ul style="list-style-type: none"> <input type="checkbox"/> factor a given expression of the form $ax^2 + bx + c$ or $a^2x^2 - b^2y^2$ <input type="checkbox"/> factor a given expression that requires the identification of monomial common factors <input type="checkbox"/> factor a given expression of the form $f^2 - g^2$, $a^2f^2 - g^2$ or $af^2 + bf + c$, where f and g are monomials and $a \neq 1$

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>Suborganizer 'Variables and Equations' continued from page 37</i></p> <p>B5 find the product of polynomials (i.e., monomials, binomials, trinomials)</p>	<ul style="list-style-type: none"> <input type="checkbox"/> multiply two given binomials <input type="checkbox"/> multiply a given monomial by a trinomial <input type="checkbox"/> apply a given product of polynomials to determine area <input type="checkbox"/> multiply three given binomials <input type="checkbox"/> multiply a given binomial by a trinomial <input type="checkbox"/> apply a given product of polynomials to determine volume
<p>B6 divide a polynomial (P or $P(x)$) by a binomial (D or $D(x)$) and express the result in the forms</p> <ul style="list-style-type: none"> – $\frac{P}{D} = Q + \frac{R}{D}$ – $P(x) = D(x)Q(x) + R$, where Q and $Q(x)$ denote the quotient and R denotes the remainder 	<ul style="list-style-type: none"> <input type="checkbox"/> determine the result of a given integral polynomial, where no coefficients are zero, divided by a given binomial of the form $(x - b)$, where b is an integer <input type="checkbox"/> determine the result of a given integral polynomial, with one or more coefficients equal to zero, divided by a given binomial of the form $(x - b)$, where b is an integer <input type="checkbox"/> determine the result of a given integral polynomial divided by a given integral divisor of the form $(ax + c)$ or $(ax^2 + c)$
<p>B7 determine equivalent forms of simple rational expressions with polynomial numerators, and denominators that are factorable monomials, binomials, or trinomials</p>	<ul style="list-style-type: none"> <input type="checkbox"/> determine the result of reducing a given rational expression involving a denominator that is monomials, a product of binomials, or a factorable trinomial of the form $ax^2 + bx + c$ <input type="checkbox"/> determine the result of reducing a given rational expression involving a denominator of the form $ax^2 + bx + c$
<p>B8 determine the non-permissible values for the variable in rational expressions with polynomial numerators, and denominators that are factorable monomials, binomials, or trinomials</p>	<ul style="list-style-type: none"> <input type="checkbox"/> determine all non-permissible values of a given rational expression where the denominator is factorable and the original expression is in its lowest terms and of the form $\frac{P(x)}{Q(x)}$ <input type="checkbox"/> determine all non-permissible values of a given rational expression of the form $\frac{P(x)}{Q(x)}$ where the denominator can be factored and the original expression is not in its lowest terms <input type="checkbox"/> determine all non-permissible values of a given rational expression of the form $\frac{P(x)}{Q(x)} \div \frac{R(x)}{T(x)}$, where $P(x)$, $Q(x)$, $R(x)$ and $T(x)$ are monomial or binomials
<p><i>Suborganizer 'Variables and Equations' continued on page 39</i></p>	

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>Suborganizer 'Variables and Equations' continued from page 38</i></p> <p>B9 perform the operations of addition, subtraction, multiplication, and division on rational expressions with polynomial numerators, and denominators that are monomials, binomials, or trinomials</p>	<ul style="list-style-type: none"> <input type="checkbox"/> write $\frac{P(x)}{Q(x)}$ in an equivalent form, by multiplying the given numerator and denominator by a monomial or a binomial <input type="checkbox"/> use the common denominator, rather than the lowest common denominator, for expressions of the type $\frac{P(x)}{Q(x)} + \frac{R(x)}{T(x)}$, with $Q(x)$ and $T(x)$ sharing a common factor <input type="checkbox"/> use the lowest common denominator for given expressions of the type $\frac{P(x)}{Q(x)} + \frac{R(x)}{T(x)}$, with $Q(x)$ and $T(x)$ sharing a common factor <input type="checkbox"/> solve a given division question, written in the form $\frac{P(x)}{Q(x)} \div \frac{R(x)}{T(x)}$, where $P(x)$ and $R(x)$ are polynomials and $Q(x)$ and $T(x)$ are monomials, binomials, or trinomials <input type="checkbox"/> solve a given multiplication question, written in the form $\frac{P(x)}{Q(x)} \cdot \frac{R(x)}{T(x)}$, where $P(x)$ and $R(x)$ are polynomials and $Q(x)$ and $T(x)$ are monomials, binomials, or trinomials <input type="checkbox"/> reduce a given or calculated answer to lowest terms
<p>B10 find and verify the solutions of rational equations that reduce to linear form</p>	<ul style="list-style-type: none"> <input type="checkbox"/> solve for x in a given rational equation containing a denominator that is a monomial or of the form $x + c$ <input type="checkbox"/> use a rational equation to solve problems stated in real-world contexts
<i>Relations and Functions</i>	
<p>B11 describe a linear function in terms of</p> <ul style="list-style-type: none"> – ordered pairs – a rule, in word or equation form – a graph 	<ul style="list-style-type: none"> <input type="checkbox"/> create a table of values and list the ordered pairs from a given linear equation <input type="checkbox"/> sketch the graph, and write a rule in words for a given linear function, using the equation
<p>B12 use function notation to evaluate and represent linear functions</p> <p><i>Suborganizer 'Relations and Functions' continued on page 40</i></p>	<ul style="list-style-type: none"> <input type="checkbox"/> evaluate a given linear function with real number inputs <input type="checkbox"/> write a given linear function using function notation

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>Suborganizer 'Relations and Functions' continued from page 39</i></p> <p>B13 determine the following characteristics of the graph of a linear function, given its equation</p> <ul style="list-style-type: none"> – x- and y-intercepts – slope – domain – range 	<ul style="list-style-type: none"> <input type="checkbox"/> identify the slope and y-intercept of a given linear function from its equation expressed in the form $y = mx + b$ <input type="checkbox"/> state the domain and range for a given graph of a linear function <input type="checkbox"/> determine the following characteristics, given the equation of a linear function in any form <ul style="list-style-type: none"> – intercepts – slope – domain – range <input type="checkbox"/> determine if a given set of data is discrete or continuous, and explain how this may affect the domain, range, and intercepts
<p>B14 sketch the graph of a linear function given its equation in the form</p> <ul style="list-style-type: none"> – $ax + by + c = 0$ (general form) – $y = mx + b$ (slope-intercept form) 	<ul style="list-style-type: none"> <input type="checkbox"/> graph a given equation using the y-intercept and slope derived from the equation <input type="checkbox"/> graph a linear function given the equation by determining the x- and y-intercepts <input type="checkbox"/> choose appropriate scales, axis labels, and graph labels when drawing graphs of linear equations <input type="checkbox"/> state the window parameters necessary to view the graph of a given linear equation using a graphing calculator <input type="checkbox"/> complete one-step or two-step manipulations of given linear equations, to enter the resulting equation into a graphing calculator <input type="checkbox"/> complete one-step or two-step manipulations of given linear equations to sketch the equation using pencil and paper
<p>B15 represent linear data, using linear function models</p>	<ul style="list-style-type: none"> <input type="checkbox"/> create a sketch of a given real-life linear relationship between variables <input type="checkbox"/> describe a real-life matching situation for the graph of given linear function, by stating the meaning of any intercepts, the slope, maxima, and/or minima
<p>B16 solve problems involving partial variation and arithmetic sequences as applications of linear functions</p>	<ul style="list-style-type: none"> <input type="checkbox"/> determine the equation for a given direct variation situation <input type="checkbox"/> determine the equation for a given partial variation situation <input type="checkbox"/> explain how direct and partial variation situations are represented by linear functions <input type="checkbox"/> graph a given direct and partial variation situation <input type="checkbox"/> explain the significance of the origin in a direct variation situation <input type="checkbox"/> solve a given real-world problem using linear equations <input type="checkbox"/> generate the graph of a given linear equation that represents an arithmetic sequence <input type="checkbox"/> explain the significance of the y-intercept given the discrete nature of given data as it relates to an arithmetic sequence

SHAPE AND SPACE

Students describe and compare everyday phenomena, using either direct or indirect measurement and describe the characteristics of 3-D objects and 2-D shapes, and analyse the relationships among them.

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>It is expected that students will:</i></p>	<p><i>The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.</i></p> <p><i>Students who have fully met the prescribed learning outcome are able to:</i></p>
<p>Measurement</p> <p>C1 solve 2-D and 3-D problems involving two right triangles</p> <hr/> <p>C2 extend the concepts of sine and cosine for angles through to 180°</p> <hr/> <p>C3 apply the sine and cosine laws to solve problems (excluding the ambiguous case)</p>	<ul style="list-style-type: none"> <input type="checkbox"/> identify a situation where basic trigonometric functions and ratios can be used to determine unknown values <input type="checkbox"/> draw additional lines that create right triangles as necessary to solve a trigonometric problem <input type="checkbox"/> apply trigonometric ratios to solve given problems; examples: <ul style="list-style-type: none"> – two right triangles, where the shared side is a given value – two-dimensional problems, where the diagram is given – two right triangles, where the shared side is determined from given information and is needed to solve the second triangle – a complex problem given in words where drawing a diagram is necessary <hr/> <ul style="list-style-type: none"> <input type="checkbox"/> use a calculator to find the values of the sine and cosine of a given angle from 0° to 180° <input type="checkbox"/> use a calculator to find the angle between 0° and 180°, given the value for the cosine of the angle <input type="checkbox"/> use a calculator to find the possible angles between 0° and 180°, given the value for the sine of the angle <input type="checkbox"/> determine the number of possible solutions for a given problem involving sine or cosine <input type="checkbox"/> use sine and cosine ratios to solve problems involving a triangle with one angle between 90° and 180° <hr/> <ul style="list-style-type: none"> <input type="checkbox"/> determine if the sine or cosine law is necessary to solve a given problem <input type="checkbox"/> use the sine or cosine law to solve problems involving triangles, given information that uniquely determines the triangle, e.g.: <ul style="list-style-type: none"> – determine any side of a triangle, given the diagram and using either the sine or cosine law provided – determine any acute angle of a triangle, given the diagram and using the sine law provided – determine any angle of a triangle, given the diagram and using the cosine law provided – determine any side or angle of a triangle, using the sine or cosine law, whether or not a diagram is given – use sine and cosine laws to solve given problems involving more than one triangle

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>3-D Objects and 2-D Shapes</i></p> <p>C4 solve problems involving distances between points in the coordinate plane</p>	<ul style="list-style-type: none"> <input type="checkbox"/> plot given points on a coordinate plane, and use the Pythagorean Theorem or distance formula to determine both exact and approximate distances between the points <input type="checkbox"/> select an appropriate strategy (e.g., using the Pythagorean Theorem or distance formula) to solve distance problems when intermediate steps are not provided
<p>C5 solve problems involving midpoints of line segments</p>	<ul style="list-style-type: none"> <input type="checkbox"/> identify problems where the solution could involve calculation of one or more midpoints <input type="checkbox"/> determine the midpoint of a line segment algebraically and graphically, given the endpoints <input type="checkbox"/> determine an endpoint of a line segment algebraically and graphically, given the other endpoint and the midpoint <input type="checkbox"/> graphically represent the solution to a problem involving midpoints
<p>C6 solve problems involving rise, run, and slope of line segments</p> <p><i>Suborganizer '3-D Objects and 2-D Shapes' continued on page 43</i></p>	<ul style="list-style-type: none"> <input type="checkbox"/> write the equation representing a given problem without using a table of values or a graph <input type="checkbox"/> find the slope of a line, given the graph of the line <input type="checkbox"/> find the slope of a line, given any two points on the line, using the slope formula $m(\overline{AB}) = \frac{y_2 - y_1}{x_2 - x_1}$ <input type="checkbox"/> explain the properties of negative and positive slopes <input type="checkbox"/> explain why vertical lines have an undefined slope and horizontal lines have a slope equal to zero <input type="checkbox"/> interpret the meaning of the slope of a given graph given the context <input type="checkbox"/> explain the meaning of the units used when stating the slope of a line

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>Suborganizer '3-D Objects and 2-D Shapes' continued from page 42</i></p> <p>C7 determine the equation of a line, given information that uniquely determines the line</p>	<ul style="list-style-type: none"> <input type="checkbox"/> write the equation of a linear function in the form $y = mx + b$, given the slope and y-intercept <input type="checkbox"/> manipulate between the given standard equation and slope-intercept form of line equations <input type="checkbox"/> identify a horizontal or vertical line given its equation in standard or slope-intercept form <input type="checkbox"/> determine the equation of a line, in slope–intercept form, given a point on the line and the slope of the line <input type="checkbox"/> determine the equation of a line, given a graph of the line with an ordered pair on the line <input type="checkbox"/> determine the slope and find the equation of the line, given two points
<p>C8 solve problems using slopes of</p> <ul style="list-style-type: none"> – parallel lines – perpendicular lines 	<ul style="list-style-type: none"> <input type="checkbox"/> determine if two lines are perpendicular or parallel, given their equations in standard form <input type="checkbox"/> determine if two lines are perpendicular or parallel, given their equations in point-intercept <input type="checkbox"/> determine if lines are parallel or perpendicular, given two points on each of the two lines <input type="checkbox"/> determine the equation of a line, given a point on the line and the equation of a parallel or perpendicular line



STUDENT ACHIEVEMENT

Principles of Mathematics 11

Key Elements: Principles of Mathematics 11

Mathematical Process (Integrated)

The following mathematical processes have been integrated within the prescribed learning outcomes and achievement indicators for the course: communication, problem solving, connections, mental mathematics and estimation, reasoning, technology, and visualization.

PATTERNS AND RELATIONS*Variable and Equations*

- linear inequalities in two variables
- systems of linear equations, including graphic and algebraic representations
- nonlinear equations
- factors of polynomials, including Remainder Theorem, Rational Zero Theorem, and Factor Theorem
- system of nonlinear equations

Relations and Functions

- characteristics of quadratic functions, including vertex, domain, range, axis of symmetry, and intercepts
- operations on functions and composition of functions
- inverse of a function, including $f^{-1}(x)$ and $\frac{1}{f(x)}$
- algebraic and graphic transformations of quadratic functions
- roots of a quadratic function, including factoring, quadratic formula, graphing, real and non-real
- characteristics of polynomial and rational functions, including domain, range, permissible and non-permissible values, and asymptotes
- solutions of absolute value equations, rational equations, and inequalities

SHAPE AND SPACE*3-D Objects and 2-D Shapes*

- distance in the coordinate plane, including point to point, point to line, and between parallel lines
- plane and coordinate geometry
- geometric properties of circles

PATTERNS AND RELATIONS

Students represent algebraic expressions in multiple ways and use algebraic and graphical models to generalize patterns, make predictions, and solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>It is expected that students will:</i></p>	<p><i>The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.</i></p> <p><i>Students who have fully met the prescribed learning outcome are able to:</i></p>
<p>Variables and Equations</p> <p>A1 graph linear inequalities, in two variables</p>	<ul style="list-style-type: none"> <input type="checkbox"/> graph the boundary line between two half planes <input type="checkbox"/> determine the solution region that satisfies a given inequality, given a boundary line using a test point (e.g., (0, 0)) <input type="checkbox"/> graph a given linear inequality expressed in the slope-intercept form (i.e., $y < mx + b$), using $<$, $>$, \leq, \geq <input type="checkbox"/> rewrite any given inequality expressed in standard form ($Ax + By < C$, using $<$, $>$, \leq, \geq) in slope-intercept form ($y < mx + b$, using $<$, $>$, \leq, \geq), where A, B, C are integral and $B > 0$ <input type="checkbox"/> explain why a solid or broken line is used to express the solution region for a given inequality <input type="checkbox"/> explain why the shaded half plane represents the solution region of the given inequality <input type="checkbox"/> rewrite any given linear inequality expressed in standard form (e.g., $Ax + By < C$) in slope-intercept form ($y < mx + b$), and graph <input type="checkbox"/> graph a given linear inequality in two variables
<p>A2 solve systems of linear equations, in two variables:</p> <ul style="list-style-type: none"> – algebraically (elimination and substitution) – graphically <p><i>Suborganizer 'Variables and Equations' continued on page 48</i></p>	<ul style="list-style-type: none"> <input type="checkbox"/> solve a given system of linear equations, using <ul style="list-style-type: none"> – elimination when only one equation needs to be multiplied by a scalar factor – substitution when one equation has a variable with a coefficient of one <input type="checkbox"/> state the solution set for a given system of parallel lines <input type="checkbox"/> state the solution set for a given system of coincident lines <input type="checkbox"/> determine the solution of a given word problem that requires a system of linear equations <input type="checkbox"/> state the window of a graphing calculator necessary to view the graphical solution of a given problem involving a system of linear equations <input type="checkbox"/> complete an elimination solution for a given problem when it is necessary to multiply both equations by a scalar factor <input type="checkbox"/> explain the method that would be the most efficient for solving a given system of linear equations and determine the solution

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>Suborganizer 'Variables and Equations' continued from page 47</i></p> <p>A3 solve systems of linear equations, in three variables:</p> <ul style="list-style-type: none"> – algebraically – with a graphing calculator 	<ul style="list-style-type: none"> <input type="checkbox"/> determine a system of equations that models a given problem <input type="checkbox"/> solve a given system of linear equations where multiplying by a constant is involved and either <ul style="list-style-type: none"> – some variables have zero as a coefficient; or – elimination can be accomplished without scalar multiplication <input type="checkbox"/> solve a given system of linear equations where <ul style="list-style-type: none"> – scalar multiplication is involved; or – no variables have zero as a coefficient
<p>A4 solve nonlinear equations</p> <ul style="list-style-type: none"> – by factoring – graphically – with a graphing calculator 	<ul style="list-style-type: none"> <input type="checkbox"/> solve a given nonlinear equation graphically by either setting one side equal to zero or finding points of intersection between the two graphs <input type="checkbox"/> solve a given polynomial equation of degree three or more using the Remainder Theorem <input type="checkbox"/> solve a given polynomial equation of degree three or more using the Factor Theorem
<p>A5 use the Remainder Theorem to evaluate polynomial expressions</p> <p>A6 use the Rational Zero Theorem and the Factor Theorem to determine factors of polynomials</p>	<ul style="list-style-type: none"> <input type="checkbox"/> determine if $(x - a)$ is a factor of a given polynomial <input type="checkbox"/> determine the remainder, if a given polynomial is divided by $(x - a)$ <input type="checkbox"/> determine one coefficient in a polynomial, given the factors <input type="checkbox"/> apply the Remainder and Factor Theorems to solve for more than one coefficient in a given polynomial
<p>A7 determine the solution to a system of nonlinear equations, using a graphing calculator as appropriate</p>	<ul style="list-style-type: none"> <input type="checkbox"/> graph a given system, and use the graphing calculator to determine an approximate solution <input type="checkbox"/> verify the given solution to a given equation algebraically
<p>Relations and Functions</p> <p>A8 determine the characteristics of the graph of a quadratic function, including</p> <ul style="list-style-type: none"> – vertex – domain and range – axis of symmetry – x- and y-intercepts <p><i>Suborganizer 'Relations and Functions' continued on page 49</i></p>	<ul style="list-style-type: none"> <input type="checkbox"/> modify the viewing window of a graphing calculator to view the characteristics of a given graph of a quadratic function, including <ul style="list-style-type: none"> – vertex – domain and range – x- and y-intercepts <input type="checkbox"/> estimate the domain and range of a given graph of a quadratic function, using the standard viewing window or the sketch of the graph <input type="checkbox"/> identify the axis of symmetry for a given graph of a quadratic function <input type="checkbox"/> write the equation for the axis of symmetry for a given graph of a quadratic function <input type="checkbox"/> determine the vertex of a given quadratic function

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>Suborganizer 'Relations and Functions' continued from page 48</i></p> <p>A9 perform operations on functions and compositions of functions</p>	<ul style="list-style-type: none"> <input type="checkbox"/> perform operations on given functions, including <ul style="list-style-type: none"> – addition – subtraction – multiplication – division <input type="checkbox"/> perform compositions of given functions (i.e., $f(g(x))$ and $(f \circ g)(x)$ for the composition of f and g) <input type="checkbox"/> explain why the domain and range may change for a composite function (e.g., composition of f and g), and identify the new domain and range
<p>A10 determine the inverse of a function</p>	<ul style="list-style-type: none"> <input type="checkbox"/> determine the inverse of a given function <input type="checkbox"/> graph $f^{-1}(x)$ and $\frac{1}{f(x)}$ of a given function and explain how they differ <input type="checkbox"/> determine if the inverse of a given function (i.e., $f^{-1}(x)$) is a function <input type="checkbox"/> determine the domain and range of the inverse function of a given function
<p>A11 connect algebraic and graphical transformations of quadratic functions, using completing the square as required</p>	<ul style="list-style-type: none"> <input type="checkbox"/> determine the values of p and q, when given the graph of the quadratic function <input type="checkbox"/> complete the square for a given quadratic function when a and p are any rational numbers <input type="checkbox"/> determine the value of a, when given the graph of the quadratic function
<p>A12 model real-world situations, using quadratic functions</p>	<ul style="list-style-type: none"> <input type="checkbox"/> determine a function that describes a given situation that can be modelled using a quadratic function <input type="checkbox"/> determine if the solution to a given situation is valid and justify
<p>A13 solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using</p> <ul style="list-style-type: none"> – factoring – the quadratic formula – graphing <p><i>Suborganizer 'Relations and Functions' continued on page 50</i></p>	<ul style="list-style-type: none"> <input type="checkbox"/> solve a given quadratic equation by factoring <input type="checkbox"/> solve a given quadratic equation using the quadratic formula <input type="checkbox"/> explain the relationship of the x-intercepts of the given graph of a function to the roots of the corresponding equation <input type="checkbox"/> determine the most efficient method (i.e., factoring, quadratic formula, graphing) to solve a given quadratic equation and explain the reasoning <input type="checkbox"/> express irrational solutions to a given quadratic equation in simplified mixed radical form

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>Suborganizer 'Relations and Functions' continued from page 49</i></p> <p>A14 determine the character of the real and non-real roots of a quadratic equation, using</p> <ul style="list-style-type: none"> – the discriminant in the quadratic formula – graphing 	<ul style="list-style-type: none"> <input type="checkbox"/> determine the discriminant for a given quadratic equation <input type="checkbox"/> describe the nature of the roots of a given quadratic equation <input type="checkbox"/> solve for parameters a, b, or c, given the nature of the roots <input type="checkbox"/> determine the values of a, b, or c in $b^2 - 4ac = 0$, where the solution has non-real roots
<p>A15 describe, graph, and analyse polynomial and rational functions, using a graphing calculator as appropriate</p>	<ul style="list-style-type: none"> <input type="checkbox"/> determine the characteristics of a given polynomial function using a graphing calculator <input type="checkbox"/> determine the characteristics of a given rational function using a graphing calculator <input type="checkbox"/> determine the domain and range of given polynomial or rational functions, including rational functions with vertical asymptotes <input type="checkbox"/> identify the horizontal and vertical asymptotes of given rational functions <input type="checkbox"/> determine the equation(s) of the horizontal or vertical asymptote(s) of given rational functions
<p>A16 formulate and apply strategies to solve absolute value equations, radical equations, rational equations, and inequalities</p>	<ul style="list-style-type: none"> <input type="checkbox"/> solve a given equation or inequalities algebraically and with the use of technology <input type="checkbox"/> solve a given radical equation that requires more than one step to eliminate the radical <input type="checkbox"/> solve a given absolute value equation that includes up to two absolute value expressions <input type="checkbox"/> solve a given polynomial inequality of the form $N(x) > k$, where k is a real number <input type="checkbox"/> solve a given two-term rational equation and verify the solution

SHAPE AND SPACE

Students describe and compare everyday phenomena, using either direct or indirect measurement, describe the characteristics of 3-D objects and 2-D shapes, and analyse the relationships among them.

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>It is expected that students will:</i></p>	<p><i>The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.</i></p> <p><i>Students who have fully met the prescribed learning outcome are able to:</i></p>
<p>3-D Objects and 2-D Shapes</p> <p>B1 solve problems involving distances between points and lines</p>	<ul style="list-style-type: none"> <input type="checkbox"/> determine the vertical distance from a given point to a line expressed in the form $y = mx + b$ <input type="checkbox"/> represent, graphically, the shortest distance from a given point to a line given in the form $y = mx + b$ <input type="checkbox"/> determine the vertical distance between two given parallel lines given in the form $y = mx + b$ <input type="checkbox"/> determine the vertical distance from a given point to a line given in the form $Ax + By + C = 0$ <input type="checkbox"/> represent, graphically, the shortest distance from a point to a line given in the form $Ax + By + C = 0$ <input type="checkbox"/> determine, algebraically, the shortest distance from a given point to a line <input type="checkbox"/> describe the procedure needed to determine the shortest distance between two parallel lines
<p>B2 verify and prove assertions in plane geometry, using coordinate geometry</p> <p><i>Suborganizer '3-D Objects and 2-D Shapes' continued on page 52</i></p>	<ul style="list-style-type: none"> <input type="checkbox"/> verify conjectures, given the coordinates of specific points, using coordinate geometry <input type="checkbox"/> differentiate between verification for special cases and proof for general cases <input type="checkbox"/> provide the diagram and complete a proof of a geometric property, using coordinate geometry, when a labelled diagram is provided <input type="checkbox"/> select and explain appropriate positions for the origin and orientations of the coordinate axes for a given problem

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p data-bbox="159 310 542 373"><i>Suborganizer '3-D Objects and 2-D Shapes' continued from page 51</i></p> <p data-bbox="159 411 568 1520"> B3 investigate the following geometric circle properties using computers with dynamic geometry software, and prove them using established concepts and theorems: <ul style="list-style-type: none"> – the perpendicular bisector of a chord contains the centre of the circle – the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc – the inscribed angles subtended by the same arc are congruent – the angle inscribed in a semicircle is a right angle – the opposite angles of a cyclic quadrilateral are supplementary – a tangent to a circle is perpendicular to the radius at the point of tangency – the tangent segments to a circle, from any external point, are congruent – the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord </p> <p data-bbox="159 1558 522 1621"><i>Suborganizer '3-D Objects and 2-D Shapes' continued on page 53</i></p>	<ul style="list-style-type: none"> <input type="checkbox"/> determine the measure of a given angle or segment when the use of one property of circles or polygons is required <input type="checkbox"/> determine the measures of given angles or segments when the use of more than one property of circles or polygons is required and the diagram is given <input type="checkbox"/> determine the measures of given angles or segments when the use of more than one property of circles or polygons is required and the diagram is not given <input type="checkbox"/> determine the measures of given angles or segments when the use of other geometric properties, such as parallel lines and congruent triangles, as well as the properties of circles or polygons are required <input type="checkbox"/> provide the verification, for one or more special cases, of the circle properties (formal proof not required) <input type="checkbox"/> demonstrate understanding of a given problem by providing a proof for the general case <input type="checkbox"/> provide reasons for the steps in a complete proof of a related conjecture

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>Suborganizer '3-D Objects and 2-D Shapes' continued from page 52</i></p> <p>B4 solve problems and justify the solution strategy using circle properties, including</p> <ul style="list-style-type: none"> – the perpendicular bisector of a chord contains the centre of the circle – the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc – the inscribed angles subtended by the same arc are congruent – the angle inscribed in a semicircle is a right angle – the opposite angles of a cyclic quadrilateral are supplementary – a tangent to a circle is perpendicular to the radius at the point of tangency – the tangent segments to a circle, from any external point, are congruent – the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord 	<ul style="list-style-type: none"> <input type="checkbox"/> provide complete solutions to a given problem requiring the properties for circles to determine the solution, and explain the strategy used <input type="checkbox"/> justify the solution strategy used to solve a given problem <input type="checkbox"/> determine the error within an incorrect solution using circle properties



STUDENT ACHIEVEMENT

Principles of Mathematics 12

Key Elements: Principles of Mathematics 12

Mathematical Process (Integrated)

The following mathematical processes have been integrated within the prescribed learning outcomes and achievement indicators for the course: communication, problem solving, connections, mental mathematics and estimation, reasoning, technology, and visualization.

PATTERNS AND RELATIONS*Patterns*

- geometric sequences
- geometric series

Variables and Equations

- solutions to exponential equations, including algebraic manipulation and substitution
- solutions to logarithmic equations, including algebraic manipulation and substitution
- exact and approximate values of trigonometric ratios using degree and radian measure
- first and second degree trigonometric equations
- general solution of trigonometric equations
- trigonometric identities and trigonometric expressions

Relations and Functions

- relationship between exponential and logarithmic functions
- exponential functions, including graphic representation, transformations, domain, range, intercepts, and asymptotes
- logarithmic functions, including bases other than 10, graphic representation, domain, range, intercepts, and asymptotes
- relationship between circle functions and the primary trigonometric ratios
- properties of the graphs of sine, cosine, and tangent functions, including amplitude, period, domain, range, asymptotes and transformations of the functions

SHAPE AND SPACE*Transformations*

- vertical and horizontal transformations of functions
- compression and expansion of functions
- reflection of functions, including reflection in the x -axis, y -axis, and line $y = x$
- properties of $y = \frac{1}{f(x)}$ including domain, range, asymptotes, and invariant points
- properties of $y = |f(x)|$ including domain, range, asymptotes, and invariant points
- single and combinations of transformations

STATISTICS AND PROBABILITY*Chance and Uncertainty*

- fundamental counting principle, permutations, combinations, binomial theorem and sample space
- independent, dependent, mutually exclusive, and complementary events
- conditional probability

PATTERNS AND RELATIONS

Students use patterns to describe the world and to solve problems; represent algebraic expressions in multiple ways; and, use algebraic and graphical models to generalize patterns, make predictions, and solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>It is expected that students will:</i></p>	<p><i>The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.</i></p> <p><i>Students who have fully met the prescribed learning outcome are able to:</i></p>
<p>Patterns</p>	
<p>A1 derive and apply expressions to represent general terms for geometric growth and to solve problems</p>	<ul style="list-style-type: none"> <input type="checkbox"/> determine the terms of a given geometric sequence <input type="checkbox"/> determine the general term of a given geometric sequence <input type="checkbox"/> solve for a, r, or n given specified terms and/or other necessary information
<p>A2 derive and apply expressions to represent sums for geometric growth and to solve problems</p>	<ul style="list-style-type: none"> <input type="checkbox"/> write the terms of a given geometric series given sigma notation <input type="checkbox"/> write the sigma notation to represent a given geometric series given the expanded terms of the series
<p>A3 estimate sums of expressions represented by infinite geometric processes where the common ratio, r, is $-1 < r < 1$</p>	<ul style="list-style-type: none"> <input type="checkbox"/> find the sum of a finite geometric sequence <input type="checkbox"/> find the sum of a geometric series given the terms or the general term <input type="checkbox"/> use sigma notation to represent the sum of a given geometric series <input type="checkbox"/> determine the number of terms in a geometric series, given the sum and other necessary information (e.g., r, a)
<p>Variables and Equations</p>	
<p>A4 solve exponential equations having bases that are powers of one another</p>	<ul style="list-style-type: none"> <input type="checkbox"/> use algebraic manipulation to solve a given exponential equation in which the base is a power of one another <input type="checkbox"/> solve for a specified term of a given geometric sequence and solve for an exponent in an exponential equation by considering tables, graphs, and common bases <input type="checkbox"/> use a graphing calculator or computer to verify the solution(s) to a given exponential equation
<p>A5 solve and verify exponential and logarithmic equations</p>	<ul style="list-style-type: none"> <input type="checkbox"/> use algebraic manipulation to solve a given exponential problem <input type="checkbox"/> use algebraic manipulation to solve a given logarithmic problem <input type="checkbox"/> verify the solution to an exponential and/or logarithmic equation by substitution
<p>A6 solve and verify exponential and logarithmic identities</p> <p><i>Suborganizer 'Variables and Equations' continued on page 58</i></p>	<ul style="list-style-type: none"> <input type="checkbox"/> use algebraic manipulation to simplify given exponential expressions and to verify identities <input type="checkbox"/> use algebraic manipulation to simplify given logarithmic expressions and to verify identities

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>Suborganizer 'Variables and Equations' continued from page 57</i></p> <p>A7 distinguish between degree and radian measure, and solve problems using both</p>	<ul style="list-style-type: none"> <input type="checkbox"/> convert a given angle measure from degrees to radians and vice versa <input type="checkbox"/> solve given problems involving arc length, radius, and angle measure in either radians or degrees
<p>A8 determine the exact and the approximate values of trigonometric ratios for any multiples of 0°, 30°, 45°, 60° and 90°, and 0 rad, $\frac{\pi}{6}$ rad, $\frac{\pi}{4}$ rad, $\frac{\pi}{3}$ rad, and $\frac{\pi}{2}$ rad</p>	<ul style="list-style-type: none"> <input type="checkbox"/> find exact values of given trigonometric ratios for special angles, θ, where $0 \leq \theta < 2\pi$ or $0^\circ \leq \theta < 360^\circ$ and verify <input type="checkbox"/> find exact values of given trigonometric ratios for any integral multiples of the special angles, θ, where $\theta < 0^\circ$ or 0 rad, or where $\theta \geq 360^\circ$ or $\theta \geq 2\pi$ rad and verify
<p>A9 solve first and second degree trigonometric equations over a specified domain</p> <ul style="list-style-type: none"> – algebraically – graphically 	<ul style="list-style-type: none"> <input type="checkbox"/> determine, algebraically and graphically <ul style="list-style-type: none"> – all solutions to first-degree equations – partial solutions to second-degree equations – partial solutions to trigonometric equations involving reciprocal trigonometric ratios – partial solutions to trigonometric equations involving multiple angles, $k\theta$, where $k = \frac{1}{3}, \frac{1}{2}, 2, 3$ in a given domain <input type="checkbox"/> explain why a given trigonometric ratio may have an undefined value <input type="checkbox"/> identify and explain the restriction on a given variable in the domain $0 \leq \theta < 2\pi$ <input type="checkbox"/> identify and explain the restrictions on a given variable in the domain $\theta \in \mathbb{R}$
<p>A10 determine the general solutions to trigonometric equations where the domain is the set of real numbers</p>	<ul style="list-style-type: none"> <input type="checkbox"/> determine the general solution of given trigonometric equations <input type="checkbox"/> determine, algebraically and graphically <ul style="list-style-type: none"> – all solutions to second-degree equations – all solutions to trigonometric equations involving reciprocal trigonometric ratios – all solutions to trigonometric equations involving multiple angles, $k\theta$, where $k = \frac{1}{3}, \frac{1}{2}, 2, 3$ in a given domain
<p>A11 analyse trigonometric identities</p> <ul style="list-style-type: none"> – graphically – algebraically for general cases <p><i>Suborganizer 'Variables and Equations' continued on page 59</i></p>	<ul style="list-style-type: none"> <input type="checkbox"/> verify given identities for a particular case <input type="checkbox"/> use algebraic manipulation to simplify and prove given identities for the general case, such as those requiring the use of conjugates or extensive use of rational operations <input type="checkbox"/> graph both sides of a given identity to verify it <input type="checkbox"/> verify sum and difference and double-angle identities of sine and cosine for a given case

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>Suborganizer 'Variables and Equations' continued from page 58</i></p> <p>A12 use sum, difference, and double angle identities for sine and cosine to verify and simplify trigonometric expressions</p>	<ul style="list-style-type: none"> <input type="checkbox"/> simplify given expressions by using sum and difference and double-angled identities of sine and cosine <input type="checkbox"/> use algebraic manipulation to prove given identities involving sum and difference and double angle identities <input type="checkbox"/> identify the values of a variable for which a given identity is undefined over the domain $0 \leq \theta < 2\pi$ or $0 \leq x < 360^\circ$
<p><i>Relations and Functions</i></p> <p>A13 change functions from exponential form to logarithmic form and vice versa</p>	<ul style="list-style-type: none"> <input type="checkbox"/> change forms between $y = b^x$ and $\log_b y = x$ <input type="checkbox"/> change forms between $y = ab^x$ and $\log_b \left(\frac{y}{a} \right) = x$ <input type="checkbox"/> show the relationship between the graph of a given exponential function and the graph of its inverse, which is a reflection in the line $y = x$ <input type="checkbox"/> explain the relationship between a given exponential function and its inverse, and determine what properties of the function have changed by taking the inverse
<p>A14 model, graph, and apply exponential functions to solve problems</p>	<ul style="list-style-type: none"> <input type="checkbox"/> find solutions to given exponential function problems when formulas are given <input type="checkbox"/> find solutions to given exponential function problems when formulas are not given and must be created algebraically <input type="checkbox"/> sketch the graph or find the equation when given a single transformation <input type="checkbox"/> sketch the graph or find the equation when given multiple transformations <input type="checkbox"/> predict the effect on the graph of a given exponential function that has had its base value altered <input type="checkbox"/> identify domain, range, x-intercepts, y-intercepts, and asymptotes from the graph or from the equation of a given exponential function
<p>A15 model, graph, and apply logarithmic functions to solve problems</p> <p><i>Suborganizer 'Relations and Functions' continued on page 60</i></p>	<ul style="list-style-type: none"> <input type="checkbox"/> identify domain, range, x- and y-intercepts, and asymptotes from the graph of a given logarithmic function of the form $y = \log_b x$ <input type="checkbox"/> graph given logarithmic functions with a base other than 10 by using technology <input type="checkbox"/> use algebraic manipulation to solve given exponential and logarithmic equations <input type="checkbox"/> recognize extraneous solutions when solving given exponential or logarithmic equations <input type="checkbox"/> complete the solution to a given problem that can be represented by logarithmic or exponential functions <input type="checkbox"/> compare values, such as earthquake intensity or pH scale, given the equation and exponents <input type="checkbox"/> solve for a value or exponent in given comparison problems

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>Suborganizer 'Relations and Functions' continued from page 59</i></p> <p>A16 describe the three primary trigonometric functions as circular functions with reference to the unit circle and an angle in standard position</p>	<ul style="list-style-type: none"> <input type="checkbox"/> find exact values for any of the six trigonometric ratios (i.e., three primary trigonometric functions plus their reciprocal functions), given a point on the terminal arm of the angle in standard position <input type="checkbox"/> find any other trigonometric ratios given one ratio and a quadrant specification <input type="checkbox"/> find other possible trigonometric ratios given one ratio and no quadrant specification <input type="checkbox"/> find an angle in standard position such that its terminal arm passes through a given point
<p>A17 sketch and analyse the graphs of sine, cosine, and tangent functions, for</p> <ul style="list-style-type: none"> – amplitude, if defined – period – domain and range – asymptotes, if any – behaviour under transformations 	<ul style="list-style-type: none"> <input type="checkbox"/> sketch a graph for given primary trigonometric functions and analyse the graph for features such as domain, range, amplitude, period, and asymptotes <input type="checkbox"/> create a graph using technology for given primary trigonometric functions and analyse the graph for features such as domain, range, amplitude, period, and asymptotes <input type="checkbox"/> sketch the graph for given reciprocal trigonometric functions <input type="checkbox"/> create a graph using technology for a given reciprocal trigonometric function <input type="checkbox"/> analyse the graph of given reciprocal trigonometric functions and state the domain, range, period, and asymptotes of the function <input type="checkbox"/> perform single transformations involving a horizontal or vertical stretch of a given reciprocal trigonometric function <input type="checkbox"/> perform both a horizontal and vertical stretch on a given reciprocal trigonometric function
<p>A18 use trigonometric functions to model and solve problems</p>	<ul style="list-style-type: none"> <input type="checkbox"/> graph a sinusoidal curve (i.e., sine or cosine) to model a problem, given the equation <input type="checkbox"/> determine the equation of a sinusoidal curve (i.e., sine or cosine), given either a description of a real-world problem or a graphical representation of the problem <input type="checkbox"/> create a graph of a sinusoidal curve (i.e., sine or cosine), given a description of a real-world application

SHAPE AND SPACE

Students perform, analyse, and create transformations of functions and relations that are described by equations or graphs.

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>It is expected that students will:</i></p>	<p><i>The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.</i></p> <p><i>Students who have fully met the prescribed learning outcome are able to:</i></p>
<p>Transformations</p> <p>B1 describe how vertical and horizontal translations of functions affect graphs and their related equations:</p> <ul style="list-style-type: none"> - $y = f(x - h)$ - $y - k = f(x)$ 	<ul style="list-style-type: none"> <input type="checkbox"/> determine and describe the effects of a given transformation on the domain, the range, and the intercepts, and identify invariant points of the relation or function <input type="checkbox"/> match a given algebraic function to its graph <input type="checkbox"/> determine the equation of a transformed function given its graph <input type="checkbox"/> use function notation to describe a given transformation <input type="checkbox"/> perform, analyse, and describe a horizontal translation ($y = f(x - h)$) and/or a vertical translation ($y - k = f(x)$) graphically or algebraically given the function f in equation or graphical form
<p>B2 describe how compressions and expansions of functions affect graphs and their related equations:</p> <ul style="list-style-type: none"> - $y = af(x)$ - $y = f(kx)$ 	<ul style="list-style-type: none"> <input type="checkbox"/> perform, analyse, and describe a horizontal stretch about the y-axis ($y = f(kx)$, where $k > 0$) or a vertical stretch about the x-axis ($y = af(x)$, where $a > 0$) graphically or algebraically given the function $y = f(x)$ in equation or graphical form <input type="checkbox"/> perform, analyse, and describe a horizontal or vertical stretch about a line other than the x- or y-axis given the function $y = f(x)$ in graphical form
<p>B3 describe how reflections of functions in both axes and in the line $y = x$ affect graphs and their related equations:</p> <ul style="list-style-type: none"> - $y = f(-x)$ - $y = -f(x)$ - $y = f^{-1}(x)$ 	<ul style="list-style-type: none"> <input type="checkbox"/> perform, analyse, and describe a horizontal stretch and reflection about the y-axis ($y = f(kx)$, where $k < 0, k \neq -1$) or a vertical stretch and reflection about the x-axis ($y = af(x)$, where $a < 0, a \neq -1$), graphically or algebraically given the function $y = f(x)$ in equation or graphical form <input type="checkbox"/> perform, analyse, and describe a reflection in the x-axis ($y = -f(x)$) and/or in the y-axis ($y = f(-x)$), or in the line $y = x$ ($y = f^{-1}(x)$) graphically or algebraically given the function $y = f(x)$ in equation or graphical form
<p>B4 using the graph and/or the equation of $f(x)$, describe and sketch $\frac{1}{f(x)}$</p> <p><i>Suborganizer 'Transformations' continued on page 62</i></p>	<ul style="list-style-type: none"> <input type="checkbox"/> sketch and describe $y = \frac{1}{f(x)}$ given the equation or graph of $f(x)$ <input type="checkbox"/> determine the domain, range, vertical asymptotes, and invariant points of $y = \frac{1}{f(x)}$ <input type="checkbox"/> analyse and determine the equation of $y = \frac{1}{f(x)}$, given the graph of $f(x)$

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>Suborganizer 'Transformations' continued from page 61</i></p> <p>B5 using the graph and/or the equation of $f(x)$, describe and sketch $f(x)$</p>	<ul style="list-style-type: none"> <input type="checkbox"/> sketch and describe $y = f(x)$ given the equation or graph of $f(x)$ <input type="checkbox"/> determine the domain, range, vertical asymptotes, and invariant points of $y = f(x)$ <input type="checkbox"/> analyse and determine the equation of $y = f(x)$, given the graph of $f(x)$
<p>B6 describe and perform single transformations and combinations of transformations on functions and relations</p>	<ul style="list-style-type: none"> <input type="checkbox"/> perform, analyse, and describe the combination of transformations on a given function or relation not involving a reflection <input type="checkbox"/> perform, analyse, and describe the combination of transformations on a given function or relation involving reflection

STATISTICS AND PROBABILITY

Students solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations, combinations, and combining of simpler probabilities.

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>It is expected that students will:</i></p>	<p><i>The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.</i></p> <p><i>Students who have fully met the prescribed learning outcome are able to:</i></p>
<p>Chance and Uncertainty</p>	
<p>C1 use the fundamental counting principle to determine the number of different ways to perform multi-step operations</p>	<ul style="list-style-type: none"> <input type="checkbox"/> apply the Fundamental Counting Principle to a given multi-step problem <input type="checkbox"/> recognize and address constraints and ambiguities within multi-step problems
<p>C2 use factorial notation to determine different ways of arranging n distinct objects in a sequence</p>	<ul style="list-style-type: none"> <input type="checkbox"/> evaluate a given factorial using technology <input type="checkbox"/> simplify a fraction that contains a factorial in both the numerator and denominator <input type="checkbox"/> use factorial notation to solve given problems
<p>C3 determine the number of permutations of n different objects taken r at a time, and use this to solve problems</p>	<ul style="list-style-type: none"> <input type="checkbox"/> write ${}_n P_r$ using factorial notation <input type="checkbox"/> calculate the number of permutations of n things taken r at a time using ${}_n P_r$ <input type="checkbox"/> obtain the solution to a given problems involving a single case or constraint <input type="checkbox"/> determine the solution to a given problem involving two or more cases or constraints
<p>C4 determine the number of combinations of n different objects taken r at a time, and use this to solve problems</p>	<ul style="list-style-type: none"> <input type="checkbox"/> write ${}_n C_r$ using factorial notation <input type="checkbox"/> calculate the number of combinations of n things taken r at a time using ${}_n C_r$
<p>C5 solve problems, using the binomial theorem where the exponent n belongs to the set of natural numbers</p>	<ul style="list-style-type: none"> <input type="checkbox"/> expand $(x + y)^n$, where $n \in N$ <input type="checkbox"/> determine specified terms of an expansion with linear terms within the binomial
<p>C6 construct a sample space for up to three events</p>	<ul style="list-style-type: none"> <input type="checkbox"/> identify all possible outcomes of an experiment that includes 1, 2, or 3 events <input type="checkbox"/> determine if a given event is included in the sample space for a probability experiment involving up to three events
<p>C7 classify events as independent or dependent</p>	<ul style="list-style-type: none"> <input type="checkbox"/> identify the characteristics of dependent and independent events <input type="checkbox"/> use the characteristics of dependent and independent events to classify a given set of events
<p><i>Suborganizer 'Chance and Uncertainty' continued on page 64</i></p>	

Prescribed Learning Outcomes	Suggested Achievement Indicators
<p><i>Suborganizer 'Chance and Uncertainty' continued from page 63</i></p> <p>C8 solve problems, using the probabilities of mutually exclusive and complementary events</p>	<ul style="list-style-type: none"> <input type="checkbox"/> identify the characteristics of mutually exclusive and complementary events <input type="checkbox"/> use the characteristics of mutually exclusive events and complementary events to classify a given set of events <input type="checkbox"/> determine the probability of two mutually exclusive or complementary events occurring (e.g., probability of event A or B: $P(A \text{ or } B) = P(A) + P(B)$ for mutually exclusive events; $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ for events that are not mutually exclusive)
<p>C9 determine the conditional probability of two events</p>	<ul style="list-style-type: none"> <input type="checkbox"/> determine the conditional probability of two events that are dependent or independent (e.g., probability of event A and B: $P(A \text{ and } B) = P(A) \times P(B A)$ for dependent events and $P(A \text{ and } B) = P(A) \times P(B)$ for independent events)
<p>C10 solve probability problems involving permutations, combinations, and conditional probability</p>	<ul style="list-style-type: none"> <input type="checkbox"/> solve for n in equations involving ${}_n C_r$ or ${}_n P_r$, given r <input type="checkbox"/> solve problems involving both permutations and combinations



LEARNING RESOURCES

Principles of Mathematics 10 to 12

This section contains general information on learning resources, and provides a link to the titles, descriptions, and ordering information for the recommended learning resources in the Principles of Mathematics 10 to 12 Grade Collections.

What Are Recommended Learning Resources?

Recommended learning resources are resources that have undergone a provincial evaluation process using teacher evaluators and have Minister's Order granting them provincial recommended status. These resources may include print, video, software and CD-ROMs, games and manipulatives, and other multimedia formats. They are generally materials suitable for student use, but may also include information aimed primarily at teachers.

Information about the recommended resources is organized in the format of a Grade Collection. A Grade Collection can be regarded as a "starter set" of basic resources to deliver the curriculum. In many cases, the Grade Collection provides a choice of more than one resource to support curriculum organizers, enabling teachers to select resources that best suit different teaching and learning styles. Teachers may also wish to supplement Grade Collection resources with locally approved materials.

How Can Teachers Choose Learning Resources to Meet Their Classroom Needs?

Teachers must use either

- provincially recommended resources OR
- resources that have been evaluated through a local, board-approved process

Prior to selecting and purchasing new learning resources, an inventory of resources that are already available should be established through consultation with the school and district resource centres. The ministry also works with school districts to negotiate cost-effective access to various learning resources.

What Are the Criteria Used to Evaluate Learning Resources?

The Ministry of Education facilitates the evaluation of learning resources that support BC curricula,

and that will be used by teachers and/or students for instructional and assessment purposes. Evaluation criteria focus on content, instructional design, technical considerations, and social considerations.

Additional information concerning the review and selection of learning resources is available from the ministry publication, *Evaluating, Selecting and Managing Learning Resources: A Guide* (Revised 2002).
www.bced.gov.bc.ca/irp/resdocs/esm_guide.pdf

What Funding is Available for Purchasing Learning Resources?

As part of the selection process, teachers should be aware of school and district funding policies and procedures to determine how much money is available for their needs. Funding for various purposes, including the purchase of learning resources, is provided to school districts. Learning resource selection should be viewed as an ongoing process that requires a determination of needs, as well as long-term planning to co-ordinate individual goals and local priorities.

What Kinds of Resources Are Found in a Grade Collection?

The Grade Collection charts list the recommended learning resources by media format, showing links to the curriculum organizers and suborganizers. Each chart is followed by an annotated bibliography. Teachers should check with suppliers for complete and up-to-date ordering information. Most suppliers maintain web sites that are easy to access.

PRINCIPLES OF MATHEMATICS 10 TO 12 GRADE COLLECTIONS

The Grade Collections for Principles of Mathematics 10 to 12 include both newly recommended learning resources, as well as relevant resources previously recommended for prior versions of the Principles of Mathematics 10 to 12 curriculum. The ministry updates the Grade Collections on a regular basis as new resources are developed and evaluated.

Please check the following ministry web site for the most current list of recommended learning resources in the Principles of Mathematics 10 to 12 Grade Collections: www.bced.gov.bc.ca/irp_resources/lr/resource/gradcoll.htm

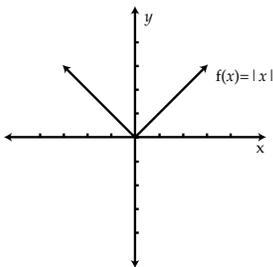


GLOSSARY

Principles of Mathematics 10 to 12

This appendix provides an illustrated glossary of terms used in this Integrated Resource Package. The terms and definitions are intended to be used by readers unfamiliar with mathematical terminology. For a more complete definition of each term, refer to a mathematical dictionary such as the *Nelson Canadian School Mathematics Dictionary* (ISBN 17-604800-6).

A



absolute value of a number

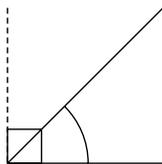
How far the number is from 0. Example: the absolute values of -4.2 and of 4.2 are each 4.2.

absolute value function

The function f defined by $f(x) = |x|$, where $|x|$ denotes the absolute value of x .

accuracy

A measure of how far an estimate is from the true value.

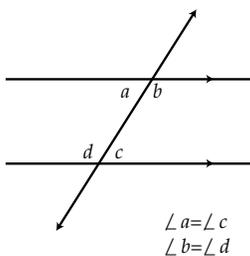


acute angle

An angle whose measure is between 0° and 90° .

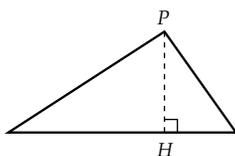
algorithm

A mechanical method for solving a certain type of problem, often a method in which one kind of step is repeated a number of times.



alternate interior angles

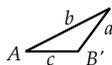
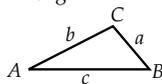
In the diagram to the left, the angles labelled a and c are alternate interior angles, as are the angles b and d .



altitude of a triangle

A line segment PH , where P is a vertex of the triangle, H lies on the line through the other two vertices, and PH is perpendicular to that line.

Given a , b , and $\angle A$
Find length c



Solution(s) $c=AB$
 $c=AB'$

ambiguous case

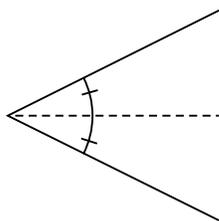
Two sides of a triangle and the angle opposite one of them are specified, and we want to calculate the remaining angles or side. There may be no solution, exactly one, or exactly two.

amplitude (of a periodic curve)

The maximum displacement from a reference level in either a positive or negative direction. That reference level is often chosen halfway between the biggest and smallest values taken on by the curve.

analytic geometry (coordinate geometry)

An approach to geometry in which position is indicated by using coordinates, lines, and curves, and other objects are represented by equations, and algebraic techniques are used to solve geometric problems.



angle bisector

A line that divides an angle into two equal parts.

antiderivative

If $f(x)$ is the derivative of $F(x)$, then $F(x)$ is an *antiderivative* of $f(x)$. *Indefinite integral* means the same thing.

antidifferentiation

The process of finding antiderivatives.

arc

A connected segment of a circle or curve.

arc sine (of x)

The angle (in radians) between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is x .
Notation: $\sin^{-1} x$ or $\arcsin x$.

arc tangent

The angle (in radians) between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose tangent is x .
Notation: $\tan^{-1} x$ or $\arctan x$.

arithmetic operation

Addition, subtraction, multiplication, and division.

arithmetic sequence (arithmetic progression)

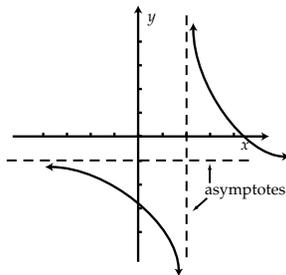
A sequence in which any term is obtained from the preceding term by adding a fixed amount, the *common difference*.

If a is the first term and d the common difference, then the sequence is $a, a + d, a + 2d, a + 3d, \dots$. The n -th term t_n is given by the formula $t_n = a + (n - 1)d$.

arithmetic series

The sum S_n of the first n terms of an arithmetic sequence. If the sequence has first term a and common difference d , then

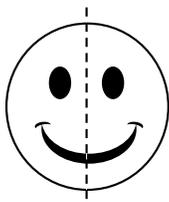
$$S_n = \frac{1}{2}n[2a + (n - 1)d] = \frac{1}{2}n(a + l)$$
 where l is $a + (n - 1)d$, the "last" term.

**asymptote (to a curve)**

A line l such that the distance from points P on the curve to l approaches zero as the distance of P from an origin increases without bound as P travels on a certain path along the curve.

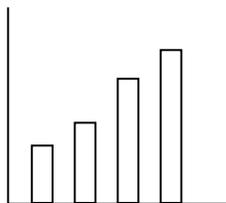
average velocity

The net change in position of a moving object divided by the elapsed time.

**axis of symmetry (of a geometric figure)**

A line such that for any point P of the figure, the mirror image of P in the line is also in the figure.

B

**bar graph**

A graph using parallel bars (vertical or horizontal) that are proportional in length to the data they represent.

base

In the expression s^t , the number or expression s is called the base, and t is the exponent. In the expression $\log_a u$, the base is a .

binomial

The sum of two monomials.

binomial distribution

The probabilities associated with the number of successes when an experiment is repeated independently a fixed number of times. For example, the number of times a six is obtained when a fair die is tossed 100 times has a binomial distribution.

Binomial Theorem

A rule for expanding expressions of the form $(x + y)^n$.

bisect

To divide into two equal parts.

broken-line graph

A graph using line segments to join the plotted points to represent data.

C

Cartesian (rectangular) coordinate system

A coordinate system in which the position of a point is specified by using its signed distances from two perpendicular reference lines (axes).

central angle

An angle determined by two radii of a given circle; equivalently, an angle whose vertex is at the centre of the circle.

chain rule

A rule for differentiating composite functions.

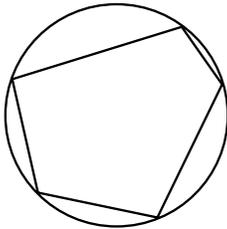
If $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$.

chord

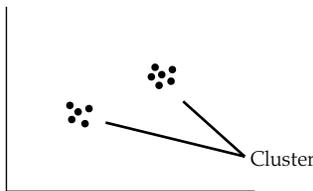
The line segment that joins two points on a curve, usually a circle.

circumference

The boundary of a closed curve, such as a circle; also, the measure (length) of that boundary. Please see *perimeter*.

**circumscribed**

The polygon P is *circumscribed* about the circle C if P is inside C and the edges of P are tangent to C . The circle C is *circumscribed* about the polygon Q if Q is inside C and the vertices of Q are on the boundary of C . The notion can be extended to other figures, and to three dimensions.

**cluster**

A collection of closely grouped data points.

coefficient

A numerical or constant multiplier in an algebraic expression. The coefficient of x^2 in $4x^2 - 2axy$ is 4, and the coefficient of xy is $-2a$.

collinear

Lying on the same line.

combination

A set of objects chosen from another set, with no attention paid to the order in which the objects are listed (see also *permutation*). The number of possible combinations of r objects selected from a set of n distinct objects is ${}_n C_r$, pronounced “ n choose r .”

common factor (CF)

A number that is a factor of two or more numbers. For example, 3 is a common factor of 6 and 12. *Common divisor* means the same thing. The term is also used with polynomials. For example, $x - 1$ is a common factor of $x^2 - x$ and $x^2 - 2x + 1$.

common fraction

A number written as $\frac{a}{b}$, where the numerator a and the denominator b are integers, and b is not zero. Examples: $\frac{4}{5}$, $\frac{-13}{6}$, $\frac{3}{1}$.

compass

An instrument for drawing circles or arcs of circles.

complementary angles

Two angles that add up to a right angle.

completing the square

Rewriting the quadratic polynomial $ax^2 + bx + c$ in the form $a(x - p)^2 + q$, perhaps to solve the equation $ax^2 + bx + c = 0$.

complex fraction

A fraction in which the numerator or the denominator, or both, contain fractions.

composite function

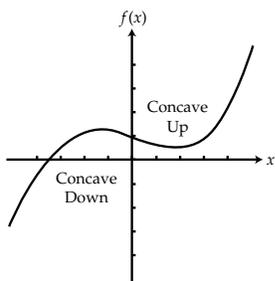
A function $h(x)$ obtained from two functions f and g by using the rule $h(x) = f(g(x))$ (first do g to x , then do f to the result).

composite number

An integer greater than 1 that is not prime, such as 9 or 14.

compound interest

The interest that accumulates over a given period when each successive interest payment is added to the principal in order to calculate the next interest payment.

**concave down (or downward)**

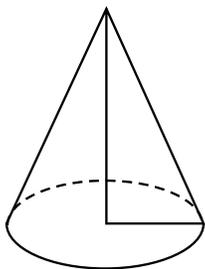
The function $f(x)$ is concave down on an interval if the graph of $y = f(x)$ lies below its tangent lines on that interval.

concave up (or upward)

The function $f(x)$ is concave up on an interval if the graph of $y = f(x)$ lies above its tangent lines on that interval.

conditional probability

The probability of an event given that another event has occurred. The (conditional) probability that someone earns more than \$200,000 a year, given that the person plays in the NHL, is different from the probability that a randomly chosen person earns more than \$200,000.

**cone (right circular)**

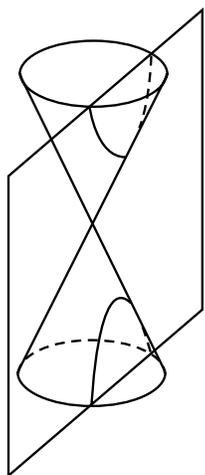
The three-dimensional object generated by rotating a right triangle about one of its legs.

confidence interval(s)

An interval that is believed, with a preassigned degree of confidence, to include the particular value of some parameter being estimated.

congruent

Having identical shape and size.

**conic section**

A curve formed by intersecting a plane and the surface of a double cone. Apart from degenerate cases, the conic sections are the ellipses, the parabola, and the hyperbolas.

conjecture

A mathematical assertion that is believed, at least by some, to be true, but has not been proved.

constant

A fixed quantity or numerical value.

continuous data

Data that can, in principle, take on any real value in some interval. For example, the exact height of a randomly chosen individual, or the exact length of life of a U-235 atom can be modelled by a continuous distribution.

continuous function

Informally, a function $f(x)$ is *continuous* at a if $f(x)$ does not make a sharp jump at a . More formally, $f(x)$ is continuous at a if $f(x)$ approaches $f(a)$ as x approaches a .

contrapositive

The contrapositive of “Whenever A is true, B must be true” is “Whenever B is false, then A must be false.” Any assertion is logically equivalent to its contrapositive, so one strategy for proving an assertion is to prove its contrapositive.

converse (of a theorem)

The assertion obtained by interchanging the premise and the conclusion. If the theorem is “Whenever A happens, B must happen,” then its converse is “Whenever B happens, A must happen.” The converse of a theorem need not be true.

coordinate geometry

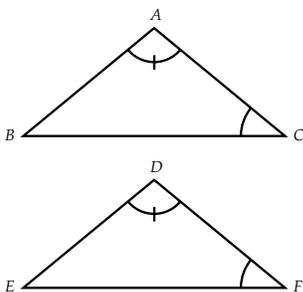
Please see *analytic geometry*.

coordinates

Numbers that uniquely identify the position of a point relative to a coordinate system.

correlation coefficient

A number (between -1 and 1) that measures the degree to which a collection of data points lies on a line.

**corresponding angles and corresponding sides**

Angles or sides that have the same relative position in geometric figures.

cosecant (of x)

This is $\frac{1}{\sin x}$. Notation: $\csc x$.

cosine law (law of cosines)

A formula used for solving triangles in plane geometry.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

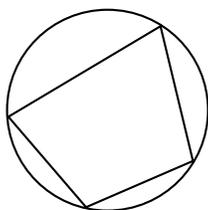
cosine (function)

See *primary trigonometric functions*.

cotangent (of x)

This is $\frac{1}{\tan x}$. Notation: $\cot x$.

D

**coterminal angles**

Angles that are rotations between the same two lines, termed the initial and terminal arms. For example: 20° , -340° , 380° are coterminal angles.

critical number (of a function)

A number where the function is defined, and where the derivative of the function is equal to 0 or doesn't exist.

cyclic (inscribed) quadrilateral

A quadrilateral whose vertices all lie on a circle.

decimal fraction

In principle, a fraction $\frac{a}{b}$ where a is an integer and b is a power of 10.

For example, $\frac{1}{4} = \frac{25}{100}$, so $\frac{1}{4}$ can be expressed as a decimal fraction, usually written as 0.25.

decreasing function

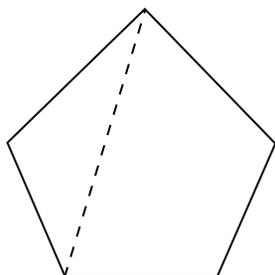
The function $f(x)$ is decreasing on an interval for any numbers s and t in that interval, if t is greater than s then $f(t)$ is less than $f(s)$.

deductive reasoning

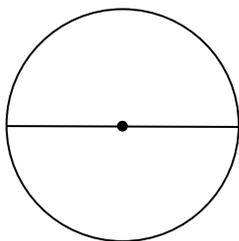
A process by which a conclusion is reached from certain assumptions by the use of logic alone.

degree

The highest power or sum of powers that occurs in any term of a given polynomial or polynomial equation. For example, $6x + 17$ has degree 1, and $2 + x^3 + 7x$ has degree 3, as does $2 + 6x + 7y + xy^2$.

**diagonal**

A line segment that joins two non-adjacent vertices in a polygon or polyhedron.

**diameter**

A line segment that joins two points on a circle or sphere and passes through the centre. All diameters of a circle or sphere have the same length. That common length is called the diameter.

difference of squares

An expression of the form $A^2 - B^2$, where A and B are numbers, polynomials, or perhaps other mathematical expressions. We can factor $A^2 - B^2$ as $(A+B)(A-B)$.

differentiable

A function is *differentiable* at $x = a$ if under extremely high magnification, the graph of the function looks almost like a straight line near a . Most familiar functions are differentiable everywhere that they are defined.

differential equation

An equation that involves only two variable quantities, say x and y , and the first derivative, or higher derivatives, of y with respect to x .

Example: $3y^2 \frac{dy}{dx} = e^x$.

differentiate; differentiation

To find the derivative; the process of finding derivatives.

direct variation

The quantity Q varies directly with x if $Q = ax$ for some constant a . This can be contrasted with *inverse* variation, in which $Q = \frac{a}{x}$ for some a .

discrete data

Data arising from situations in which the possible outcomes lie in a finite or infinite sequence.

discriminant

The *discriminant* of the quadratic polynomial $ax^2 + bx + c$ (or of the equation $ax^2 + bx + c = 0$) is $b^2 - 4ac$.

displacement

Position, as measured from some reference point.

distance formula

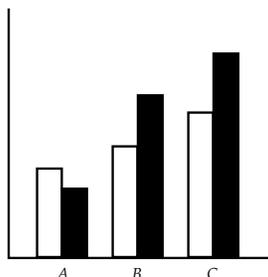
The formula used in coordinate geometry to find the distance between two points. If A has coordinates (x_1, y_1) and B has coordinates (x_2, y_2) ,

then the distance from A to B is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

domain (of a function)

The set of numbers where the function is defined. For example, if

$f(x) = \frac{\sqrt{x-2}}{x-5}$, then the domain of $f(x)$ consists of all real numbers greater than or equal to 2, except for the number 5.

**double bar graph**

A bar graph that uses bars to represent two sets of data visually.

E**edge**

The straight line segment that is formed where two faces of a polyhedron meet.

ellipse

A closed curve obtained by intersecting the surface of a cone with a plane. Please see *conic section*.

equation

A statement that two mathematical expressions are equal, such as $3x + y = 7$.

equidistant

Having equal distances from some specified object, point, or line.

estimate

v. To approximate a quantity, perhaps only roughly.

n. The result of estimating. Also, an approximation, based on sampling, to some number associated with a population, such as the average age.

Euclidean geometry

Geometry based on the definitions and axioms set out in Euclid's Elements.

event

A subset of the sample space of all possible outcomes of an experiment.

experimental probability

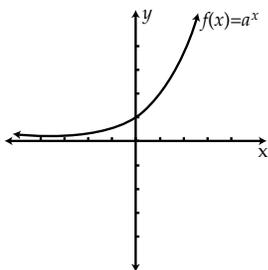
An estimate of the probability of an event obtained by repeating an experiment many times. If the event occurred in k of the n experiments, it has experimental probability k / n .

exponent

The number that indicates the power to which the base is raised. For example: 3^4 : exponent is 4.

exponential decay

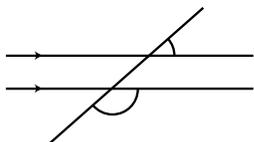
A quantity undergoes exponential decay if its rate of decrease at any time is proportional to its size at the time. Exponential decay models well the decay of radioactive substances.

**exponential function**

An exponential function is a function of the form $f(x) = a^x$, where $a > 0$ and the variable x occurs as the exponent. The exponential function is the function $f(x) = e^x$, where e is a mathematical constant roughly equal to 2.7182818284.

exponential growth

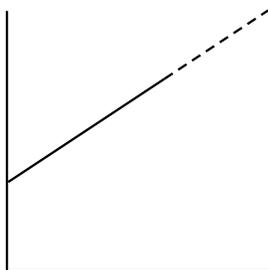
A quantity undergoes exponential growth if its rate of increase at any time is proportional to its size at the time. Exponential growth models well the growth of a population of bacteria under ideal conditions.

**exterior angles on the same side of the transversal**

A transversal of two parallel lines forms two supplementary exterior angles.

extraneous root

A spurious root obtained by manipulating an equation. For example, if we square both sides of $1 - x + \sqrt{x - 1}$ and simplify, we obtain $(x - 1)(x - 2) = 0$, that is, $x = 1$ or $x = 2$. Since 2 is not a root of the original equation, it is sometimes called an extraneous root.

**extrapolate**

Estimate the value of a function at a point from values at places on one side of the point only.

extreme values

The highest and lowest numbers in a set.

F

face

One of the plane surfaces of a polyhedron.

factor

n. A *factor* of a number n is a number (usually taken to be positive) that divides n exactly. For example, the factors of 18 are 1, 2, 3, 6, 9, and 18. Similarly, a factor of a polynomial $P(x)$ is a polynomial that divides $P(x)$ exactly. Thus x and $x - 1$ are two of the factors of $x^3 - x$.

v. To *factor* a number or polynomial is to express it as a product of basic terms. For example, $x^3 - x$ factors as $x(x - 1)(x + 1)$.

Factor Theorem

If $P(x)$ is a polynomial, and a is a root of the equation $P(x) = 0$, then $x - a$ is a factor of $P(x)$.

factor(s)

Numbers multiplied to produce a specific product. For example: $2 \times 3 \times 3 = 18$: factors are 2 and 3; $(x - 2)$ and $(x + 1)$ are factors of $x^2 + x - 2$.

first-hand data

Data collected by an individual directly from observations or measurements.

flip

Another word for *reflection*.

frequency diagram

A diagram used to record the number of times various events occurred.

function

A rule that produces, for any element x of a certain set A , an object $f(x)$. The set A is the *domain* of the function; the set of values taken on by $f(x)$ is the *range* of the function.

More formally, a function is a collection of ordered pairs (x, y) such that the second entry y is completely determined by the first entry x .

function notation

If a quantity y is completely determined by a quantity x , then y is called a function of x . For example, the area of a circle of general radius x might be denoted by $A(x)$ (pronounced "A of x .) In this case, $A(x) = \pi x^2$.

fundamental counting principle

If an event can happen in x different ways, and for each of these ways a second event can happen in y different ways, then the two events can happen in $x \times y$ different ways.

G

general polynomial equation

An equation of the form $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n = 0$.

general term (of a sequence)

If n is unspecified, a_n is called the *general term* of the sequence a_1, a_2, a_3, \dots . Sometimes there is an explicit formula for a_n in terms of n .

geometric sequence (progression)

A sequence in which each term except the first is a fixed multiple of the preceding term. If the first term is a , and each term is r times the previous one (r is the *common ratio*), then the general term t_n is given by $t_n = ar^{n-1}$.

geometric series

The sum S_n of the first n terms of a geometric sequence. If a is the first

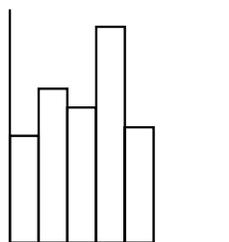
term, r the common ratio, where $r \neq 1$, then $S_n = \frac{a(1-r^n)}{1-r}$.

See also *infinite geometric series*.

greatest common factor (GCF)

The largest positive integer that divides two or more given numbers. For example, the GCF of 12 and 18 is 6. The GCF is also called the greatest common divisor (GCD).

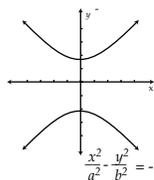
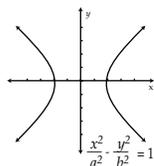
H

**higher derivatives**

The derivative of the derivative of $f(x)$, the derivative of the derivative of the derivative of $f(x)$, and so on.

histogram

A bar graph showing the frequency in each class using class intervals of the same length.

**hyperbola**

A curve with two branches where a plane and a circular conical surface meet. Please see *conic section*.

I

hypotenuse

The side opposite the right angle in a right triangle.

hypothesis

A statement or condition from which consequences are derived.

identity

A statement that two mathematical expressions are equal for all values of their variables.

if-then proposition

A mathematical statement that asserts that *if* certain conditions hold, *then* certain other conditions hold.

Imperial measure

The system of units (foot, pound, and so on) for measuring length, mass, and so on that was once the legal standard in Great Britain.

implicit function

A function y of x defined by a formula of shape $H(x, y) = 0$.

For example, $y^3 - x^2 + 1 = 0$ defines y implicitly as a function of x .

In this case, y is given *explicitly* by $y = (x^2 - 1)^{\frac{1}{3}}$. But often (example: $H(x, y) = y^7 + (x^2 + 1)y - 1$), it is not possible to give an explicit formula for y .

improper fraction

A *proper fraction* is a fraction whose numerator is less in absolute value than its denominator. An *improper fraction* is a fraction that is not a proper fraction.

increasing function

The function $f(x)$ is increasing on an interval for any numbers s and t in that interval, if t is greater than s then $f(t)$ is greater than $f(s)$.

indefinite integral

Another word for *antiderivative*.

independent events

Two events are *independent* if whether or not one of them occurs has no effect on the probability that the other occurs.

inductive reasoning

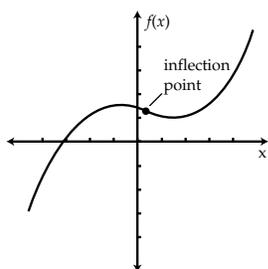
A form of reasoning in which the truth of an assertion in some particular cases is used to leap to the (tentative) conclusion that the assertion is true in general.

inequality

A mathematical statement that one quantity is greater than or less than the other. The statement $s > t$ means that s is greater than t , while $s < t$ means that s is less than t .

infinite geometric series

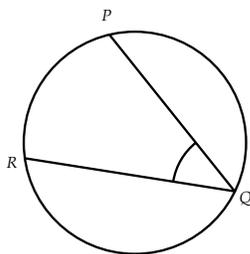
The sum $a + ar + ar^2 + \dots + ar^{n-1} + \dots$ of all of the terms of a geometric sequence. If $|r| < 1$, then this sum is equal to $\frac{a}{1-r}$.

**inflection point**

A point on a curve that separates a part of the curve that is concave up from one that is concave down.

initial value problem

A function is described by specifying a differential equation that it satisfies, together with the value of the function at some "initial" point; the problem is to find the function.

**inscribed angle**

The angle PQR , where P , Q , and R are three points on a curve, in most cases a circle.

instantaneous velocity (at a particular time)

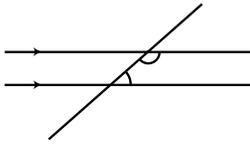
The exact rate at which the position is changing at that time.

integer

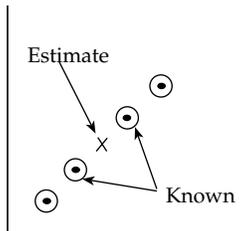
One of 0, 1, -1, 2, -2, 3, -3, 4, -4, and so on.

integration

In part, the process of finding antiderivatives.

**interior angles on the same side of the transversal**

The transversal of two parallel lines forms interior supplementary angles.

**interpolate**

Estimate the value of a function at a point from values of the function at places on both sides of the point.

intersection

The point or points where two curves meet.

interval

The set of all real numbers between two given numbers, which may or may not be included. The set of all real numbers from a given point on, or up to a given point, is also an interval, as is the set of all real numbers.

inverse (of a function)

The function $g(x)$ is the inverse of the function $f(x)$ if $f(g(x)) = x$ and $g(f(x)) = x$ for all x , or more informally if each function undoes what the other did.

inverse operations

Operations that counteract each other. For example, addition and subtraction are inverse operations.

inverse trigonometric functions

Inverses of the six basic trigonometric functions. For the two most commonly used, please see *arc sine* and *arc tan*.

irrational number

A number that cannot be expressed as a quotient of two integers. For example, $\sqrt{2}$, π , and e are irrational numbers.

irregular

Lacking in symmetry or pattern.

isosceles triangle

A triangle that has two or more equal sides. Occasionally defined as a triangle that has *exactly* two equal sides.

L

least squares

A criterion used to find the line of best fit, namely that the sum of the squares of the differences between “predicted values” and actual values should be as small as possible.

limit

The *limit* of $f(x)$ as x approaches a (notation: $\lim_{x \rightarrow a} f(x)$) is the number that $f(x)$ tends to as x moves closer and closer to a . There may not be such a number. For example, if x is measured in radians, $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, but $\lim_{x \rightarrow 0} \frac{1}{x}$ does not exist.

line of best fit

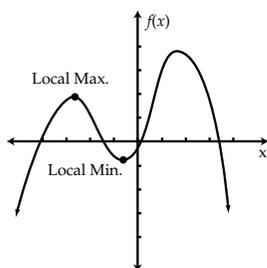
For a collection of points in the plane obtained from an experiment, a line that comes in some sense closest to the points. Please see *least squares*.

linear function

A function f given by a formula of the type $f(x) = ax + b$, where a and b are specific numbers.

linear programming

Finding the largest or smallest value taken on by a given function $a_1x_1 + a_2x_2 + \dots + a_nx_n$ (the *objective function*) given that x_1, x_2, \dots, x_n satisfy certain *linear constraints*. The constraints are inequalities of the form $b_1x_1 + b_2x_2 + \dots + b_nx_n \geq c$. Many applied problems, such as designing the cheapest animal feed that meets given nutritional goals, can be formulated as linear programming problems.

**local maximum**

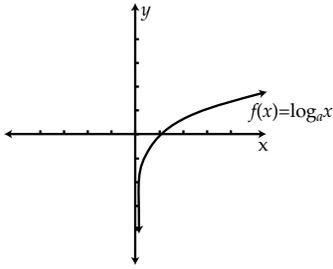
The function $f(x)$ is said to reach a local maximum at $x = a$ if there is a neighbourhood of a such that $f(x) \leq f(a)$ for any x in the neighbourhood; informally, $(a, f(a))$ is at the top of a hill.

local minimum

The function $f(x)$ is said to reach a local minimum at $x = a$ if there is a neighbourhood of a such that $f(x) \geq f(a)$ for any x in the neighbourhood; informally, $(a, f(a))$ is at the bottom of a valley.

logarithmic differentiation

The process of differentiating a product/quotient of functions by finding the logarithm and then differentiating. For example, let $y = \frac{(1+x)^2}{(1+3x)}$. Then $\ln y = 2 \ln(1+x) = \ln(1+3x)$ and $\frac{1}{y} \frac{dy}{dx} = \frac{2}{1+x} - \frac{3}{1+3x}$.

**logarithmic function**

Let a be positive and not equal to 1. The logarithm of x to the base a is the number u such that $a^u = x$, and is denoted by $\log_a x$. Any function of the form $f(x) = \log_a x$ is called a logarithmic function.

lowest common multiple (LCM)

The smallest positive integer that is a multiple of two or more given positive integers. For example, the LCM of 3, 4, and 6 is 12. The LCM is often called the *least* common multiple.

M**matrix**

A rectangular array of numbers. For example:

$$\begin{pmatrix} 3 & 4 \\ -2 & 5 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$$

2×2 matrix 3×1 matrix

maximum point (or value)

The greatest value of a function.

mean (of a sequence of numerical data)

A measure of the average value, obtained by adding up the terms of the sequence and dividing by the number of items.

median (of a sequence of numerical data)

The “middle value” when the data are arranged in order of size. If there is an even number of data, then the average of the two middle values. For example, the median of 5, 3, 7.4, 5, 8, is 5, while the median of 5, 7.4, 5, and 8 is 6.2.

median (of a triangle)

The line segment that joins a vertex of the triangle to the midpoint of the opposite side.

minimum point (or value)

The lowest value of a function.

mixed number

A number that is expressed as the sum of a whole number and a fraction.

For example: $3\frac{2}{5}$

mode

The value that occurs most often in a sequence of data.

monomial

An algebraic expression that is a product of variables and constants.

Examples: $6x^2$, 1 , $\frac{3}{4}x^2y$

multiple (of an integer)

The result obtained when the given integer is multiplied by some integer. Equivalently, an integer that has the given integer as a factor. (Often negative integers are not allowed.)

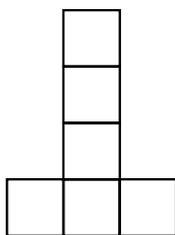
N

natural logarithm

Logarithm to the base e , where e is a fundamental mathematical constant roughly equal to 2.7182818284. The natural logarithm of x is usually written $\ln x$.

natural number (counting number)

One of the numbers 1, 2, 3, 4, . . . *Positive integer* means the same thing.

**net**

A flat diagram consisting of plane faces arranged so that it may be folded to form a solid.

Newton's Law of Cooling

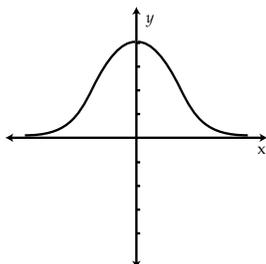
The assertion that if a warm object is placed in a cool room, its temperature decreases at a rate proportional to the difference in temperature between the object and its surroundings.

Newton's Method

An often highly efficient iterative method for approximating the roots of $f(x) = 0$. If r_n is the current estimate, then the next estimate is the x -intercept of the tangent line to $y = f(x)$ at $x = r_n$.

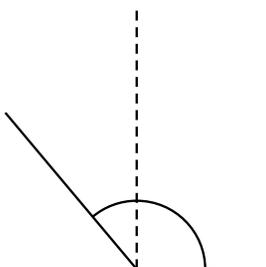
non-differentiable (function)

A function $f(x)$ is non-differentiable at $x = a$ if it does not have a derivative there. Example: if $f(x) = |x|$ then $f(x)$ is not differentiable at $x = 0$, basically because the curve $y = |x|$ has a sharp kink there.

**normal distribution curve**

The standard normal distribution curve has equation $y = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$.

The general normal is obtained by shifting the standard normal to the left or right, and/or rescaling. These curves are sometimes called bell-shaped curves. They figure importantly in probability, statistics, and signal processing.

O**obtuse angle**

An angle whose measure is between 90° and 180° .

one-sided limit

Sometimes $f(x)$ exhibits different behaviour depending on whether x approaches a from the right (through values of x greater than a) or from the left.

For example, let $f(x) = \frac{1}{\left(1 + 2^{\frac{1}{x}}\right)}$. As x approaches 0 from

the right, $f(x)$ approaches 0 (notation: $\lim_{x \rightarrow 0^+} f(x) = 0$) while $f(x)$ approaches 1 as x approaches 0 from the left.

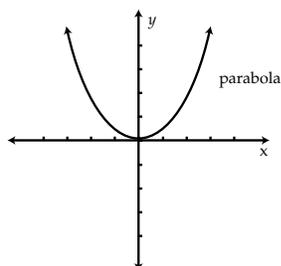
optimization problem

A problem, often of an applied nature, in which we need to find the largest or smallest possible value of a quantity; also called a *max/min problem*.

ordered pair

A sequence of length 2. Ordered pairs (x, y) of real numbers are used to indicate the x and y coordinates of a point in the plane.

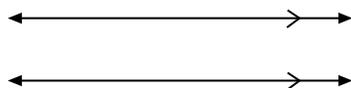
P

**ordinal number**

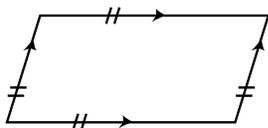
A number designating the place occupied by an item in an ordered sequence (e.g., first, second, and third).

origin

The point in a coordinate system at the intersection of the axes.

**parallel lines**

Two lines in the *plane* are *parallel* if they do not meet. In three-dimensional space, two lines are parallel if they do not meet *and* there is a plane that contains them both. Alternately, in the plane or in space, two lines are parallel if they stay a constant distance apart.

**parallelogram**

A quadrilateral such that pairs of opposite sides are parallel.

percentage

In a problem such as “Find 15% of 400,” the number 400 is sometimes called the *base*, 15% or 0.15 is called the *rate*, and the answer 60 is sometimes called the *percentage*.

percent error

The relative error expressed in parts per hundred. Let A be an estimate of a quantity whose true value is T . Then $A - T$ is the *error*, and $(A - T)/T$ is the *relative error*.

percentile

The k -th *percentile* of a sequence of numerical data is the number x such that k percent of the data points are less than or equal to x . (Often x is not precisely determined, particularly if the data set is not large.)

perimeter

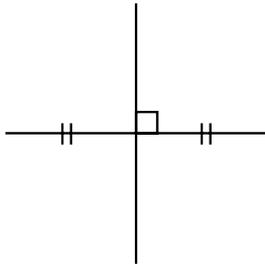
The length of the boundary of a closed figure.

period

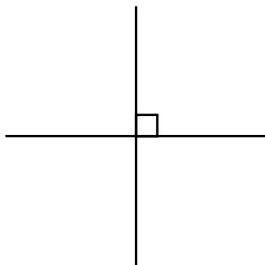
The interval taken to make one complete oscillation or cycle.

permutation

An ordered arrangement of objects. The number of ways of producing a permutation of r (distinct) objects from a collection of n objects is ${}_n P_r$, where ${}_n P_r = n(n-1)(n-2)\dots(n-r+1)$.

**perpendicular bisector**

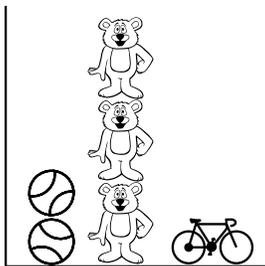
A line that intersects a line segment at a right angle and divides the line segment into two equal parts.

**perpendicular line**

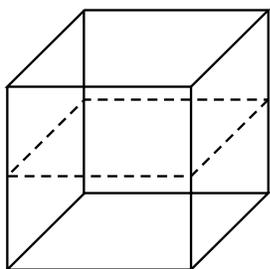
Two lines that intersect at a right angle.

phase shift

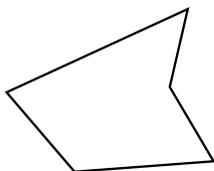
A horizontal translation of a periodic function. For example, the function $\cos 2\left(x - \frac{\pi}{3}\right)$ is $\cos 2x$ with a phase shift of $\frac{\pi}{3}$.

**pictograph**

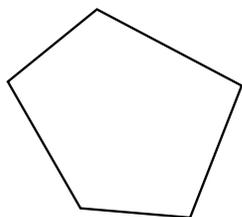
A graph that uses pictures or symbols to represent similar data.

**plane of symmetry**

A 2-D flat surface that cuts through a 3-D object, forming two parts that are mirror images.

**polygon**

A closed curve formed by line segments that do not intersect other than at the vertices.

**polygonal region**

A part of a plane that has a polygon as a boundary.

polyhedron

A solid bounded by plane polygonal regions.

polynomial

A mathematical expression that is a sum of monomials. Examples:

$$4x^3 - 3x - \frac{1}{2}\pi x^2 + 2\pi xy, xyz$$

population

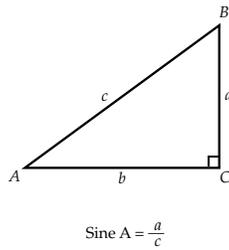
The items, actual or theoretical, from which a sample is drawn.

power

A power of q is any term of the form q^k . Often but not always, k is taken to be a positive integer.

precision

A measure of the estimated degree of repeatability of a measurement, often described by a phrase such as correct to two decimal places.

**primary trigonometric functions**

$$\sin A = \frac{a}{c} = \frac{\textit{opposite}}{\textit{hypotenuse}}$$

$$\cos A = \frac{b}{c} = \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

$$\tan A = \frac{a}{b} = \frac{\textit{opposite}}{\textit{adjacent}}$$

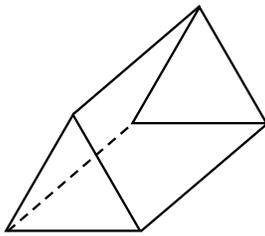
Functions of angles defined, for an acute angle, as ratios of sides in a right triangle.

prime

A positive integer that is divisible by exactly two positive integers, namely 1 and itself. The first few primes are 2, 3, 5, 7, 11, and 13.

prime factorization (of a positive integer)

The given integer expressed as a product of primes. For example, $2 \times 5 \times 3 \times 2$ is a prime factorization of 60. Usually the primes are listed in increasing order. The standard prime factorization of 60 is $2^2 \times 3 \times 5$.

**prism**

A solid with two parallel and congruent bases in the shape of polygons; the other faces are parallelograms.

probability (of an event)

A number between 0 and 1 that measures the likelihood that the event will occur. $P(A)$ often denotes the probability of the event A .

product

The *product* of two or more objects (numbers, functions, etc.) is the result of multiplying these objects together.

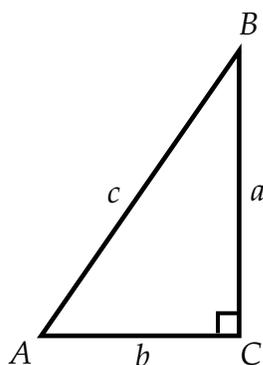
product rule

The rule for finding the derivative of a product of two functions.

$$\text{If } p(x) = f(x)g(x) \text{ then } p'(x) = f(x)g'(x) + g(x)f'(x).$$

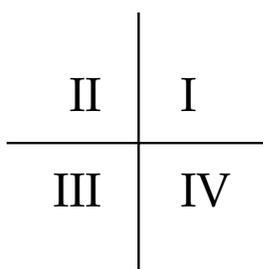
pyramid

A polyhedron one of whose faces is an arbitrary polygon (called the *base*) and whose remaining faces are triangles with a common vertex called the *apex*.

**Pythagorean theorem**

In a right-angled triangle, the sum of the squares of the lengths of the sides containing the right angle is equal to the square of the hypotenuse ($a^2 + b^2 = c^2$).

Q

**quadrant**

One of the four regions that the plane is divided into by two perpendicular lines. When these lines are the usual coordinate axes, the quadrants are called the first quadrant, the second quadrant, and so on as in the diagram.

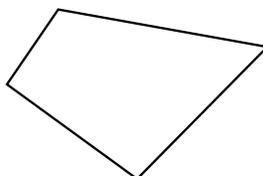
quadratic formula

A formula used to determine the roots of a quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

quadratic function

A function of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. The graph of such a function is a parabola.

**quadrilateral**

A polygon with four sides.

quartile

The 25th percentile is the first quartile, the 50th percentile is the second quartile (or median), and the 75th percentile is the third quartile. Please see *percentile*.

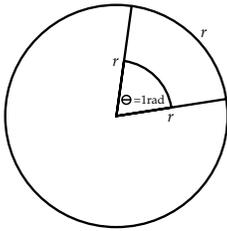
quotient

The result of dividing one object (number, function) by another.

quotient rule

The rule for differentiating the quotient of two functions.

$$\text{If } q(x) = \frac{f(x)}{g(x)} \text{ then } q'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}.$$

R**radian**

Equal to the central angle subtended by an arc of unit length at the centre of a circle of unit radius.

radical

The square root, or cube root, and so on of a quantity. For example, the cube root of the quantity Q is the quantity R such that R^3 (the cube of R) is equal to Q . The square root of Q is written \sqrt{Q} ($\sqrt{\quad}$ is the *radical sign*). The cube root of Q is written $\sqrt[3]{Q}$.

radius

A line segment that joins the centre of a circle or sphere to a point on the boundary. All radii of a circle or sphere have the same length. That common length is called the *radius*.

range

A measure of variability of a sequence of data, defined to be the difference between the extremes in the sequence. For example, if the data are 27, 22, 27, 20, 35, and 34, then the range is 15.

range (of a function)

The set of values taken on by a function. Please see *function*.

rank ordering

Ordering (of a sample) according to the value of some statistical characteristic.

rate

A comparison of two measurements with different units. For example, the speed of an object measured in kilometres per hour.

rate of change (of a function at a point)

How fast the function is changing. If $f(x)$ is the function, its rate of change with respect to x at $x = a$ is the derivative of $f(x)$ at $x = a$.

ratio

Another word for quotient. Also, an indication of the relative size of two quantities. We say that P and Q are in the ratio $a:b$ if the size of A divided by the size of B is a / b .

rational expression

The quotient of two polynomials.

rational number

A number that can be expressed as a / b , where a and b are integers.

rationalize the denominator

Transform a quotient P / Q where the denominator Q involves radicals into an equivalent expression with the denominator free of radicals.

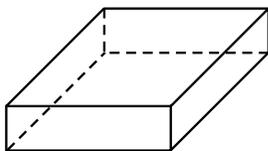
$$\text{For example: } \frac{4}{4 - \sqrt{7}} = \frac{(4)(4 + \sqrt{7})}{(4 - \sqrt{7})(4 + \sqrt{7})} = \frac{(4)(4 + \sqrt{7})}{9}.$$

real number

An indicator of location on a line with respect to an origin; a quantity represented by an arbitrary decimal expansion.

reciprocal

The number or expression produced by dividing 1 by a given number or expression.

**rectangular prism**

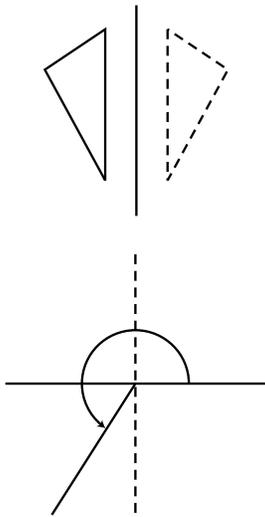
A prism whose bases are congruent rectangles.

recursive definition (of a sequence)

A way of defining a sequence by possibly specifying some terms directly, and giving an algorithm by which any term can be obtained from its predecessors. For example, the Fibonacci sequence is defined recursively by the rules $F_0 = F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$.

reference angle

The acute angle between the ray line and the x -axis. For example, the reference angles of a 165° and of a 195° angle are each 15° angles.

**reflection (in a line)**

The transformation that takes any 2-dimensional object to the object that is symmetrical to it with respect to the line, that is, to its mirror image in the line. *Flip* means the same thing. In three-dimensions, we can define analogously *reflection in a plane*.

reflex angle

An angle greater than 180° and less than 360° .

relative maximum, minimum

Please see *local maximum*, *local minimum*.

Remainder Theorem

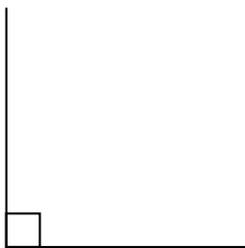
If we divide the polynomial $P(x)$ by $x - a$, the remainder is equal to $P(a)$.

repeating decimal

A decimal expansion that has a block of digits that ultimately cycles forever. For example, $\frac{23}{22}$ has the decimal expansion $1.0454545\dots$, with the block 45 ultimately cycling forever. A terminating decimal like 0.25 is usually viewed as being a repeating decimal, indeed in two ways: as $0.25000\dots$ and $0.24999\dots$

resultant

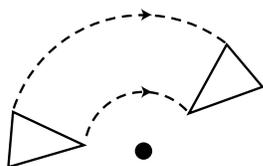
The sum of two or more vectors.

**right angle**

An angle whose measure is 90° .

root of an equation (in one variable)

If the equation has the form $F(x) = G(x)$, a root of the equation is a number a such that $F(a) = G(a)$.

**rotation (in the plane)**

A transformation in which an object is turned through some angle about a point. An analogous notion can be defined in three dimensions; there the turn is about a line.

rounding

A process to follow when making an approximation to a given number by using fewer significant figures.

S

sample

A selection from a population.

sample space

The set of all possible outcomes of an experiment.

scalar

A number. Usually used in contexts where there are also vectors around, or functions. Examples of usage: “the length of a vector is a scalar”; “ $-3 \sin x$ is a scalar multiple of $\sin x$.”

scatter plot

If each item in a sample yields two measurements, such as the height x and weight y of the individual chosen, the point with coordinates (x,y) is plotted. If this is repeated for all members of the sample, the resulting collection of points is a scatter plot.

secant (of x)

This is $\frac{1}{\cos x}$. Notation: $\sec x$.

secant line

A line that passes through two points on a curve.

second derivative

The second derivative of $f(x)$ is the derivative of the derivative of $f(x)$.

Two common notations: $f''(x)$ and $\frac{d^2 f}{dx^2}$.

second derivative test

Suppose that $f'(a) = 0$. The second derivative test gives a way of checking whether at $x = a$ the function $f(x)$ reaches a local minimum or a local maximum.

second-hand data

Data not collected directly by the researcher. For example: encyclopedia.

semicircle

A half-circle; any diameter cuts a circle into two semicircles.

sequence

A finite ordered list t_1, t_2, \dots, t_n of terms (finite sequence) or a list $t_1, t_2, \dots, t_n, \dots$ that goes on forever (an infinite sequence).

series

Any sum of $t_1 + t_2 + \dots + t_n$, the first n terms of a sequence. The sum $t_1 + t_2 + \dots + t_n + \dots$ of all the terms of an infinite sequence is an *infinite series*. The concept of limit is required to define the sum of infinitely many terms.

SI measure

Abbreviation for *Système International d'Unités* – International System of Units – kilogram, second, ampere, kelvin, candela, mole, radian, and so on.

side (of a polygon)

Any of the line segments that make up the boundary of the polygon.

sigma notation

The use of the sign (Greek capital sigma) to denote sum.

For example: $\sum_{i=1}^5 a_i = a_1 + a_2 + a_3 + a_4 + a_5$.

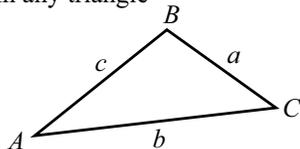
simple interest

Interest computed only on the original principal of a loan or bank deposit.

sine (function)

Please see *primary trigonometric functions*.

In any triangle

**sine law (law of sines)**

A formula used for solving triangles in plane trigonometry.

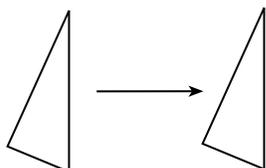
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

skeleton

A representation of the edges of a polyhedron.

skip counting

Counting by multiples of a number. For example: 2, 4, 6, 8.

**slide**

A transformation of a figure by moving it up/down and/or left/right without any rotation. The word is a synonym for the more standard mathematical term translation.

slope

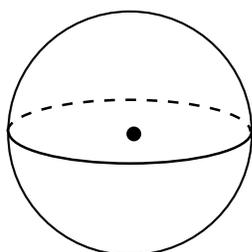
The slope of a (non-vertical) line is a measure of how fast the line is climbing. It can be defined as the change in the y -coordinate of a point on the line when the x -coordinate is increased by 1. If a curve has a (non-vertical) tangent line at the point, the slope of the curve at the point is defined to be the slope of that tangent line.

slope-intercept form (of the equation of a line)

An equation of the form $y = mx + b$. All lines in the plane except for vertical lines can be written in this form. The number m is the slope and b is the y -intercept.

solution (of a differential equation)

A function that satisfies the differential equation. For example, for any constant C , the function given $y = (x^2 + C)^{\frac{1}{3}}$ is a solution of the differential equation $3y^2 \frac{dy}{dx} = 2x$.

**sphere**

A solid whose surface is all points equidistant from a centre point.

square root

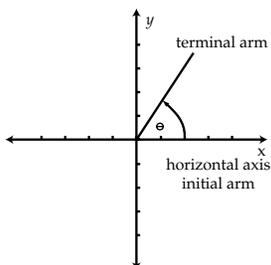
The square root of x is the non-negative number that when multiplied by itself produces x . For example, 5 is the square root of 25. In general the square root of x^2 is $|x|$, the absolute value of x .

standard deviation

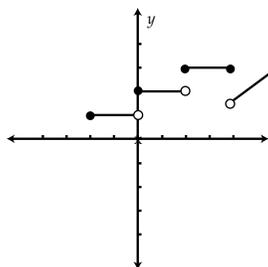
Sample standard deviation is the square root of the sample variance. Population standard deviation is the square root of the population variance.

standard form

The usual form of an equation. For example, the standard form of the equation of a circle is $(x - a)^2 + (y - b)^2 = r^2$, because it reveals geometrically important features, the centre and the radius.

**standard position (angle in)**

The initial arm of the angle is the positive horizontal axis (x -axis.) Counterclockwise rotation gives a positive angle.

**step function**

A function whose graph is flat except at a finite number of points, where it takes a sudden jump.

supplementary angles

Two angles whose sum is 180° .

symmetrical (has symmetry)

A geometrical figure is symmetrical if there is a rotation reflection, or combination of these that takes the figure to itself but moves some points. For example, a square has symmetry because it is taken to itself by a rotation about its centre through 90° .

system of equations

A set of equations. A *solution* of the system is an assignment of values to the variables such that all of the equations are (simultaneously) satisfied. For example, $x = 1$, $y = 2$, $z = -3$ is a solution of the system $x + y + z = 0$, $x - y - 4z = 11$.

T**tangent (function)**

Please see *primary trigonometric functions*.

tangent (to a curve)

A line is tangent to a curve at the point P if under very high magnification, the line is nearly indistinguishable from the curve at points close to P . A tangent line to a circle can be thought of as a line that meets the circle at only one point.

tangent line approximation

If P is a point on a curve, then close to P the curve can be approximated by the tangent line at P . In symbols, if x is close to a , then $f(x)$ is very closely approximated by $f(a) + (x - a)f'(a)$.

tangram

A square cut into seven shapes: two large triangles, one medium triangle, two small triangles, one square, and one parallelogram.

term

Part of an algebraic expression. For example, x^3 and $5x$ are terms of the polynomial $x^3 + 3x^2 + 5x - 1$.

terminating decimal

A decimal expansion that (ultimately) ends. Example: 3.73.

tessellation

A covering of a surface (usually the entire plane) without overlap or bare spots, by copies of a given geometric figure or of a finite number of given geometric figures. The word comes from *tessala*, the Latin word for a small tile.

theoretical probability (of an event)

A numerical measure of the likelihood that the event will occur, based on a probability model. If, as can happen with dice or coins, an experiment has only a finite number n of possible outcomes, all equally likely, and in k of these the event occurs, then the theoretical probability of the event is k / n .

tolerance (interval)

The set of numbers that are considered acceptable as the dimension of an item. Example: a manufacturer's tolerance interval for the weight of a "400 gram" box of cereal might be from 395 grams to 420 grams.

transformation

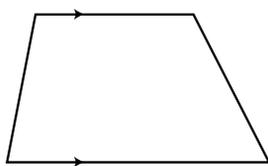
A change in the position of an object, and/or a change in size, and related changes. Also, a change in the form of a mathematical expression.

translation

Please see *slide*.

transversal

A line that intersects two or more lines at different points.

**trapezoid**

A quadrilateral that has two parallel sides. Some definitions require that the remaining two sides not be parallel.

tree diagram

A pictorial way of representing the outcomes of an experiment that involves more than one step.

trigonometry

The branch of mathematics concerned with the properties and applications of the trigonometric functions, in particular their use in “solving” triangles, in surveying, in the study of periodic phenomena, and so on.

trinomial

A polynomial that has three terms. For example: $ax^2 + bx + c$.

turn

Please see *rotation*.

U

unbiased

A sampling procedure for estimating a population parameter (like the proportion of BC teenagers who smoke) is unbiased if on average it should yield the correct value. At a more informal level, a polling procedure is unbiased if proper randomization procedures are used to select the sample, the wording of the questions is neutral, and so on.

unit circle

A circle of radius 1.

unit vector

A vector of length 1.

V

variable

A mathematical entity that can stand for any of the members of a given set.

variance

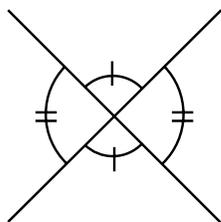
Sample variance is a measure of the variability of a sample, based on the sum of the squared deviations of the data values about the mean. *Population variance* is a theoretical measure of the variability of a population.

vector

A directed line segment (arrow) used to describe a quantity that has direction as well as magnitude.

vertex (pl. vertices)

In a polygon, a point of intersection of two sides. In a polyhedron, a vertex of a face.

**vertically opposite angles**

Opposite (and equal) angles resulting from the intersection of two lines.

W**whole number**

One of the counting numbers 0, 1, 2, 3, 4, and so on; a non-negative integer.

X **x -intercept(s)**

The point(s) at which a curve meets the x -axis (horizontal axis).

Y **y -intercept(s)**

The point(s) at which a curve meets the y -axis (vertical axis).

Z**z-score**

If x is the numerical value of one observation in a sample, the z-score of x is $\frac{(x - \bar{x})}{s}$, where \bar{x} is the sample mean and s is the sample standard deviation. The z-score measures how far x is from the mean.

zero (root) of $f(x)$

Any number a such that $f(a)=0$.