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PAMPHLET NO. # 53 DATE: January 10, 1996

SUBJECT: Balanced Incomplete Block (BIB) Study Designs

When treatment plots must be large, as in mixedwood and silviculture systems studies, it may be difficult to find blocks large enough to accomodate all the treatments. Designs with **incomplete blocks**, that is designs where all the treatments do not occur in each block, may be suitable. They may even be preferable, if the plots within the smaller blocks can be substantially more homogenous (similar) than is possible for the plots within larger blocks. Incomplete block designs are called Balanced Incomplete Block (BIB) designs[1] if:

i) each block has the same number of plots (k);
ii) each treatment occurs the same number of times (r);
iii) each pair of treatments occur together in a block the same number of times (λ).

Balanced incomplete block designs estimate all the treatment means with the same precision and are straightforward to analyse. However, a Balanced Incomplete Block design of reasonable size is not always possible. The following relationship must be met for a Balanced Incomplete Block design to be possible:

$$\#\text{treatments} * \#\text{replications} = \#\text{blocks} * \#\text{plots/block}$$
$$t * r = b * k$$

but this is **not** a guarantee that a suitable design is available. A shorthand notation for these designs is BIBD(t,b,k,r;λ). These designs are enumerated and fully described in many textbooks (see the references for a short list).

Brief descriptions of possible Balanced Incomplete Block Designs for studies with four or five treatments may help show what these designs are like. For example, a complete randomized block design for a mixedwood replacement series study that uses four mixtures of two species as treatments (Df stands for Douglas-fir and Dr for Red Alder) can be represented by:

(Note that the numbers under the Df/Dr column represent Percent Composition for each species and that √ denotes treatments that occur in that block.)

Df / Dr	Block 1	Block 2	Block 3
100/0	√	√	√
70/30	√	√	√
40/60	√	√	√
0/100	√	√	√

There are the only two types of BIB designs available for 4 treatments; one with four blocks and the other with six. Different randomizations of the design allow for many variations. The treatment assignment to blocks for the two types can be represented by:

[1] Note that complete block designs also fulfill these requirements.

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BIBD(4,4,3,3;2):

Df / Dr	Block 1	2	3	4
100/0	√	√	√	
70/30	√	√		√
40/60	√		√	√
0/100		√	√	√

BIBD(4,6,2,3;1):

Df / Dr	Block 1	2	3	4	5	6
100/0	√		√		√	
70/30	√			√		√
40/60		√	√			√
0/100		√		√	√	

If more than three replicates are required, then more complete sets can be installed in the study. The six block design has an interesting property: pairs of blocks, blocks 1 & 2, 3 & 4, 5 & 6, each form what are traditionally called replicates because each pair contains all four treatments. This is known as a **resolvable design** and can be useful if, for instance, a study was to be conducted using two cutblocks within each of three watersheds. Each watershed could then be assigned one of the replicates, and each cutblock could receive two of the treatments.

The following represents a complete randomized block design for a mixedwood additive design study with five treatments:

Df 1200 s/ha Dr (s/ha)	Block 1	2	3	4
400	√	√	√	√
200	√	√	√	√
100	√	√	√	√
50	√	√	√	√
0	√	√	√	√

There are the only three types of BIB designs available for five treatments. The treatment assignment for a design with five blocks and four plots per block is:

BIBD(5,5,4,4;3):

Df 1200 s/ha Dr (s/ha)	Block 1	2	3	4	5
400	√	√	√	√	
200	√	√	√		√
100	√	√		√	√
50	√		√	√	√
0		√	√	√	√

The two possible arrangements with 10 blocks can be represented by:

BIBD(5,10,3,6;3):

Df 1200 s/ha	Block (k=3)									
Dr (s/ha)	1	2	3	4	5	6	7	8	9	10
400	√	√	√			√	√	√		
200	√	√		√		√			√	√
100	√			√	√		√	√	√	
50			√	√	√	√	√			√
0		√	√		√			√	√	√

BIBD(5,10,2,4;1):

Df 1200 s/ha	Block (k=2)									
Dr (s/ha)	1	2	3	4	5	6	7	8	9	10
400	√			√		√			√	
200	√		√				√			√
100		√		√			√	√		
50		√			√	√				√
0			√		√			√	√	

As with all study designs, proper random assignment of treatments to experimental units is important, although the procedure is a more complicated for incomplete block designs. Cochran and Cox (1957, pg 442) and Cox (1958, pg 227) describe the steps involved[2]. These steps are:

1. Rearrange the block numbers in the diagram at random and assign them to real blocks[3].
2. Randomly assign the treatments to the rows in the diagram.
3. Randomly assign the treatments allocated to a particular block to its plots.

For an example, suppose that we are planning to use the BIBD(4,4,3,3;2) described on the previous page. This design has four blocks with four treatments replicated three times. First, the block numbers are randomly ordered. Then the treatments are randomly ordered against the rows. One possible randomization is:

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[2] This randomization can also be done using Proc Plan. See the SAS/STAT manual (Vers. 6.03 or later) for details.

[3] If the design is arranged in complete replications, the blocks are randomized within each replication to keep the replications separate.

STEP 1:

Block 2	Block 1	Block 4	Block 3
√	√	√	
√	√		√
√		√	√
	√	√	√

STEP 2:

Df / Dr	Block 2	Block 1	Block 4	Block 3
40/60	√	√	√	
100/0	√	√		√
70/30	√		√	√
0/100		√	√	√

In the field, the first block would contain treatments 0/100, 100/0 and 40/60. The third step is to randomly assign and apply these three treatments to the three plots in block 1. For block 2, treatments 40/60, 100/0, and 70/30 are randomly assigned and applied to its three plots. Treatments are assigned similarly to plots in blocks 3 and 4.

There are many possible variations on the Balanced Incomplete Block Design: For instance, a complete factorial arrangement of treatments, all assigned the same experimental units (plots within blocks), could be used as the treatment factor; A second variable, such as moisture or slope position that should be balanced in the design, can be added to some of the BIB designs to form a variation of the Latin Square called a **Youden Square** (see, for example, Mead, 1988); The plots within each block can be split for treatments requiring smaller plots. The design then becomes a **balanced incomplete block split-splot design**, and the plots within each block would be the mainplots.

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References[4]:

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4 Note: Kuehl and Mead have particularily good discussions of Incomplete Block Designs.