Pre-Calculus

11 and 12 (2008)

PRE-CALCULUS
GRADE 11

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Algebra and Number	**General Outcome:** Develop algebraic reasoning and number sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
A1. Demonstrate an understanding of the absolute value of real numbers. [R, V]	1.1 Determine the distance of two real numbers of the form $\pm a, a \in R$, from 0 on a number line, and relate this to the absolute value of a ($\lvert a \rvert$). 1.2 Determine the absolute value of a positive or negative real number. 1.3 Explain, using examples, how distance between two points on a number line can be expressed in terms of absolute value. 1.4 Determine the absolute value of a numerical expression. 1.5 Compare and order the absolute values of real numbers in a given set.
A2. Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands. [CN, ME, PS, R, T]	2.1 Compare and order radical expressions with numerical radicands in a given set. 2.2 Express an entire radical with a numerical radicand as a mixed radical. 2.3 Express a mixed radical with a numerical radicand as an entire radical. 2.4 Perform one or more operations to simplify radical expressions with numerical or variable radicands. 2.5 Rationalize the denominator of a rational expression with monomial or binomial denominators. 2.6 Describe the relationship between rationalizing a binomial denominator of a rational expression and the product of the factors of a difference of squares expression. 2.7 Explain, using examples, that $(-x)^2 = x^2$, $\sqrt{x^2} = \lvert x \rvert$ and $\sqrt{x^2} \neq \pm x$; e.g., $\sqrt{9} \neq \pm 3$. 2.8 Identify the values of the variable for which a given radical expression is defined. 2.9 Solve a problem that involves radical expressions.

[C] Communication	**[PS]** Problem Solving
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Algebra and Number (continued)	**General Outcome:** Develop algebraic reasoning and number sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
A3. Solve problems that involve radical equations (limited to square roots). [C, PS, R]	*(It is intended that the equations will have no more than two radicals.)* 3.1 Determine any restrictions on values for the variable in a radical equation. 3.2 Determine the roots of a radical equation algebraically, and explain the process used to solve the equation. 3.3 Verify, by substitution, that the values determined in solving a radical equation algebraically are roots of the equation. 3.4 Explain why some roots determined in solving a radical equation algebraically are extraneous. 3.5 Solve problems by modelling a situation using a radical equation.
A4. Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials). [C, ME, R]	4.1 Compare the strategies for writing equivalent forms of rational expressions to the strategies for writing equivalent forms of rational numbers. 4.2 Explain why a given value is non-permissible for a given rational expression. 4.3 Determine the non-permissible values for a rational expression. 4.4 Determine a rational expression that is equivalent to a given rational expression by multiplying the numerator and denominator by the same factor (limited to a monomial or a binomial), and state the non-permissible values of the equivalent rational expression. 4.5 Simplify a rational expression. 4.6 Explain why the non-permissible values of a given rational expression and its simplified form are the same. 4.7 Identify and correct errors in a simplification of a rational expression, and explain the reasoning.

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Algebra and Number (continued)	**General Outcome:** Develop algebraic reasoning and number sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
A5. Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials). [CN, ME, R]	5.1 Compare the strategies for performing a given operation on rational expressions to the strategies for performing the same operation on rational numbers. 5.2 Determine the non-permissible values when performing operations on rational expressions. 5.3 Determine, in simplified form, the sum or difference of rational expressions with the same denominator. 5.4 Determine, in simplified form, the sum or difference of rational expressions in which the denominators are not the same and which may or may not contain common factors. 5.5 Determine, in simplified form, the product or quotient of rational expressions. 5.6 Simplify an expression that involves two or more operations on rational expressions.
A6. Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials or trinomials). [C, PS, R]	*(It is intended that the rational equations be those that can be simplified to linear and quadratic equations.)* 6.1 Determine the non-permissible values for the variable in a rational equation. 6.2 Determine the solution to a rational equation algebraically, and explain the process used to solve the equation. 6.3 Explain why a value obtained in solving a rational equation may not be a solution of the equation. 6.4 Solve problems by modelling a situation using a rational equation.

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Trigonometry	**General Outcome:** Develop trigonometric reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
B1. Demonstrate an understanding of angles in standard position [0° to 360°]. [R, V]	1.1 Sketch an angle in standard position, given the measure of the angle. 1.2 Determine the reference angle for an angle in standard position. 1.3 Explain, using examples, how to determine the angles from 0° to 360° that have the same reference angle as a given angle. 1.4 Illustrate, using examples, that any angle from 90° to 360° is the reflection in the *x*-axis and/or the *y*-axis of its reference angle. 1.5 Determine the quadrant in which a given angle in standard position terminates. 1.6 Draw an angle in standard position given any point $\mathrm{P}(x, y)$ on the terminal arm of the angle. 1.7 Illustrate, using examples, that the points $\mathrm{P}(x, y)$, $\mathrm{P}(-x, y)$, $\mathrm{P}(-x, -y)$ and $\mathrm{P}(x, -y)$ are points on the terminal sides of angles in standard position that have the same reference angle.

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Trigonometry (continued)	**General Outcome:** Develop trigonometric reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
B2. Solve problems, using the three primary trigonometric ratios for angles from 0° to 360° in standard position. [C, ME, PS, R, T, V]	2.1 Determine, using the Pythagorean theorem or the distance formula, the distance from the origin to a point $P(x, y)$ on the terminal arm of an angle. 2.2 Determine the value of $\sin\theta$, $\cos\theta$ or $\tan\theta$, given any point $P(x, y)$ on the terminal arm of angle θ. 2.3 Determine, without the use of technology, the value of $\sin\theta$, $\cos\theta$ or $\tan\theta$, given any point $P(x, y)$ on the terminal arm of angle θ, where θ = 0º, 90º, 180º, 270º or 360º. 2.4 Determine the sign of a given trigonometric ratio for a given angle, without the use of technology, and explain. 2.5 Solve, for all values of θ, an equation of the form $\sin\theta = a$ or $\cos\theta = a$, where $-1 \le a \le 1$, and an equation of the form $\tan\theta = a$, where a is a real number. 2.6 Determine the exact value of the sine, cosine or tangent of a given angle with a reference angle of 30º, 45º or 60º. 2.7 Describe patterns in and among the values of the sine, cosine and tangent ratios for angles from 0° to 360°. 2.8 Sketch a diagram to represent a problem. 2.9 Solve a contextual problem, using trigonometric ratios.
B3. Solve problems, using the cosine law and sine law, including the ambiguous case. [C, CN, PS, R, T]	3.1 Sketch a diagram to represent a problem that involves a triangle without a right angle. 3.2 Solve, using primary trigonometric ratios, a triangle that is not a right triangle. 3.3 Explain the steps in a given proof of the sine law or cosine law. 3.4 Sketch a diagram and solve a problem, using the cosine law. 3.5 Sketch a diagram and solve a problem, using the sine law. 3.6 Describe and explain situations in which a problem may have no solution, one solution or two solutions.

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Relations and Functions	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
C1. Factor polynomial expressions of the form: • $ax^2+bx+c,\ a\neq 0$ • $a^2x^2-b^2y^2,\ a\neq 0,\ b\neq 0$ • $a\left(f\left(x\right)\right)^2+b\left(f\left(x\right)\right)+c,\ a\neq 0$ • $a^2\left(f\left(x\right)\right)^2-b^2\left(g\left(y\right)\right)^2,\ a\neq 0,\ b\neq 0$ where a, b and c are rational numbers. [CN, ME, R]	1.1 Factor a given polynomial expression that requires the identification of common factors. 1.2 Determine whether a given binomial is a factor for a given polynomial expression, and explain why or why not. 1.3 Factor a given polynomial expression of the form: • $ax^2+bx+c,\ a\neq 0$ • $a^2x^2-b^2y^2,\ a\neq 0,\ b\neq 0$. 1.4 Factor a given polynomial expression that has a quadratic pattern, including: • $a\left(f\left(x\right)\right)^2+b\left(f\left(x\right)\right)+c,\ a\neq 0$ • $a^2\left(f\left(x\right)\right)^2-b^2\left(g\left(y\right)\right)^2,\ a\neq 0,\ b\neq 0$.

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Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
C2. Graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems. [C, PS, R, T, V]	2.1 Create a table of values for $y=\left\|f\left(x\right)\right\|$, given a table of values for $y=f\left(x\right)$. 2.2 Generalize a rule for writing absolute value functions in piecewise notation. 2.3 Sketch the graph of $y=\left\|f\left(x\right)\right\|$; state the intercepts, domain and range; and explain the strategy used. 2.4 Solve an absolute value equation graphically, with or without technology. 2.5 Solve, algebraically, an equation with a single absolute value, and verify the solution. 2.6 Explain why the absolute value equation $\left\|f\left(x\right)\right\|<0$ has no solution. 2.7 Determine and correct errors in a solution to an absolute value equation. 2.8 Solve a problem that involves an absolute value function.

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Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
C3. Analyze quadratic functions of the form $y = a(x-p)^2 + q$ and determine the: • vertex • domain and range • direction of opening • axis of symmetry • x- and y-intercepts. [CN, R, T, V]	3.1 Explain why a function given in the form $y = a(x-p)^2 + q$ is a quadratic function. 3.2 Compare the graphs of a set of functions of the form $y = ax^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of a. 3.3 Compare the graphs of a set of functions of the form $y = x^2 + q$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of q. 3.4 Compare the graphs of a set of functions of the form $y = (x-p)^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of p. 3.5 Determine the coordinates of the vertex for a quadratic function of the form $y = a(x-p)^2 + q$, and verify with or without technology. 3.6 Generalize, using inductive reasoning, a rule for determining the coordinates of the vertex for quadratic functions of the form $y = a(x-p)^2 + q$. 3.7 Sketch the graph of $y = a(x-p)^2 + q$, using transformations, and identify the vertex, domain and range, direction of opening, axis of symmetry and x- and y-intercepts. 3.8 Explain, using examples, how the values of a and q may be used to determine whether a quadratic function has zero, one or two x-intercepts. 3.9 Write a quadratic function in the form $y = a(x-p)^2 + q$ for a given graph or a set of characteristics of a graph.

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Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
C4. Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including: • vertex • domain and range • direction of opening • axis of symmetry • x- and y-intercepts and to solve problems. [CN, PS, R, T, V]	4.1 Explain the reasoning for the process of completing the square as shown in a given example. 4.2 Write a quadratic function given in the form $y = ax^2 + bx + c$ as a quadratic function in the form $y = a(x - p)^2 + q$ by completing the square. 4.3 Identify, explain and correct errors in an example of completing the square. 4.4 Determine the characteristics of a quadratic function given in the form $y = ax^2 + bx + c$, and explain the strategy used. 4.5 Sketch the graph of a quadratic function given in the form $y = ax^2 + bx + c$. 4.6 Verify, with or without technology, that a quadratic function in the form $y = ax^2 + bx + c$ represents the same function as a given quadratic function in the form $y = a(x - p)^2 + q$. 4.7 Write a quadratic function that models a given situation, and explain any assumptions made. 4.8 Solve a problem, with or without technology, by analyzing a quadratic function.

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Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
C5. Solve problems that involve quadratic equations. [C, CN, PS, R, T, V]	5.1 Explain, using examples, the relationship among the roots of a quadratic equation, the zeros of the corresponding quadratic function and the *x*-intercepts of the graph of the quadratic function. 5.2 Derive the quadratic formula, using deductive reasoning. 5.3 Solve a quadratic equation of the form $ax^2+bx+c=0$ by using strategies such as: • determining square roots • factoring • completing the square • applying the quadratic formula • graphing its corresponding function. 5.4 Select a method for solving a quadratic equation, justify the choice, and verify the solution. 5.5 Explain, using examples, how the discriminant may be used to determine whether a quadratic equation has two, one or no real roots; and relate the number of zeros to the graph of the corresponding quadratic function. 5.6 Identify and correct errors in a solution to a quadratic equation. 5.7 Solve a problem by: • analyzing a quadratic equation • determining and analyzing a quadratic equation.

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Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
C6. Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables. [CN, PS, R, T, V]	*(It is intended that the quadratic equations be limited to those that correspond to quadratic functions.)* 6.1 Model a situation, using a system of linear-quadratic or quadratic-quadratic equations. 6.2 Relate a system of linear-quadratic or quadratic-quadratic equations to the context of a given problem. 6.3 Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations graphically, with technology. 6.4 Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations algebraically. 6.5 Explain the meaning of the points of intersection of a system of linear-quadratic or quadratic-quadratic equations. 6.6 Explain, using examples, why a system of linear-quadratic or quadratic-quadratic equations may have zero, one, two or an infinite number of solutions. 6.7 Solve a problem that involves a system of linear-quadratic or quadratic-quadratic equations, and explain the strategy used.

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Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
C7. Solve problems that involve linear and quadratic inequalities in two variables. [C, PS, T, V]	7.1 Explain, using examples, how test points can be used to determine the solution region that satisfies an inequality. 7.2 Explain, using examples, when a solid or broken line should be used in the solution for an inequality. 7.3 Sketch, with or without technology, the graph of a linear or quadratic inequality. 7.4 Solve a problem that involves a linear or quadratic inequality.
C8. Solve problems that involve quadratic inequalities in one variable. [CN, PS, V]	8.1 Determine the solution of a quadratic inequality in one variable, using strategies such as case analysis, graphing, roots and test points, or sign analysis; and explain the strategy used. 8.2 Represent and solve a problem that involves a quadratic inequality in one variable. 8.3 Interpret the solution to a problem that involves a quadratic inequality in one variable.
C9. Analyze arithmetic sequences and series to solve problems. [CN, PS, R, T]	9.1 Identify the assumption(s) made when defining an arithmetic sequence or series. 9.2 Provide and justify an example of an arithmetic sequence. 9.3 Derive a rule for determining the general term of an arithmetic sequence. 9.4 Describe the relationship between arithmetic sequences and linear functions. 9.5 Determine t_1, d, n or t_n in a problem that involves an arithmetic sequence. 9.6 Derive a rule for determining the sum of n terms of an arithmetic series. 9.7 Determine t_1, d, n or S_n in a problem that involves an arithmetic series. 9.8 Solve a problem that involves an arithmetic sequence or series.

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Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
C10. Analyze geometric sequences and series to solve problems. [PS, R, T]	10.1 Identify assumptions made when identifying a geometric sequence or series. 10.2 Provide and justify an example of a geometric sequence. 10.3 Derive a rule for determining the general term of a geometric sequence. 10.4 Determine t_1, r, n or t_n in a problem that involves a geometric sequence. 10.5 Derive a rule for determining the sum of n terms of a geometric series. 10.6 Determine t_1, r, n or S_n in a problem that involves a geometric series. 10.7 Generalize, using inductive reasoning, a rule for determining the sum of an infinite geometric series. 10.8 Explain why a geometric series is convergent or divergent. 10.9 Solve a problem that involves a geometric sequence or series.
C11. Graph and analyze reciprocal functions (limited to the reciprocal of linear and quadratic functions). [CN, R, T, V]	11.1 Compare the graph of $y = \frac{1}{f(x)}$ to the graph of $y = f(x)$. 11.2 Identify, given a function $f(x)$, values of x for which $y = \frac{1}{f(x)}$ will have vertical asymptotes; and describe their relationship to the non-permissible values of the related rational expression. 11.3 Graph, with or without technology, $y = \frac{1}{f(x)}$, given $y = f(x)$ as a function or a graph, and explain the strategies used. 11.4 Graph, with or without technology, $y = f(x)$, given $y = \frac{1}{f(x)}$ as a function or a graph, and explain the strategies used.

PRE-CALCULUS
GRADE 12

[C] Communication	**[PS]** Problem Solving
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Trigonometry	**General Outcome:** Develop trigonometric reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
A1. Demonstrate an understanding of angles in standard position, expressed in degrees and radians. [CN, ME, R, V]	1.1 Sketch, in standard position, an angle (positive or negative) when the measure is given in degrees. 1.2 Describe the relationship among different systems of angle measurement, with emphasis on radians and degrees. 1.3 Sketch, in standard position, an angle with a measure of 1 radian. 1.4 Sketch, in standard position, an angle with a measure expressed in the form $k\pi$ radians, where $k \in Q$. 1.5 Express the measure of an angle in radians (exact value or decimal approximation), given its measure in degrees. 1.6 Express the measure of an angle in degrees, given its measure in radians (exact value or decimal approximation). 1.7 Determine the measures, in degrees or radians, of all angles in a given domain that are coterminal with a given angle in standard position. 1.8 Determine the general form of the measures, in degrees or radians, of all angles that are coterminal with a given angle in standard position. 1.9 Explain the relationship between the radian measure of an angle in standard position and the length of the arc cut on a circle of radius *r*, and solve problems based upon that relationship.

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Trigonometry (continued)	**General Outcome:** Develop trigonometric reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
A2. Develop and apply the equation of the unit circle. [CN, R, V]	2.1 Derive the equation of the unit circle from the Pythagorean theorem. 2.2 Describe the six trigonometric ratios, using a point P (x, y) that is the intersection of the terminal arm of an angle and the unit circle. 2.3 Generalize the equation of a circle with centre (0, 0) and radius r.
A3. Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees. [ME, PS, R, T, V]	3.1 Determine, with technology, the approximate value of a trigonometric ratio for any angle with a measure expressed in either degrees or radians. 3.2 Determine, using a unit circle or reference triangle, the exact value of a trigonometric ratio for angles expressed in degrees that are multiples of 0º, 30º, 45º, 60º or 90º, or for angles expressed in radians that are multiples of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ or $\frac{\pi}{2}$, and explain the strategy. 3.3 Determine, with or without technology, the measures, in degrees or radians, of the angles in a specified domain, given the value of a trigonometric ratio. 3.4 Explain how to determine the exact values of the six trigonometric ratios, given the coordinates of a point on the terminal arm of an angle in standard position. 3.5 Determine the measures of the angles in a specified domain in degrees or radians, given a point on the terminal arm of an angle in standard position. 3.6 Determine the exact values of the other trigonometric ratios, given the value of one trigonometric ratio in a specified domain. 3.7 Sketch a diagram to represent a problem that involves trigonometric ratios. 3.8 Solve a problem, using trigonometric ratios.

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Trigonometry (continued)	**General Outcome:** Develop trigonometric reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
A4. Graph and analyze the trigonometric functions sine, cosine and tangent to solve problems. [CN, PS, T, V]	4.1 Sketch, with or without technology, the graph of $y = \sin x$, $y = \cos x$ or $y = \tan x$. 4.2 Determine the characteristics (amplitude, asymptotes, domain, period, range and zeros) of the graph of $y = \sin x$, $y = \cos x$ or $y = \tan x$. 4.3 Determine how varying the value of a affects the graphs of $y = a \sin x$ and $y = a \cos x$. 4.4 Determine how varying the value of d affects the graphs of $y = \sin x + d$ and $y = \cos x + d$. 4.5 Determine how varying the value of c affects the graphs of $y = \sin (x + c)$ and $y = \cos (x + c)$. 4.6 Determine how varying the value of b affects the graphs of $y = \sin bx$ and $y = \cos bx$. 4.7 Sketch, without technology, graphs of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$, using transformations, and explain the strategies. 4.8 Determine the characteristics (amplitude, asymptotes, domain, period, phase shift, range and zeros) of the graph of a trigonometric function of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$. 4.9 Determine the values of a, b, c and d for functions of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$ that correspond to a given graph, and write the equation of the function. 4.10 Determine a trigonometric function that models a situation to solve a problem. 4.11 Explain how the characteristics of the graph of a trigonometric function relate to the conditions in a problem situation. 4.12 Solve a problem by analyzing the graph of a trigonometric function.

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Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
A5. Solve, algebraically and graphically, first and second degree trigonometric equations with the domain expressed in degrees and radians. [CN, PS, R, T, V]	5.1 Verify, with or without technology, that a given value is a solution to a trigonometric equation. 5.2 Determine, algebraically, the solution of a trigonometric equation, stating the solution in exact form when possible. 5.3 Determine, using technology, the approximate solution of a trigonometric equation in a restricted domain. 5.4 Relate the general solution of a trigonometric equation to the zeros of the corresponding trigonometric function (restricted to sine and cosine functions). 5.5 Determine, using technology, the general solution of a given trigonometric equation. 5.6 Identify and correct errors in a solution for a trigonometric equation.
A6. Prove trigonometric identities, using: • reciprocal identities • quotient identities • Pythagorean identities • sum or difference identities (restricted to sine, cosine and tangent) • double-angle identities (restricted to sine, cosine and tangent). [R, T, V]	6.1 Explain the difference between a trigonometric identity and a trigonometric equation. 6.2 Verify a trigonometric identity numerically for a given value in either degrees or radians. 6.3 Explain why verifying that the two sides of a trigonometric identity are equal for given values is insufficient to conclude that the identity is valid. 6.4 Determine, graphically, the potential validity of a trigonometric identity, using technology. 6.5 Determine the non-permissible values of a trigonometric identity. 6.6 Prove, algebraically, that a trigonometric identity is valid. 6.7 Determine, using the sum, difference and double-angle identities, the exact value of a trigonometric ratio.

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[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
B1. Demonstrate an understanding of operations on, and compositions of, functions. [CN, R, T, V]	1.1 Sketch the graph of a function that is the sum, difference, product or quotient of two functions, given their graphs. 1.2 Write the equation of a function that is the sum, difference, product or quotient of two or more functions, given their equations. 1.3 Determine the domain and range of a function that is the sum, difference, product or quotient of two functions. 1.4 Write a function $h(x)$ as the sum, difference, product or quotient of two or more functions. 1.5 Determine the value of the composition of functions when evaluated at a point, including: • $f(f(a))$ • $f(g(a))$ • $g(f(a))$. 1.6 Determine, given the equations of two functions $f(x)$ and $g(x)$, the equation of the composite function: • $f(f(x))$ • $f(g(x))$ • $g(f(x))$ and explain any restrictions.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
	1.7 Sketch, given the equations of two functions $f(x)$ and $g(x)$, the graph of the composite function: • $f\left(f\left(x\right)\right)$ • $f\left(g\left(x\right)\right)$ • $g\left(f\left(x\right)\right)$. 1.8 Write a function *h(x)* as the composition of two or more functions. 1.9 Write a function *h(x)* by combining two or more functions through operations on, and compositions of, functions.
B2. Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations. [C, CN, R, V]	2.1 Compare the graphs of a set of functions of the form $y - k = f(x)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of k. 2.2 Compare the graphs of a set of functions of the form $y = f\left(x - h\right)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of h. 2.3 Compare the graphs of a set of functions of the form $y - k = f(x - h)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effects of h and k. 2.4 Sketch the graph of $y - k = f(x)$, $y = f\left(x - h\right)$ or $y - k = f(x - h)$ for given values of h and k, given a sketch of the function $y = f(x)$, where the equation of $y = f(x)$ is not given. 2.5 Write the equation of a function whose graph is a vertical and/or horizontal translation of the graph of the function $y = f(x)$.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
B3. Demonstrate an understanding of the effects of horizontal and vertical stretches on the graphs of functions and their related equations. [C, CN, R, V]	3.1 Compare the graphs of a set of functions of the form $y=af(x)$ to the graph of $y=f(x)$, and generalize, using inductive reasoning, a rule about the effect of a. 3.2 Compare the graphs of a set of functions of the form $y=f(bx)$ to the graph of $y=f(x)$, and generalize, using inductive reasoning, a rule about the effect of b. 3.3 Compare the graphs of a set of functions of the form $y=af(bx)$ to the graph of $y=f(x)$, and generalize, using inductive reasoning, a rule about the effects of a and b. 3.4 Sketch the graph of $y=af(x)$, $y=f(bx)$ or $y=af(bx)$ for given values of a and b, given a sketch of the function $y=f(x)$, where the equation of $y=f(x)$ is not given. 3.5 Write the equation of a function, given its graph which is a vertical and/or horizontal stretch of the graph of the function $y=f(x)$.
B4. Apply translations and stretches to the graphs and equations of functions. [C, CN, R, V]	4.1 Sketch the graph of the function $y-k=af(b(x-h))$ for given values of a, b, h and k, given the graph of the function $y=f(x)$, where the equation of $y=f(x)$ is not given. 4.2 Write the equation of a function, given its graph which is a translation and/or stretch of the graph of the function $y=f(x)$.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
B5. Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections through the: • x-axis • y-axis • line $y = x$. [C, CN, R, V]	5.1 Generalize the relationship between the coordinates of an ordered pair and the coordinates of the corresponding ordered pair that results from a reflection through the x-axis, the y-axis or the line $y = x$. 5.2 Sketch the reflection of the graph of a function $y = f(x)$ through the x-axis, the y-axis or the line $y = x$, given the graph of the function $y = f(x)$, where the equation of $y = f(x)$ is not given. 5.3 Generalize, using inductive reasoning, and explain rules for the reflection of the graph of the function $y = f(x)$ through the x-axis, the y-axis or the line $y = x$. 5.4 Sketch the graphs of the functions $y = -f(x)$, $y = f(-x)$ and $x = -f(y)$, given the graph of the function $y = f(x)$, where the equation of $y = f(x)$ is not given. 5.5 Write the equation of a function, given its graph which is a reflection of the graph of the function $y = f(x)$ through the x-axis, the y-axis or the line $y = x$.
B6. Demonstrate an understanding of inverses of relations. [C, CN, R, V]	6.1 Explain how the graph of the line $y = x$ can be used to sketch the inverse of a relation. 6.2 Explain how the transformation $(x, y) \Rightarrow (y, x)$ can be used to sketch the inverse of a relation. 6.3 Sketch the graph of the inverse relation, given the graph of a relation. 6.4 Determine if a relation and its inverse are functions. 6.5 Determine restrictions on the domain of a function in order for its inverse to be a function. 6.6 Determine the equation and sketch the graph of the inverse relation, given the equation of a linear or quadratic relation. 6.7 Explain the relationship between the domains and ranges of a relation and its inverse. 6.8 Determine, algebraically or graphically, if two functions are inverses of each other.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
B7. Demonstrate an understanding of logarithms. [CN, ME, R]	7.1 Explain the relationship between logarithms and exponents. 7.2 Express a logarithmic expression as an exponential expression and vice versa. 7.3 Determine, without technology, the exact value of a logarithm, such as $\log_2 8$. 7.4 Estimate the value of a logarithm, using benchmarks, and explain the reasoning; e.g., since $\log_2 8 = 3$ and $\log_2 16 = 4$, $\log_2 9$ is approximately equal to 3.1.
B8. Demonstrate an understanding of the product, quotient and power laws of logarithms. [C, CN, R, T]	8.1 Develop and generalize the laws for logarithms, using numeric examples and exponent laws. 8.2 Derive each law of logarithms. 8.3 Determine, using the laws of logarithms, an equivalent expression for a logarithmic expression. 8.4 Determine, with technology, the approximate value of a logarithmic expression, such as $\log_2 9$.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
B9. Graph and analyze exponential and logarithmic functions. [C, CN, T, V]	9.1 Sketch, with or without technology, a graph of an exponential function of the form $y = a^x$, $a > 0$. 9.2 Identify the characteristics of the graph of an exponential function of the form $y = a^x$, $a > 0$, including the domain, range, horizontal asymptote and intercepts, and explain the significance of the horizontal asymptote. 9.3 Sketch the graph of an exponential function by applying a set of transformations to the graph of $y = a^x$, $a > 0$, and state the characteristics of the graph. 9.4 Sketch, with or without technology, the graph of a logarithmic function of the form $y = \log_b x$, $b > 1$. 9.5 Identify the characteristics of the graph of a logarithmic function of the form $y = \log_b x$, $b > 1$, including the domain, range, vertical asymptote and intercepts, and explain the significance of the vertical asymptote. 9.6 Sketch the graph of a logarithmic function by applying a set of transformations to the graph of $y = \log_b x$, $b > 1$, and state the characteristics of the graph. 9.7 Demonstrate, graphically, that a logarithmic function and an exponential function with the same base are inverses of each other.
B10. Solve problems that involve exponential and logarithmic equations. [C, CN, PS, R]	10.1 Determine the solution of an exponential equation in which the bases are powers of one another. 10.2 Determine the solution of an exponential equation in which the bases are not powers of one another, using a variety of strategies. 10.3 Determine the solution of a logarithmic equation, and verify the solution. 10.4 Explain why a value obtained in solving a logarithmic equation may be extraneous. 10.5 Solve a problem that involves exponential growth or decay. 10.6 Solve a problem that involves the application of exponential equations to loans, mortgages and investments. 10.7 Solve a problem that involves logarithmic scales, such as the Richter scale and the pH scale. 10.8 Solve a problem by modelling a situation with an exponential or a logarithmic equation.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
B11. Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree $\leq$ 5 with integral coefficients). [C, CN, ME]	11.1 Explain how long division of a polynomial expression by a binomial expression of the form $x-a, a \in I$, is related to synthetic division. 11.2 Divide a polynomial expression by a binomial expression of the form $x-a, a \in I$, using long division or synthetic division. 11.3 Explain the relationship between the linear factors of a polynomial expression and the zeros of the corresponding polynomial function. 11.4 Explain the relationship between the remainder when a polynomial expression is divided by $x-a, a \in I$, and the value of the polynomial expression at $x = a$ (remainder theorem). 11.5 Explain and apply the factor theorem to express a polynomial expression as a product of factors.
B12. Graph and analyze polynomial functions (limited to polynomial functions of degree ≤ 5). [C, CN, T, V]	12.1 Identify the polynomial functions in a set of functions, and explain the reasoning. 12.2 Explain the role of the constant term and leading coefficient in the equation of a polynomial function with respect to the graph of the function. 12.3 Generalize rules for graphing polynomial functions of odd or even degree. 12.4 Explain the relationship between: • the zeros of a polynomial function • the roots of the corresponding polynomial equation • the x-intercepts of the graph of the polynomial function. 12.5 Explain how the multiplicity of a zero of a polynomial function affects the graph. 12.6 Sketch, with or without technology, the graph of a polynomial function. 12.7 Solve a problem by modelling a given situation with a polynomial function and analyzing the graph of the function.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
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Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
B13. Graph and analyze radical functions (limited to functions involving one radical). [CN, R, T, V]	13.1 Sketch the graph of the function $y = \sqrt{x}$, using a table of values, and state the domain and range. 13.2 Sketch the graph of the function $y - k = a\sqrt{b(x-h)}$ by applying transformations to the graph of the function $y = \sqrt{x}$, and state the domain and range. 13.3 Sketch the graph of the function $y = \sqrt{f(x)}$, given the graph of the function $y = f(x)$, and explain the strategies used. 13.4 Compare the domain and range of the function $y = \sqrt{f(x)}$, to the domain and range of the function $y = f(x)$, and explain why the domains and ranges may differ. 13.5 Describe the relationship between the roots of a radical equation and the x-intercepts of the graph of the corresponding radical function. 13.6 Determine, graphically, an approximate solution of a radical equation.
B14. Graph and analyze rational functions (limited to numerators and denominators that are monomials, binomials or trinomials). [CN, R, T, V]	14.1 Graph, with or without technology, a rational function. 14.2 Analyze the graphs of a set of rational functions to identify common characteristics. 14.3 Explain the behaviour of the graph of a rational function for values of the variable near a non-permissible value. 14.4 Determine if the graph of a rational function will have an asymptote or a hole for a non-permissible value. 14.5 Match a set of rational functions to their graphs, and explain the reasoning. 14.6 Describe the relationship between the roots of a rational equation and the x-intercepts of the graph of the corresponding rational function. 14.7 Determine, graphically, an approximate solution of a rational equation.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Permutations, Combinations and Binomial Theorem	**General Outcome:** Develop algebraic and numeric reasoning that involves combinatorics.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
C1. Apply the fundamental counting principle to solve problems. [C, PS, R, V]	1.1 Count the total number of possible choices that can be made, using graphic organizers such as lists and tree diagrams. 1.2 Explain, using examples, why the total number of possible choices is found by multiplying rather than adding the number of ways the individual choices can be made. 1.3 Solve a simple counting problem by applying the fundamental counting principle.
C2. Determine the number of permutations of *n* elements taken *r* at a time to solve problems. [C, PS, R, V]	2.1 Count, using graphic organizers such as lists and tree diagrams, the number of ways of arranging the elements of a set in a row. 2.2 Determine, in factorial notation, the number of permutations of *n* different elements taken *n* at a time to solve a problem. 2.3 Determine, using a variety of strategies, the number of permutations of *n* different elements taken *r* at a time to solve a problem. 2.4 Explain why *n* must be greater than or equal to *r* in the notation ${}_nP_r$. 2.5 Solve an equation that involves ${}_nP_r$ notation, such as ${}_nP_2 = 30$. 2.6 Explain, using examples, the effect on the total number of permutations when two or more elements are identical.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Permutations, Combinations and Binomial Theorem (continued)	**General Outcome:** Develop algebraic and numeric reasoning that involves combinatorics.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
C3. Determine the number of combinations of *n* different elements taken *r* at a time to solve problems. [C, PS, R, V]	3.1 Explain, using examples, the difference between a permutation and a combination. 3.2 Determine the number of ways that a subset of *k* elements can be selected from a set of *n* different elements. 3.3 Determine the number of combinations of *n* different elements taken *r* at a time to solve a problem. 3.4 Explain why *n* must be greater than or equal to *r* in the notation ${}_nC_r$ or $\binom{n}{r}$. 3.5 Explain, using examples, why ${}_nC_r = {}_nC_{n-r}$ or $\binom{n}{r} = \binom{n}{n-r}$. 3.6 Solve an equation that involves ${}_nC_r$ or $\binom{n}{r}$ notation, such as ${}_nC_2 = 15$ or $\binom{n}{2} = 15$.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Permutations, Combinations and Binomial Theorem (continued)	**General Outcome:** Develop algebraic and numeric reasoning that involves combinatorics.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
C4. Expand powers of a binomial in a variety of ways, including using the binomial theorem (restricted to exponents that are natural numbers). [CN, R, V]	4.1 Explain the patterns found in the expanded form of $(x+y)^n$, $n \leq 4$ and $n \in N$, by multiplying n factors of $(x+y)$. 4.2 Explain how to determine the subsequent row in Pascal's triangle, given any row. 4.3 Relate the coefficients of the terms in the expansion of $(x+y)^n$ to the $(n+1)$ row in Pascal's triangle. 4.4 Explain, using examples, how the coefficients of the terms in the expansion of $(x+y)^n$ are determined by combinations. 4.5 Expand, using the binomial theorem, $(x+y)^n$. 4.6 Determine a specific term in the expansion of $(x+y)^n$.