MATHEMATICS 8 AND 9

Integrated Resource Package 2008

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Many people contributed their expertise to this document through the development of the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework (CCF) for Kindergarten to Grade 9 Mathematics, on which this IRP is based. The Project Co-ordinator was Mr. Richard DeMerchant of the Ministry of Education, working with other ministry personnel and our partners in education.

This Integrated Resource Package (IRP) provides basic information teachers will require in order to implement Mathematics 8 and 9. Once fully implemented, this document will supersede the Mathematics 8 and 9 *Integrated Resource Package* (2001).

The information contained in this document is also available on the Internet at www.bced.gov.bc.ca/irp/irp.htm

The following paragraphs provide brief descriptions of the components of the IRP.

INTRODUCTION

The Introduction provides general information about Mathematics 8 and 9, including special features and requirements.

Included in this section are
- a rationale for teaching Mathematics 8 and 9 in BC schools
- goals for Mathematics 8 and 9
- descriptions of the curriculum organizers and suborganizers – groupings for Prescribed Learning Outcomes that share a common focus
- key concepts of Mathematics 8 and 9
- information about learning resources and the Mathematics 8 and 9 Grade Collections
- a suggested timeframe for each grade
- additional information that sets the context for teaching Mathematics 8 and 9

CONSIDERATIONS FOR PROGRAM DELIVERY

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners.

PRESCRIBED LEARNING OUTCOMES

This section contains the Prescribed Learning Outcomes, the legally required content standards for the provincial education system. The Prescribed Learning Outcomes define the required knowledge, skills, and attitudes for each subject. They are statements of what students are expected to know and be able to do by the end of each grade.

The Prescribed Learning Outcomes for the Mathematics 8 and 9 IRP are based on the learning outcomes contained within the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework (CCF) for K to 9 Mathematics, available at www.wncp.ca.

STUDENT ACHIEVEMENT

This section of the IRP contains information about classroom assessment and the measurement of student achievement, including sets of specific Suggested Achievement Indicators for each Prescribed Learning Outcome. Suggested Achievement Indicators are statements that describe what students are able to do in order to demonstrate that they fully meet the expectations set out by the Prescribed Learning Outcomes. Suggested Achievement Indicators are not mandatory; they are provided to assist in the assessment of how well students achieve the Prescribed Learning Outcomes.

The Suggested Achievement Indicators for the Mathematics 8 and 9 IRP are based on the achievement indicators contained within the WNCP Common Curriculum Framework for K to 9 Mathematics.

The WNCP CCF for K to 9 Mathematics is available online at www.wncp.ca under "Mathematics."

Also included in this section are Key Elements – descriptions of content that help determine the intended depth and breadth of Prescribed Learning Outcomes.

GLOSSARY

The glossary section provides a link to an online glossary that contains definitions for selected terms used in this Integrated Resource Package.

INTRODUCTION

Mathematics 8 and 9

This Integrated Resource Package (IRP) sets out the provincially prescribed curriculum for Mathematics 8 and 9. The development of this IRP has been guided by the principles of learning:

- Learning requires the active participation of the student.
- People learn in a variety of ways and at different rates.
- Learning is both an individual and a group process.

In addition to these three principles, this document recognizes that British Columbia's schools include young people of varied backgrounds, interests, abilities, and needs. Wherever appropriate for this curriculum, ways to meet these needs and to ensure equity and access for all learners have been integrated as much as possible into the Prescribed Learning Outcomes and Suggested Achievement Indicators.

Mathematics 8 and 9 is based on the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework (CCF) for Kindergarten to Grade 9 Mathematics (May 2006). A complete list of references used to inform the revisions of the WNCP CCF for K to 9 Mathematics as well as this IRP can be found at the end of this Introduction.

RATIONALE

The aim of Mathematics 8 and 9 is to provide students with the opportunity to further their knowledge, skills, and attitudes related to mathematics.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Numeracy can be defined as the combination of mathematical knowledge, problem solving, and communication skills required by all persons to function successfully within our technological world. Numeracy is more than knowing about numbers and number operations (British Columbia Association of Mathematics Teachers 1998).

Students learn by attaching meaning to what they do, and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of a variety of manipulatives and pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics. Information gathered from these discussions can be used for formative assessment to guide instruction.

As facilitators of learning, educators are encouraged to highlight mathematics concepts as they occur within the Grades 8 and 9 school environment and within home environments. Mathematics concepts are present within every school subject, and drawing students' attention to these concepts as they occur can provide "teachable moments."

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary. Positive learning experiences build self-confidence and develop attitudes that value learning mathematics.

ABORIGINAL PERSPECTIVE

Aboriginal students in British Columbia come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings, including urban, rural, and isolated communities. Teachers need to understand the diversity of cultures and experiences of students.

Aboriginal students come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is also vital that teachers

understand and respond to non-verbal cues so that student learning and mathematical understanding are optimized. Depending on their learning styles, students may look for connections in learning and learn best when mathematics is contextualized and not taught as discrete components.

A variety of teaching and assessment strategies is required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences, and learning styles of students. *The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks & Banks 1993).*

AFFECTIVE DOMAIN

Bloom's taxonomy of learning behaviours identified three domains of educational activities: affective (growth in feelings or emotional areas – attitude), cognitive (mental skills – knowledge), and psychomotor (manual or physical skills – skills). The affective domain involves the way in which we perceive and respond to things emotionally, such as feelings, values, appreciation, enthusiasms, motivations, and attitudes.

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help students develop and maintain positive attitudes and self-confidence. Research has shown that students who are more engaged with school and with mathematics are far more likely to be successful in school and in learning mathematics (Nardi & Steward 2003). Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

Substantial progress has been made in the last decade in research that has examined the importance and use of the affective domain as part of the learning process. In addition, a parallel increase in specific research involving the affective domain and its relationship to the learning of mathematics has provided powerful evidence of the importance of this area to the learning of mathematics (McLeod 1988, 1992 & 1994; Hannula 2002 & 2006; Malmivuori 2001 & 2006). Teachers, students, and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Students who feel more comfortable with a subject demonstrate more confidence and have the opportunity for greater academic achievement (Denton & McKinney 2004; Hannula 2006; Smith et al. 1998). Educators can include opportunities for active and co-operative learning in their mathematics lessons, which has been shown in research to promote greater conceptual understanding, more positive attitudes, and subsequently improved academic achievement (Denton & McKinney 2004). By allowing sharing and discussion of answers and strategies used in mathematics, educators are providing rich opportunities for students' mathematical development. Educators can foster greater conceptual understanding in students by having them practise mathematics topics and concepts in a meaningful and engaging manner.

It is important for educators, students, and parents to recognize the relationship between the affective and cognitive domains and attempt to nurture those aspects of the affective domain that contribute to positive attitudes and success in learning.

NATURE OF MATHEMATICS

Mathematics is one way of trying to understand, interpret, and describe our world. Components integral to the nature of mathematics include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty. These components are woven throughout this curriculum.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, p. 184)

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (The Primary Program 2000, p. 146).

A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms.

Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment.

Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems.

Learning to work with patterns in the early grades helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations, and identify relationships to mathematical strands.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

GOALS FOR MATHEMATICS 8 AND 9

Mathematics 8 and 9 continues the path students begin in Mathematics K to 7 toward becoming lifelong learners of mathematics.

GOALS FOR MATHEMATICS 8 AND 9

The Mathematics 8 and 9 curriculum is meant to reinforce the main goals of mathematics education:

- using mathematics confidently to solve problems
- using mathematics to better understand the world around us
- communicating and reasoning mathematically
- appreciating and valuing mathematics
- making connections between mathematics and its applications
- committing themselves to lifelong learning
- becoming mathematically literate and using mathematics to participate in, and contribute to, society

Students who have met these goals will:

- gain an understanding and appreciation of the contributions of mathematics as a science, philosophy, and art
- be able to use mathematics to make and justify decisions about the world around us
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

CURRICULUM ORGANIZERS

A curriculum organizer consists of a set of Prescribed Learning Outcomes that share a common focus. The Prescribed Learning Outcomes for Mathematics 8 and 9 progress in age-appropriate ways, and are grouped under the following curriculum organizers and suborganizers:

Curriculum Organizers and Suborganizers
MATHEMATICS 8 AND 9
NUMBER
PATTERNS AND RELATIONS • Patterns • Variables and Equations
SHAPE AND SPACE • Measurement • 3-D Objects and 2-D Shapes • Transformations
STATISTICS AND PROBABILITY • Data Analysis • Chance and Uncertainty

These curriculum organizers reflect the main areas of mathematics that students are expected to address. The ordering of organizers, suborganizers, and outcomes in the Mathematics 8 and 9 curriculum does not imply an order of instruction. The order in which various topics are addressed is left to the professional judgment of teachers. Mathematics teachers are encouraged to integrate topics throughout the curriculum and within other subject areas to emphasize the connections between mathematics concepts.

Number

Students develop their concept of the number system and relationships between numbers. Concrete, pictorial, and symbolic representations are used to help students develop their number sense. Computational fluency, the ability to connect understanding of the concepts with accurate, efficient, and flexible computation strategies for multiple purposes, is stressed throughout the Number organizer with an emphasis on the development of personal strategies, mental mathematics, and estimation strategies.

The Number organizer does not contain any suborganizers.

Patterns and Relations

Students develop their ability to recognize, extend, create, and use numerical and non-numerical patterns to better understand the world around them as well as the world of mathematics. This organizer provides opportunities for students to look for relationships in the environment and to describe the relationships. These relationships should be examined in multiple sensory forms.

The Patterns and Relations organizer includes the following suborganizers:
- Patterns
- Variables and Equations

Shape and Space

Students develop their understanding of objects and shapes in the environment around them. This includes recognition of attributes that can be measured, measurement of these attributes, description of these attributes, identification and use of referents, and positional change of 3-D objects and 2-D shapes on the environment and on the Cartesian plane.

The Shape and Space organizer includes the following suborganizers:
- Measurement
- 3-D Objects and 2-D Shapes
- Transformations

Statistics and Probability

Students collect, interpret, and present data sets in relevant contexts to make decisions. The development of concepts involving probability is also presented as a means to make decisions.

The Shape and Space organizer includes the following suborganizers:
- Data Analysis
- Chance and Uncertainty

MATHEMATICS 8 AND 9: KEY CONCEPTS

* Grade 7 Key Concepts have been included to show the progression of Mathematics concepts into Grade 8.

	Grade 7 *	Grade 8	Grade 9
Number	• divisibility rules • addition, subtraction, multiplication, and division of numbers • percents from 1% to 100% • decimal and fraction relationships for repeating and terminating decimals • addition and subtraction of positive fractions and mixed numbers • addition and subtraction of integers	• perfect squares and square roots • percents greater than or equal to 0% • rate, ratio, and proportional reasoning • multiplication and division of fractions and mixed numbers • multiplication and division of integers	• powers • rational numbers • square root of positive rational numbers
Patterns and Relations *Patterns*	• table of values and graphs of linear relations	• two-variable linear relations	• graph of linear relations • interpolation and extrapolation
Patterns and Relations *Variables and Equations*	• preservation of equality • expressions and equations • one-step linear equations	• linear equations	• linear equations • single variable linear inequalities • operations on polynomials
Shape and Space *Measurement*	• properties of circles • area of triangles, parallelograms, and circles	• Pythagorean theorem • 3-D object nets • surface area of right prisms and cylinders • formulas for right prisms and cylinders	• circle properties
Shape and Space *3-D Objects and 2-D Shapes*	• geometric constructions	• top, front, and side view of 3-D objects	• surface area of composite 3-D objects • similarity of polygons
Shape and Space *Transformations*	• four quadrants of the Cartesian plane • transformations in the four quadrants of the Cartesian plane	• tessellations	• scale diagrams of 2-D shapes • line and rotational symmetry
Statistics and Probability *Data Analysis*	• central tendency, outliers, and range • circle graphs	• presentation of data	• data collection • population and sample data • project plan for collection, display, and analysis of data
Statistics and Probability *Chance and Uncertainty*	• ratios, fractions, and percents to express probabilities • two independent events • tree diagrams for two independent events	• independent events	• probability in society

MATHEMATICS 8 AND 9: KEY CONCEPTS

In Grade 10, students may choose to take Apprenticeship and Workplace Mathematics 10, Foundations of Mathematics and Pre-Calculus 10, or both courses.

* Grade 10 Key Concepts have been included to show the progression of Mathematics concepts from Grade 9.

	Apprenticeship and Workplace Mathematics 10*
Measurement	• SI and Imperial measurement systems • regular, composite, and irregular 2-D shape and 3-D objects
Geometry	• puzzles and games involving spatial reasoning • Pythagorean theorem • similarity of regular and irregular convex polygons • primary trigonometric ratios • parallel, perpendicular, transversal lines, and resulting angles • acute, obtuse, straight, and reflex angles
Number	• proportional reasoning through unit pricing and currency exchange • forms of income
Algebra	• manipulation and application of formulas

	Foundations of Mathematics and Pre-Calculus 10*
Measurement	• SI and Imperial measurement systems • surface area and volume of 3-D objects
Algebra and Number	• factors of whole numbers • irrational numbers • powers with integral and rational exponents • multiplication of polynomial expressions • common and trinomial factors
Relations and Functions	• relationships between data, graphs, and situations • relations and functions • slope • characteristics of linear relations and functions

MATHEMATICAL PROCESSES

Students must encounter critical components in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to
- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The following seven mathematical processes should be integrated within Mathematics 8 and 9.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students need to be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine 1991, p. 5).

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhances flexible thinking and number sense. It is calculating mentally without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility.

Even more important than performing computational procedures or using calculators is the greater facility that students need – more than ever before – with estimation and mental mathematics (NCTM May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein 2001).

Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating.

Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you...?" or "How could you...?" the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong 1993, p. 10). The use of visualization in the study of mathematics provides students with the opportunity to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to decide when to measure, when to estimate, and to know several estimation strategies (Shaw & Cliatt 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

LEARNING RESOURCES

For the current list of Mathematics 8 and 9 recommended learning resources, please check the Learning Resource website: www.bced.gov.bc.ca/irp_resources/lr/resource/gradcoll.htm

The Grade Collection chart lists the recommended learning resources by media format, showing links to the curriculum organizers and sub-organizers. The chart is followed by an annotated bibliography. Teachers should check with suppliers for complete and up-to-date ordering information. Math resources approved by WNCP are recommended for use for Mathematics 8 and 9 in BC. See www.wncp.ca for details.

Ministry policy concerning Learning Resources can be found on the ministry's policy website: www.bced.gov.bc.ca/policy/policies

SUGGESTED TIMEFRAME

Provincial curricula are developed in accordance with the amount of instructional time recommended by the Ministry of Education for each subject area. Teachers may choose to combine various curricula to enable students to integrate ideas and make meaningful connections.

For Mathematics 8 and 9, the Ministry of Education recommends a time allotment of approximately 20% of the total instructional time for each school year. This includes both mathematics instruction time and incorporation of mathematics content within other subject areas.

For both of these grades, this recommended time allotment includes a minimum of 100 hours of mathematics instruction per year. In addition, it is suggested that mathematics content be incorporated and applied within other subject areas to reinforce concepts. Suggested contexts where mathematics can be incorporated into other subject areas can be found on page 35 of this IRP in the Applying Mathematics section of the Considerations for Program Delivery.

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CONSIDERATIONS FOR PROGRAM DELIVERY

Mathematics 8 and 9

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners. Included in this section is information about

- Alternative Delivery policy
- addressing local contexts
- involving parents and guardians
- establishing a positive classroom climate
- confidentiality
- inclusion, equity, and accessibility for all learners
- working with the school and community
- working with the Aboriginal community
- information and communications technology
- copyright and responsibility
- fostering the development of positive attitudes in mathematics
- instructional focus
- applying mathematics
-

ALTERNATIVE DELIVERY POLICY

The Alternative Delivery policy does not apply to the Mathematics 8 and 9 curriculum.

The Alternative Delivery policy outlines how students, and their parents or guardians, in consultation with their local school authority, may choose means other than instruction by a teacher within the regular classroom setting for addressing Prescribed Learning Outcomes contained in the Health curriculum organizer of the following curriculum documents:

- Health and Career Education K to 7
- Health and Career Education 8 and 9
- Planning 10

The policy recognizes the family as the primary educator in the development of children's attitudes, standards, and values, but the policy still requires that all Prescribed Learning Outcomes be addressed and assessed in the agreed-upon alternative manner of delivery.

It is important to note the significance of the term "alternative delivery" as it relates to the Alternative Delivery policy. The policy does not permit schools to omit addressing or assessing any of the Prescribed Learning Outcomes within the health and career education curriculum.

Neither does it allow students to be excused from meeting any Prescribed Learning Outcomes related to health. It is expected that students who arrange for alternative delivery will address the health-related Prescribed Learning Outcomes and will be able to demonstrate their understanding of these Prescribed Learning Outcomes.

For more information about policy relating to alternative delivery, refer to www.bced.gov.bc.ca/policy/

ADDRESSING LOCAL CONTEXTS

The Mathematics 8 and 9 curriculum includes opportunities for individual teacher and student choice in specific activities to meet certain Prescribed Learning Outcomes. This flexibility allows educators to plan their programs to meet the particular requirements of their students and to respond to local contexts. It may be appropriate to incorporate student input when selecting relevant activities.

INVOLVING PARENTS AND GUARDIANS

The family is the primary educator in the development of students' attitudes and values. The school plays a supportive role by focussing on the Prescribed Learning Outcomes in the Mathematics 8 and 9 curriculum. Parents and guardians can support, enrich, and extend the curriculum at home.

It is highly recommended that schools inform parents and guardians about the Mathematics 8 and 9 curriculum. Teachers (along with school and district administrators) may choose to do so by

- informing parents/guardians and students of the Prescribed Learning Outcomes for the course (e.g., by sending home class letters, providing an overview during parent-teacher interviews)
- responding to parent and guardian requests to discuss the course, unit plans, and learning resources

ESTABLISHING A POSITIVE CLASSROOM CLIMATE

Teachers are responsible for setting and promoting a classroom climate in which students feel comfortable learning about and discussing topics in Mathematics 8 and 9.

The following guidelines may help educators establish and promote a positive classroom climate:

- Allow class members sufficient time and opportunities to become comfortable with each other before engaging in group discussion. It is important that the classroom climate encourages students to relate to one another in positive, respectful, and supportive ways. Be prepared to facilitate any potentially controversial discussions.
- Establish clear ground rules for class discussions that demonstrate respect for privacy, for diversity, and for the expression of differing viewpoints.
- Ensure that any external groups or organizations making a presentation to students have met the district's guidelines for presenting. There should be a direct relationship between the content of the presentation and the Prescribed Learning Outcomes. Review any materials they may use, especially handouts, for appropriateness.
- Become familiar with
  - relevant legislation (e.g., *Human Rights Code; Child, Family and Community Services Act*)
  - relevant initiatives (e.g., *Safe, Caring and Orderly Schools: A Guide* and *Diversity in BC Schools: A Framework*)
  - provincial and district policies and protocols concerning topics such as disclosure related to child abuse and protection of privacy

Further information about these policies and initiatives is available online:

BC Handbook for Action on Child Abuse and Neglect
www.mcf.gov.bc.ca/child_protection/pdf/handbook_action_child_abuse.pdf

Safe, Caring and Orderly Schools: A Guide
www.bced.gov.bc.ca/sco/

Diversity in BC Schools: A Framework
www.bced.gov.bc.ca/diversity/diversity_framework.pdf

Human Rights Code
www.qp.gov.bc.ca/statreg/stat/H/96210_01.htm

Child, Family and Community Services Act
www.qp.gov.bc.ca/statreg/stat/C/96046_01.htm

CONFIDENTIALITY

The *Freedom of Information and Protection of Privacy Act* (FOIPPA) applies to students, to school districts, and to all curricula. Teachers, administrators, and district staff should consider the following:

- Be aware of district and school guidelines regarding the provisions of FOIPPA and how it applies to all subjects, including Mathematics 8 and 9.
- Do not use students' Personal Education Numbers (PENs) on any assignments that students wish to keep confidential.
- Ensure students are aware that if they disclose personal information that indicates they are at risk for harm, then that information cannot be kept confidential.
- Inform students of their rights under FOIPPA, especially the right to have access to their own personal information in their school records. Inform parents of their rights to access their children's school records.
- Minimize the type and amount of personal information collected, and ensure that it is used only for purposes that relate directly to the reason for which it is collected.
- Inform students that they will be the only ones recording personal information about themselves unless they, or their parents, have consented to teachers collecting that information from other people (including parents).
- Provide students and their parents with the reason(s) they are being asked to provide personal information in the context of the Mathematics 8 and 9 curriculum.
- Inform students and their parents that they can ask the school to correct or annotate any of the personal information held by the school, in accordance with Section 29 of FOIPPA.
- Ensure students are aware that their parents may have access to the schoolwork they create only insofar as it pertains to students' progress.
- Ensure that any information used in assessing students' progress is up-to-date, accurate, and complete.

For more information about confidentiality, refer to
www.cio.gov.bc.ca/services/privacy/

INCLUSION, EQUITY, AND ACCESSIBILITY FOR ALL LEARNERS

British Columbia's schools include young people of varied backgrounds, interests, and abilities. The Kindergarten to Grade 12 school system focusses on meeting the needs of all students. When selecting specific topics, activities, and resources to support the implementation of Mathematics 8 and 9, teachers are encouraged to ensure that these choices support inclusion, equity, and accessibility for all students. In particular, teachers should ensure that classroom instruction, assessment, and resources reflect sensitivity to diversity and incorporate positive role portrayals, relevant issues, and themes such as inclusion, respect, and acceptance.

Government policy supports the principles of integration and inclusion of students for whom English is a second language and of students with special needs. Most of the Prescribed Learning Outcomes and Suggested Achievement Indicators in this IRP can be met by all students, including those with special needs and/or ESL needs. Some strategies may require adaptations to ensure that those with special and/or ESL needs can successfully achieve the Prescribed Learning Outcomes. Where necessary, modifications can be made to the Prescribed Learning Outcomes for students with Individual Education Plans (IEPs).

For more information about resources and support for students with special needs, refer to www.bced.gov.bc.ca/specialed/

For more information about resources and support for ESL students, refer to www.bced.gov.bc.ca/esl/

WORKING WITH THE SCHOOL AND COMMUNITY

This curriculum addresses a wide range of skills and understandings that students are developing in other areas of their lives. It is important to recognize that learning related to this curriculum extends beyond the classroom.

School and district-wide programs and initiatives may support and extend learning in Mathematics 8 and 9. Community organizations may also support the curriculum with locally developed learning resources, guest speakers, workshops, and field studies. Teachers may wish to draw on the expertise of these community organizations and members.

Bringing outside resource people into the classroom is an effective way of reinforcing content, emphasizing and practising listening skills, exposing students to diverse points of view, providing opportunities for discussion and debate, providing a departure point for writing and other activities, and making learning more concrete and relevant. A panel discussion also provides an opportunity for several viewpoints on an issue to be presented at the same time.

To help achieve a successful guest speaker activity, consider the following:

- Determine the nature of the presentation (e.g., lecture, question-and-answer, debate, response to students' presentations, facilitation of a simulation or case study). Ensure that guest speakers are clear about their purpose, the structure, and the time allotted. Also ensure that guests understand the skill and developmental levels of students. Review any materials speakers may use, especially any handouts, for appropriateness.
- Be aware of any district guidelines for external presenters, and ensure that guests have met these guidelines.
- Where appropriate, have students take responsibility for contacting the guest(s) beforehand and making any logistical arrangements.

WORKING WITH THE ABORIGINAL COMMUNITY

The Ministry of Education is dedicated to ensuring that the cultures and contributions of Aboriginal peoples in BC are reflected in all provincial curricula. To address these topics in the classroom in a way that is accurate and that respectfully reflects Aboriginal concepts of teaching and learning, teachers are strongly encouraged to seek the advice and support of local Aboriginal communities. Aboriginal communities are diverse in terms of language, culture, and available resources, and each community will have its own unique protocol to gain support for integration of local knowledge and expertise. To begin discussion of possible instructional and assessment activities, teachers should first contact Aboriginal education co-ordinators, teachers, support workers, and counsellors in their district who will be able to facilitate the identification of local resources and contacts such as elders, chiefs, tribal or band councils, Aboriginal cultural centres, Aboriginal Friendship Centres, and Métis or Inuit organizations.

In addition, teachers may wish to consult the various Ministry of Education publications available, including the "Planning Your Program" section of the resource, *Shared Learnings* (2006). This resource was developed to help all teachers provide students with knowledge of, and opportunities to share experiences with, Aboriginal peoples in BC.

For more information about these documents, consult the Aboriginal Education web site: www.bced.gov.bc.ca/abed/welcome.htm

INFORMATION AND COMMUNICATIONS TECHNOLOGY

The study of information and communications technology is increasingly important in our society. Students need to be able to acquire and analyse information, to reason and communicate, to make informed decisions, and to understand and use information and communications technology for a variety of purposes. Development of these skills is important for students in their education, their future careers, and their everyday lives.

Literacy in the area of information and communications technology can be defined as the ability to obtain and share knowledge through investigation, study, instruction, or transmission of information by means of media technology. Becoming literate in this area involves finding, gathering, assessing, and communicating information using electronic means, as well as developing the knowledge and skills to use and solve problems effectively with the technology. Literacy also involves a critical examination and understanding of the ethical and social issues related to the use of information and communications technology. Mathematics 8 and 9 provides opportunities for students to develop literacy in relation to information and communications technology sources, and to reflect critically on the role of these technologies in society.

COPYRIGHT AND RESPONSIBILITY

Copyright is the legal protection of literary, dramatic, artistic, and musical works; sound recordings; performances; and communications signals. Copyright provides creators with the legal right to be paid for their work and the right to say how their work is to be used. The law permits certain exceptions for schools (i.e., specific things permitted) but these are very limited, such as copying for private study or research. The copyright law determines how resources can be used in the classroom and by students at home.

In order to respect copyright it is necessary to understand the law. It is unlawful to do the following, unless permission has been given by a copyright owner:

- photocopy copyrighted material to avoid purchasing the original resource for any reason
- photocopy or perform copyrighted material beyond a very small part – in some cases the copyright law considers it "fair" to copy whole works, such as an article in a journal or a photograph, for purposes of research and private study, criticism, and review
- show recorded television or radio programs to students in the classroom unless these are cleared for copyright for educational use (there are exceptions such as for news and news commentary taped within one year of broadcast that by law have record-keeping requirements – see the web site at the end of this section for more details)
- photocopy print music, workbooks, instructional materials, instruction manuals, teacher guides, and commercially available tests and examinations
- show video recordings at schools that are not cleared for public performance
- perform music or copyrighted material for entertainment (i.e., for purposes other than a specific educational objective)
- copy work from the Internet without an express message that the work can be copied

Permission from or on behalf of the copyright owner must be given in writing. Permission may also be given to copy or use all or some portion of copyrighted work through a licence or agreement. Many creators, publishers, and producers have formed groups or "collectives" to negotiate royalty payments and copying conditions for educational institutions. It is important to know what licences are in place and how these affect the activities schools are involved in. Some licences may also require royalty payments that are determined by the quantity of photocopying or the length of performances. In these cases, it is important to assess the educational value and merits of copying or performing certain works to protect the school's financial exposure (i.e., only copy or use the portion that is absolutely necessary to meet an educational objective).

It is important for education professionals, parents, and students to respect the value of original thinking and the importance of not plagiarizing the work of others. The works of others should not be used without their permission.

For more information about copyright, refer to www.cmec.ca/copyright/indexe.stm

FOSTERING THE DEVELOPMENT OF POSITIVE ATTITUDES IN MATHEMATICS

A positive attitude toward mathematics is often a result of a learning environment in the classroom that encourages students' own mathematical thinking and contributions to classroom activities and discussions. Teachers should use a variety of instructional approaches in the classroom in order to reach students with a variety of learning styles and dispositions. These include experiences that encourage students to

- enjoy and value mathematics
- develop mathematical habits of mind
- explore
- take risks
- exhibit curiosity
- make and correct errors
- persevere
- experience mathematics in non-threatening, engaging ways
- understand and appreciate the role of mathematics in human affairs

These learning opportunities enable students to gain confidence in their abilities to solve complex problems.

The assessment of attitudes is indirect, and based on inferences drawn from students' behaviour. We can see what students do and hear what they say, and from these observations make inferences and draw conclusions about their attitudes.

It is important for teachers to consider their role in developing a positive attitude in mathematics. Teachers and parents are role models from whom students begin to develop their disposition toward mathematics. Teachers need to model these attitudes in order to help students develop them (Burns 2000). In this manner teachers need to "present themselves as problem solvers, as active learners who are seekers, willing to plunge into new situations, not always knowing the answer or what the outcome will be" (p. 29).

INSTRUCTIONAL FOCUS

The Mathematics 8 and 9 courses are arranged into a number of organizers with mathematical processes integrated throughout. Students learn in different ways and at different rates. As in other subject areas, it is essential when teaching mathematics that concepts are introduced to students in a variety of ways. Students should hear explanations, watch demonstrations, draw to represent their thinking, engage in experiences with concrete materials, and be encouraged to visualize and discuss their understanding of concepts. Most students need a range of concrete or representational experiences with mathematics concepts before they develop symbolic or abstract understanding. The development of conceptual understanding should be emphasized throughout the curriculum as a means to help students become mathematical problem solvers.

Teaching through Problem Solving

Problem solving should be an integral part of all mathematics classrooms. Teachers are encouraged to weave problem solving throughout all curriculum organizers in the grades 8 and 9 mathematics curriculum on a regular basis. Problem solving provides a way of helping students learn mathematics.

Hiebert et al. (1996) encourage teachers to make mathematics problematic. A problem can be defined as any task or activity for which the students have not memorized a method or rule, nor is there an assumption by the students that there is only one correct way to solve the problem (Hiebert et al. 1997). Van de Walle (2006, p.11) notes that "a problem for learning mathematics also has these features:

- The problem must begin where the students are.
- The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn.
- The problem must require justifications and explanations for answers and methods."

The research suggests that teaching through problem solving has the following advantages:

- The math makes more sense. When using real-world math problems, students are able to make the connections between what math is and how they can apply it.
- Problems are more motivating when they are challenging. Although some students are anxious when they are not directed by the

teacher, most enjoy a challenge they can be successful in solving.
- Problem solving builds confidence. It maximizes the potential for understanding as each child makes his or her own sense out of the problem and allows for individual strategies.
- Problem solving builds perseverance. Because an answer is not instantaneous, many children think they are unable to do the math. Through the experience of problem solving they learn to apply themselves for longer periods of time and not give up.
- Problems can provide practice with concepts and skills. Good problems enable students to learn and apply the concepts in a meaningful way and an opportunity to practise the skills.
- Problem solving provides students with insight into the world of mathematics. Mathematicians struggle to find solutions to many problems and often need to go down more than one path to arrive at a solution. This is a creative process that is difficult to understand if one has never had to struggle.
- Problem solving provides the teacher with insight into a student's mathematical thinking. As students choose strategies and solve problems, the teacher has evidence of their thinking and can inform instruction based on this.
- Students need to practise problem solving. If we are expecting students to confront new situations involving mathematics, they need practice to become independent problem solvers (Small 2005).

Polya (1957) characterized a general method that can be used to solve problems, and to describe how problem solving should be taught and learned. He advocated for the following steps in solving a mathematical problem:

- Understand the problem – What is unknown? What is known? Is enough information provided to determine the solution? Can a figure or model be used to represent the situation?
- Make a plan – Is there a similar problem that has been solved before? Can the problem be restated so it makes more sense?
- Carry out the plan – Have all of the steps been completed correctly?
- Look back – Do the results look correct? Is there another way to solve the problem that would verify the results?

While there are a number of variations of the problem-solving model proposed by Polya (Van de Walle 2006, Small 2006, Burns 2000) they all have similar characteristics. The incorporation of a wide variety of strategies to solve problems is essential to developing students' ability to be flexible problem solvers.

The Mathematics 8 and 9 (2001) IRP provides a number of useful strategies that students can use to increase their flexibility in solving problems. These include:
- look for a pattern
- construct a table
- make an organized list
- act it out
- draw a picture
- use objects
- guess and check
- work backward
- write an equation
- solve a simpler (or similar) problem
- make a model (BC Ministry of Education 2001)

During problem-solving experiences, students are encouraged to solve problems using ways that make sense to them. As students share different ways of solving problems they can learn strategies from each other. Teachers are encouraged to facilitate this process to in an open and non-threatening environment. In this manner, students can develop a repertoire of strategies from which to draw upon when mathematical problems are presented to them.

Problem solving requires a shift in student attitudes and how teachers model these attitudes in the classroom. In order to be successful, students must develop, and teachers model, the following characteristics:
- interest in finding solutions to problems
- confidence to try various strategies
- willingness to take risks
- ability to accept frustration when not knowing
- understanding the difference between not knowing the answer and not having found it yet (Burns 2000)

Problems should not just be simple computations embedded in a story or be contrived, that is, existing only in the math classroom. Students will be engaged if the problems relate to their lives – their culture, interests, families, and current events. Good problems are tasks that are rich and open-ended so there is more than one

way of arriving at a solution, or multiple answers. They should allow for every student in the class to demonstrate his or her knowledge, skill, or understanding. The student should not know the answer immediately. Problem solving takes time and effort on the part of the student and the teacher. Teaching though problem solving is one of the ways teachers can bring increased depth to the Mathematics 8 and 9 curriculum.

Instruction should provide an emphasis on mental mathematics and estimation to check the reasonableness of paper and pencil exercises, and the solutions to problems that are determined through the use of technology, including calculators and computers. (It is assumed that all students have regular access to appropriate technology such as calculators, or computers with graphing software and standard spreadsheet programs.) Concepts should be introduced using manipulatives, and gradually developed from the concrete to the pictorial to the symbolic.

APPLYING MATHEMATICS

For students to view mathematics as relevant and useful, they must see how it can be applied in a variety of contexts. Mathematics helps students understand and interpret their world and solve problems that occur in their daily lives both within and outside of the school context.

Teachers are encouraged to incorporate, and make explicit, mathematics concepts that naturally occur across the subject areas. Possible situations where cross-curricular integration may occur in grades 8 and 9 include the following:

Fine Arts
- creating nets to represent objects
- drawing views of objects from different orientations
- examining the similarity of objects and lines
- identifying the fractions used in a particular rhythm and metre
- identifying geometric shapes in visual arts, drama, and dance
- explaining symmetry and unison
- creating transformations such as tessellations
- measuring and proportional reasoning for mixing and applying materials in visual arts

Health and Career Education
- creating tables and schedules
- interpreting statistical data to make decisions
- collecting, organizing, and interpreting data charts, graphs, diagrams, and tables
- using mathematics to develop a logical argument to support a position on a topic or issue

Language Arts
- reading literature with a mathematics theme
- creating a picture book or writing a story with mathematical content
- listening to stories to decode mathematical contexts
- examining the plot of a story from a mathematical perspective
- creating graphic organizers to provide an explanation, proof, or justification for an argument
- role-playing or presenting oral presentations of problems and solutions
- creating word walls, personal dictionaries, or glossaries of mathematics terms
- researching and examining the roots of mathematical terms and personalities

Physical Education
- examining the benefits of various physical activities (e.g., burning calories)
- examining patterns in physical movement
- measuring distances that require mathematical calculations (e.g., the height a ball is thrown)
- estimating distances and other quantities using referents
- reading and recording dates and time
- gathering and interpreting data relating to healthy lifestyles and exercise

Science
- discussing the magnitude of numbers including large and small quantities
- drawing representations of experimental observations
- determining and interpreting patterns in charts and representing them algebraically
- examining patterns to make a hypothesis
- conducting and recording measurements
- using referents for estimating measurement
- making conversions between units
- solving formulas to determine the answer to questions
- reading and writing quantities in multiple formats (e.g., numerals, words)
- collecting, organizing, and interpreting data charts, graphs, diagrams, and tables

- creating a logical argument to support a hypothesis
- using mental mathematics for calculations

Social Studies
- discussing the magnitude of numbers and building referents for numbers
- graphing using the Cartesian plane
- using circle concepts to explain latitude and longitude, time zones, and great circle routes
- interpreting statistical data and making inferences
- defending or rejecting a historical decision based on statistics
- collecting, organizing, and interpreting data charts, graphs, diagrams, and tables
- examining the history of mathematics in the context of world events
- explaining the role of probability in society
- using mathematics to develop a logical argument to support a position on a topic or issue

Students can also be encouraged to identify and examine the mathematics around them. In this way, students will come to see that mathematics is present outside of the classroom. Students may encounter mathematics in many aspects of their daily lives, such as
- making decisions that impact the environment
- making purchases
- reading bus schedules
- reading sports statistics
- interpreting newspaper and media sources
- following a recipe
- estimating time to complete tasks
- estimating quantities
- using symmetry when doodling

Making these connections explicit for students helps to solidify the importance of mathematics.

PRESCRIBED LEARNING OUTCOMES

Mathematics 8 and 9

Prescribed Learning Outcomes are content standards for the provincial education system; they are the prescribed curriculum. Clearly stated and expressed in measurable and observable terms, Prescribed Learning Outcomes set out the required knowledge, skills, and attitudes – what students are expected to know and be able to do – by the end of the specified subject and grade.

UNDERSTANDING THE PRESCRIBED LEARNING OUTCOMES

Schools have the responsibility to ensure that all Prescribed Learning Outcomes in this curriculum are met; however, schools have flexibility in determining how delivery of the curriculum can best take place.

It is expected that student achievement will vary in relation to the Prescribed Learning Outcomes. Evaluation, reporting, and student placement with respect to these outcomes are dependent on the professional judgment and experience of teachers, guided by provincial policy.

Prescribed Learning Outcomes for Mathematics 8 and 9 are presented by grade and by curriculum organizer and suborganizer, and are coded alphanumerically for ease of reference. This arrangement, however, is not intended to imply a required instructional sequence.

Wording of the Prescribed Learning Outcomes

All Prescribed Learning Outcomes complete the stem, "It is expected that students will …."

When used in a Prescribed Learning Outcome, the word "including" indicates that any ensuing item must be addressed. Lists of items introduced by the word "including" represent a set of minimum requirements associated with the general requirement set out by the outcome. The lists are not necessarily exhaustive, however, and teachers may choose to address additional items that also fall under the general requirement set out by the outcome.

Conversely, the abbreviation "e.g.," (for example) in a Prescribed Learning Outcome indicates that the ensuing items are provided for illustrative purposes or clarification, and are not required. Presented in parentheses, the list of items introduced by "e.g.," is neither exhaustive nor prescriptive, nor is it put forward in any special order of importance or priority. Teachers are free to substitute items of their own choosing that they feel best address the intent of the Prescribed Learning Outcome.

DOMAINS OF LEARNING

Prescribed Learning Outcomes in BC curricula identify required learning in relation to one or more of the three domains of learning: cognitive, psychomotor, and affective. The following definitions of the three domains are based on Bloom's taxonomy.

The **cognitive domain** deals with the recall or recognition of knowledge and the development of intellectual abilities. The cognitive domain can be further specified as including three cognitive levels: knowledge, understanding and application, and higher mental processes. These levels are determined by the verb used in the Prescribed Learning Outcome, and illustrate how student learning develops over time.

- *Knowledge* includes those behaviours that emphasize the recognition or recall of ideas, material, or phenomena.
- *Understanding and application* represents a comprehension of the literal message contained in a communication, and the ability to apply an appropriate theory, principle, idea, or method to a new situation.
- *Higher mental processes* include analysis, synthesis, and evaluation. The higher mental processes level subsumes both the knowledge and the understanding and application levels.

The **affective domain** concerns attitudes, beliefs, and the spectrum of values and value systems.

The **psychomotor domain** includes those aspects of learning associated with movement and skill demonstration, and integrates the cognitive and affective consequences with physical performances.

Domains of learning as represented in Mathematics 8 and 9 are addressed in the Assessment Overview Tables provided for each grade in the Student Achievement section.

PRESCRIBED LEARNING OUTCOMES

By Grade

GRADE 8

It is expected that students will:

NUMBER

A1 demonstrate an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers)
[C, CN, R, V]

A2 determine the approximate square root of numbers that are not perfect squares (limited to whole numbers)
[C, CN, ME, R, T]

A3 demonstrate an understanding of percents greater than or equal to 0%
[CN, PS, R, V]

A4 demonstrate an understanding of ratio and rate
[C, CN, V]

A5 solve problems that involve rates, ratios, and proportional reasoning
[C, CN, PS, R]

A6 demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically
[C, CN, ME, PS]

A7 demonstrate an understanding of multiplication and division of integers, concretely, pictorially, and symbolically
[C, CN, PS, R, V]

PATTERNS AND RELATIONS

Patterns

B1 graph and analyse two-variable linear relations
[C, ME, PS, R, T,V]

Variables and Equations

B2 model and solve problems using linear equations of the form
- $ax = b$
- $\frac{x}{a} = b, a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c, a \neq 0$
- $a(x + b) = c$

concretely, pictorially, and symbolically, where a, b, and c are integers
[C, CN, PS, V]

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

SHAPE AND SPACE

Measurement

C1 develop and apply the Pythagorean theorem to solve problems
[CN, PS, R, V, T]

C2 draw and construct nets for 3-D objects
[C, CN, PS, V]

C3 determine the surface area of
- right rectangular prisms
- right triangular prisms
- right cylinders

to solve problems
[C, CN, PS, R, V]

C4 develop and apply formulas for determining the volume of right prisms and right cylinders
[C, CN, PS, R, V]

3-D Objects and 2-D Shapes

C5 draw and interpret top, front, and side views of 3-D objects composed of right rectangular prisms
[C, CN, R, T, V]

Transformations

C6 demonstrate an understanding of tessellation by
- explaining the properties of shapes that make tessellating possible
- creating tessellations
- identifying tessellations in the environment

[C, CN, PS, T, V]

STATISTICS AND PROBABILITY

Data Analysis

D1 critique ways in which data is presented
[C, R, T, V]

Chance and Uncertainty

D2 solve problems involving the probability of independent events
[C, CN, PS, T]

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

GRADE 9

It is expected that students will:

NUMBER

A1 demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by
- representing repeated multiplication using powers
- using patterns to show that a power with an exponent of zero is equal to one
- solving problems involving powers

[C, CN, PS, R]

A2 demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents
[C, CN, PS, R, T]

A3 demonstrate an understanding of rational numbers by
- comparing and ordering rational numbers
- solving problems that involve arithmetic operations on rational numbers

[C, CN, PS, R, T, V]

A4 explain and apply the order of operations, including exponents, with and without technology
[PS, T]

A5 determine the square root of positive rational numbers that are perfect squares
[C, CN, PS, R, T]

A6 determine an approximate square root of positive rational numbers that are non-perfect squares
[C, CN, PS, R, T]

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

PATTERNS AND RELATIONS

Patterns

B1 generalize a pattern arising from a problem-solving context using linear equations and verify by substitution
[C, CN, PS, R, V]

B2 graph linear relations, analyse the graph, and interpolate or extrapolate to solve problems
[C, CN, PS, R, T, V]

Variables and Equations

B3 model and solve problems using linear equations of the form
- $ax = b$
- $\frac{x}{a} = b, a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c, a \neq 0$
- $ax = b + cx$
- $a(x + b) = c$
- $ax + b = cx + d$
- $a(bx + c) = d(ex + f)$
- $\frac{a}{x} = b, x \neq 0$

where $a, b, c, d, e,$ and f are rational numbers
[C, CN, PS, V]

B4 explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context
[C, CN, PS, R, V]

B5 demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2)
[C, CN, R, V]

B6 model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2)
[C, CN, PS, R, V]

B7 model, record, and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially, and symbolically
[C, CN, R, V]

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

SHAPE AND SPACE

Measurement

C1 solve problems and justify the solution strategy using circle properties, including
- the perpendicular from the centre of a circle to a chord bisects the chord
- the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
- the inscribed angles subtended by the same arc are congruent
- a tangent to a circle is perpendicular to the radius at the point of tangency

[C, CN, PS, R, T, V]

3-D Objects and 2-D Shapes

C2 determine the surface area of composite 3-D objects to solve problems
[C, CN, PS, R, V]

C3 demonstrate an understanding of similarity of polygons
[C, CN, PS, R, V]

Transformations

C4 draw and interpret scale diagrams of 2-D shapes
[CN, R, T, V]

C5 demonstrate an understanding of line and rotation symmetry
[C, CN, PS, V]

STATISTICS AND PROBABILITY

Data Analysis

D1 describe the effect of
- bias
- use of language
- ethics
- cost
- time and timing
- privacy
- cultural sensitivity

on the collection of data
[C, CN, R, T]

D2 select and defend the choice of using either a population or a sample of a population to answer a question
[C, CN, PS, R]

D3 develop and implement a project plan for the collection, display, and analysis of data by
- formulating a question for investigation
- choosing a data collection method that includes social considerations
- selecting a population or a sample
- collecting the data
- displaying the collected data in an appropriate manner
- drawing conclusions to answer the question

[C, PS, R, T, V]

Chance and Uncertainty

D4 demonstrate an understanding of the role of probability in society
[C, CN, R, T]

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

PRESCRIBED LEARNING OUTCOMES

By Curriculum Organizer

NUMBER

It is expected that students will:

GRADE 8

A1 demonstrate an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers)
[C, CN, R, V]

A2 determine the approximate square root of numbers that are not perfect squares (limited to whole numbers)
[C, CN, ME, R, T]

A3 demonstrate an understanding of percents greater than or equal to 0%
[CN, PS, R, V]

A4 demonstrate an understanding of ratio and rate
[C, CN, V]

A5 solve problems that involve rates, ratios, and proportional reasoning
[C, CN, PS, R]

A6 demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically
[C, CN, ME, PS]

A7 demonstrate an understanding of multiplication and division of integers, concretely, pictorially, and symbolically
[C, CN, PS, R, V]

GRADE 9

A1 demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by
- representing repeated multiplication using powers
- using patterns to show that a power with an exponent of zero is equal to one
- solving problems involving powers

[C, CN, PS, R]

A2 demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents
[C, CN, PS, R, T]

A3 demonstrate an understanding of rational numbers by
- comparing and ordering rational numbers
- solving problems that involve arithmetic operations on rational numbers

[C, CN, PS, R, T, V]

A4 explain and apply the order of operations, including exponents, with and without technology
[PS, T]

A5 determine the square root of positive rational numbers that are perfect squares
[C, CN, PS, R, T]

A6 determine an approximate square root of positive rational numbers that are non-perfect squares
[C, CN, PS, R, T]

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

PATTERNS AND RELATIONS

It is expected that students will:

GRADE 8

Patterns

B1 graph and analyse two-variable linear relations
[C, ME, PS, R, T, V]

Variables and Equations

B2 model and solve problems using linear equations of the form:
- $ax = b$
- $\frac{x}{a} = b, a \neq 0$
- $ax + b = c$
- $a(x + b) = c$

concretely, pictorially, and symbolically, where a, b, and c are integers
[C, CN, PS, V]

GRADE 9

Patterns

B1 generalize a pattern arising from a problem-solving context using linear equations and verify by substitution
[C, CN, PS, R, V]

B2 graph linear relations, analyse the graph, and interpolate or extrapolate to solve problems
[C, CN, PS, R, T, V]

Variables and Equations

B3 model and solve problems using linear equations of the form
- $ax = b$
- $\frac{x}{a} = b, a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c, a \neq 0$
- $ax = b + cx$
- $a(x + b) = c$
- $ax + b = cx + d$
- $a(bx + c) = d(ex + f)$
- $\frac{a}{x} = b, x \neq 0$

where a, b, c, d, e, and f are rational numbers
[C, CN, PS, V]

B4 explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context
[C, CN, PS, R, V]

B5 demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2)
[C, CN, R, V]

B6 model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2)
[C, CN, PS, R, V]

B7 model, record, and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially, and symbolically
[C, CN, R, V]

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

SHAPE AND SPACE

It is expected that students will:

GRADE 8

Measurement

C1 develop and apply the Pythagorean theorem to solve problems
[CN, PS, R, V, T]

C2 draw and construct nets for 3-D objects
[C, CN, PS, V]

C3 determine the surface area of
- right rectangular prisms
- right triangular prisms
- right cylinders

to solve problems
[C, CN, PS, R, V]

C4 develop and apply formulas for determining the volume of right prisms and right cylinders
[C, CN, PS, R, V]

3-D Objects and 2-D Shapes

C5 draw and interpret top, front, and side views of 3-D objects composed of right rectangular prisms
[C, CN, R, T, V]

Transformations

C6 demonstrate an understanding of tessellation by
- explaining the properties of shapes that make tessellating possible
- creating tessellations
- identifying tessellations in the environment

[C, CN, PS, T, V]

GRADE 9

Measurement

C1 solve problems and justify the solution strategy using circle properties, including
- the perpendicular from the centre of a circle to a chord bisects the chord
- the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
- the inscribed angles subtended by the same arc are congruent
- a tangent to a circle is perpendicular to the radius at the point of tangency

[C, CN, PS, R, T, V]

3-D Objects and 2-D Shapes

C2 determine the surface area of composite 3-D objects to solve problems
[C, CN, PS, R, V]

C3 demonstrate an understanding of similarity of polygons
[C, CN, PS, R, V]

Transformations

C4 draw and interpret scale diagrams of 2-D shapes
[CN, R, T, V]

C5 demonstrate an understanding of line and rotation symmetry
[C, CN, PS, V]

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

STATISTICS AND PROBABILITY

It is expected that students will:

GRADE 8

Data Analysis

D1 critique ways in which data is presented
[C, R, T, V]

Chance and Uncertainty

D2 solve problems involving the probability of independent events
[C, CN, PS, T]

GRADE 9

Data Analysis

D1 describe the effect of
- bias
- use of language
- ethics
- cost
- time and timing
- privacy
- cultural sensitivity

on the collection of data
[C, CN, R, T]

D2 select and defend the choice of using either a population or a sample of a population to answer a question
[C, CN, PS, R]

D3 develop and implement a project plan for the collection, display, and analysis of data by
- formulating a question for investigation
- choosing a data collection method that includes social considerations
- selecting a population or a sample
- collecting the data
- displaying the collected data in an appropriate manner
- drawing conclusions to answer the question

[C, PS, R, T, V]

Chance and Uncertainty

D4 demonstrate an understanding of the role of probability in society
[C, CN, R, T]

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

STUDENT ACHIEVEMENT

Mathematics 8 to 9

This section of the IRP contains information about classroom assessment and student achievement, including specific achievement indicators that may be used to assess student performance in relation to each Prescribed Learning Outcome. Also included in this section are Key Elements – descriptions of content that help determine the intended depth and breadth of Prescribed Learning Outcomes.

UNDERSTANDING THE KEY ELEMENTS

Key Elements provide an overview of content in each curriculum organizer. They can be used to determine the expected depth and breadth of the Prescribed Learning Outcomes.

Note that some topics appear at both grade levels in order to emphasize their importance and to allow for developmental learning.

UNDERSTANDING THE ACHIEVEMENT INDICATORS

To support the assessment of provincially prescribed curricula, this IRP includes sets of achievement indicators in relation to each Prescribed Learning Outcome.

Achievement indicators, taken together as a set, define the specific level of knowledge acquired, skills applied, or attitudes demonstrated by the student in relation to a corresponding Prescribed Learning Outcome. They describe what evidence to look for to determine whether or not the student has fully met the intent of the Prescribed Learning Outcome. Each achievement indicator defines only one aspect of the corresponding Prescribed Learning Outcome. It should be noted that the achievement indicators are designed to be considered as an entire set when determining whether students have fully met the Prescribed Learning Outcome.

In some cases, achievement indicators may also include suggestions as to the type of task that would provide evidence of having met the Prescribed Learning Outcome (e.g., a constructed response such as a list, comparison, or analysis; a product created and presented such as a report, poster, letter, or model; a particular skill demonstrated such as map making or critical thinking).

Achievement indicators support the principles of assessment *for* learning, assessment *as* learning, and assessment *of* learning. They provide teachers and parents with tools that can be used to reflect on what students are learning, as well as provide students with a means of self-assessment and ways of defining how they can improve their own achievement.

Achievement indicators are not mandatory; they are suggestions only, provided to assist in the assessment of how well students achieve the Prescribed Learning Outcomes.

The following pages contain the suggested achievement indicators corresponding to each Prescribed Learning Outcome for the Mathematics 8 and 9 curriculum. The achievement indicators are arranged by curriculum organizer and suborganizer for each grade; however, this order is not intended to imply a required sequence of instruction and assessment.

CLASSROOM ASSESSMENT AND EVALUATION

Assessment is the systematic gathering of information about what students know, are able to do, and are working toward. Assessment evidence can be collected using a wide variety of methods, such as

- observation
- student self-assessments and peer assessments
- quizzes and tests (written, oral, practical)
- samples of student work
- projects and presentations
- oral and written reports
- journals and learning logs
- performance reviews
- portfolio assessments

Assessment of student achievement is based on the information collected through assessment activities. Teachers use their insight, knowledge about learning, and experience with students, along with the specific criteria they establish, to make judgments about student performance in relation to Prescribed Learning Outcomes.

Three major types of assessment can be used in conjunction with each other to support student achievement.

- Assessment **for** learning is assessment for the purpose of greater learning achievement.
- Assessment **as** learning is assessment as a process of developing and supporting

students' active participation in their own learning.
- Assessment **of** learning is assessment for the purpose of providing evidence of achievement for reporting.

Assessment for Learning

Classroom assessment for learning provides ways to engage and encourage students to become involved in their own day-to-day assessment – to acquire the skills of thoughtful self-assessment and to promote their own achievement.

This type of assessment serves to answer the following questions:
- What do students need to learn to be successful?
- What does the evidence of this learning look like?

Assessment for learning is criterion-referenced, in which a student's achievement is compared to established criteria rather than to the performance of other students. Criteria are based on Prescribed Learning Outcomes, as well as on Suggested Achievement Indicators or other learning expectations.

Students benefit most when assessment feedback is provided on a regular, ongoing basis. When assessment is seen as an opportunity to promote learning rather than as a final judgment, it shows students their strengths and suggests how they can develop further. Students can use this information to redirect their efforts, make plans, communicate with others (e.g., peers, teachers, parents) about their growth, and set future learning goals.

Assessment for learning also provides an opportunity for teachers to review what their students are learning and what areas need further attention. This information can be used to inform teaching and create a direct link between assessment and instruction. Using assessment as a way of obtaining feedback on instruction supports student achievement by informing teacher planning and classroom practice.

Assessment as Learning

Assessment as learning actively involves students in their own learning processes. With support and guidance from their teacher, students take responsibility for their own learning, constructing meaning for themselves. Through a process of continuous self-assessment, students develop the ability to take stock of what they have already learned, determine what they have not yet learned, and decide how they can best improve their own achievement.

Although assessment as learning is student-driven, teachers can play a key role in facilitating how this assessment takes place. By providing regular opportunities for reflection and self-assessment, teachers can help students develop, practise, and become comfortable with critical analysis of their own learning.

Assessment of Learning

Assessment of learning can be addressed through summative assessment, including large-scale assessments and teacher assessments. These summative assessments can occur at the end of the year or at periodic stages in the instructional process.

Large-scale assessments, such as Foundation Skills Assessment (FSA) and Graduation Program exams, gather information on student performance throughout the province and provide information for the development and revision of curriculum. These assessments are used to make judgments about students' achievement in relation to provincial and national standards.

Assessment of learning is also used to inform formal reporting of student achievement.

For Ministry of Education reporting policy, refer to www.bced.gov.bc.ca/policy/policies/student_reporting.htm

Assessment *for* Learning	Assessment *as* Learning	Assessment *of* Learning
Formative assessment is *ongoing in the classroom*	**Formative assessment is *ongoing in the classroom***	**Summative assessment occurs *at end of year or at key stages***
• teacher assessment, student self-assessment, and/or student peer assessment • criterion-referenced – criteria based on Prescribed Learning Outcomes identified in the provincial curriculum, reflecting performance in relation to a specific learning task • involves both teacher and student in a process of continual reflection and review about progress • teachers adjust their plans and engage in corrective teaching in response to formative assessment	• self-assessment • provides students with information on their own achievement and prompts them to consider how they can continue to improve their learning • student-determined criteria based on previous learning and personal learning goals • students use assessment information to make adaptations to their learning process and to develop new understandings	• teacher assessment • may be either criterion-referenced (based on Prescribed Learning Outcomes) or norm-referenced (comparing student achievement to that of others) • information on student performance can be shared with parents/guardians, school and district staff, and other education professionals (e.g., for the purpose of curriculum development) • used to make judgments about students' performance in relation to provincial standards

For more information about assessment for, as, and of learning, refer to *Rethinking Assessment with Purpose in Mind*, developed by the Western and Northern Canadian Protocol (WNCP).

This resource is available online at www.wncp.ca

In addition, the BC Performance Standards describe levels of achievement in key areas of learning (reading, writing, numeracy, social responsibility, and information and communications technology integration) relevant to all subject areas. Teachers may wish to use the Performance Standards as resources to support ongoing formative assessment in mathematics.

BC Performance Standards are available at www.bced.gov.bc.ca/perf_stands/

Criterion-Referenced Assessment and Evaluation

In criterion-referenced evaluation, a student's performance is compared to established criteria rather than to the performance of other students. Evaluation in relation to prescribed curriculum requires that criteria be established based on the Prescribed Learning Outcomes.

Criteria are the basis for evaluating student progress. They identify, in specific terms, the critical aspects of a performance or product that indicate how well the student is meeting the Prescribed Learning Outcomes. For example, weighted criteria, rating scales, or scoring guides (reference sets) are ways that student performance can be evaluated using criteria.

Wherever possible, students should be involved in setting the assessment criteria. This helps students develop an understanding of what high-quality work or performance looks like.

Criterion-referenced assessment and evaluation may involve these steps:

Step 1	Identify the Prescribed Learning Outcomes and Suggested Achievement Indicators (as articulated in this IRP) that will be used as the basis for assessment.
Step 2	Establish criteria. When appropriate, involve students in establishing criteria.
Step 3	Plan learning activities that will help students gain the knowledge, skills, or attitudes outlined in the criteria.
Step 4	Prior to the learning activity, inform students of the criteria against which their work will be evaluated.
Step 5	Provide examples of the desired levels of performance.
Step 6	Conduct the learning activities.
Step 7	Use appropriate assessment instruments (e.g., rating scale, checklist, scoring guide) and methods (e.g., observation, collection, self-assessment) based on the particular assignment and student.
Step 8	Review the assessment data and evaluate each student's level of performance or quality of work in relation to criteria.
Step 9	Where appropriate, provide feedback and/or a letter grade to indicate how well the criteria are met.
Step 10	Communicate the results of the assessment and evaluation to students and parents/guardians.

STUDENT ACHIEVEMENT

Grade 8

GRADE 8: ASSESSMENT OVERVIEW TABLE

The purpose of this table is to provide teachers with suggestions and guidelines for types of formative and summative classroom-based assessment and grading of Mathematics 8.

Curriculum Organizers	Suggested Assessment Activities	Suggested Weight for Grading	Number of Outcomes	Number of Outcomes by Domain*		
				K	U&A	HMP
Number	• journals • observations • pictorial representations • Venn diagrams • Carroll diagrams • student interviews • Frayer models • sort and classify • interviews • similarities and differences • student centres • portfolio • charts • projects • self-evaluation • posters • open-ended problems • error correction • graphic organizers • technology • problem solving • concept maps • peer evaluation • sketches • student work • drawing • concrete representations • symbolic representations • geometric constructions • self-directed assignments • presentations • constructions • experiments • oral reports	**40-50%**	**7**	**4**	**2**	**1**
Patterns and Relations		**10-20%**	**2**	**1**	**0**	**1**
Shape and Space		**20-30%**	**6**	**2**	**1**	**2**
Statistics and Probability		**10-20%**	**2**	**0**	**1**	**1**
Totals		**100%**	**17**	**7**	**4**	**5**

* The following abbreviations are used to represent the three cognitive levels within the cognitive domain:
K = Knowledge; U&A = Understanding and Application; HMP = Higher Mental Processes.

KEY ELEMENTS: GRADE 8

MATHEMATICAL PROCESS (INTEGRATED)

The following mathematical processes have been integrated within the Prescribed Learning Outcomes and Suggested Achievement Indicators for the grade: communication, connections, mental mathematics and estimation, problem solving, reasoning, technology, and visualization.

NUMBER – develop number sense

- perfect squares, square roots, and approximate square roots
- percents greater than or equal to 0%
- rates, ratio, and proportional reasoning
- multiplication and division of positive fractions
- multiplication and division of integers

PATTERNS AND RELATIONS – use patterns to describe the world and solve problems

Patterns

- two variable linear relations

Variables and Equations

- linear equations

SHAPE AND SPACE – use direct and indirect measurement to solve problems

Measurement

- Pythagorean theorem
- nets for 3-D objects
- surface area of right rectangular prisms, right triangular prisms, and right cylinders
- formulas for volume of right prisms and cylinders

3-D Objects and 2-D Shapes

- top, front, and side view of 3-D objects composed of right rectangular prisms

Transformations

- tessellations

STATISTICS AND PROBABILITY – collect, display, and analyse data to solve problems

Data Analysis

- presentation of data

Chance and Uncertainty

- probability of independent events

NUMBER

General Outcome: Develop number sense.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
A1 demonstrate an understanding of perfect square and square root, concretely, pictorially, and symbolically (limited to whole numbers) [C, CN, R, V]	❑ represent a given perfect square as a square region using materials such as grid paper or square shapes ❑ determine the factors of a given perfect square, and explain why one of the factors is the square root and the others are not ❑ determine whether or not a given number is a perfect square using materials and strategies such as square shapes, grid paper, or prime factorization, and explain the reasoning ❑ determine the square root of a given perfect square and record it symbolically ❑ determine the square of a given number
A2 determine the approximate square root of numbers that are not perfect squares (limited to whole numbers) [C, CN, ME, R, T]	❑ estimate the square root of a given number that is not a perfect square using the roots of perfect squares as benchmarks ❑ approximate the square root of a given number that is not a perfect square using technology (e.g., calculator, computer) ❑ explain why the square root of a number shown on a calculator may be an approximation ❑ identify a number with a square root that is between two given numbers

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

A3 demonstrate an understanding of percents greater than or equal to 0% [CN, PS, R, V]	❑ provide a context where a percent may be more than 100% or between 0% and 1% ❑ represent a given fractional percent using grid paper ❑ represent a given percent greater than 100 using grid paper ❑ determine the percent represented by a given shaded region on a grid, and record it in decimal, fractional, and percent form ❑ express a given percent in decimal or fractional form ❑ express a given decimal in percent or fractional form ❑ express a given fraction in decimal or percent form ❑ solve a given problem involving percents ❑ solve a given problem involving combined percents (e.g., addition of percents, such as GST + PST) ❑ solve a given problem that involves finding the percent of a percent (e.g., A population increased by 10% one year and then 15% the next year. Explain why there was not a 25% increase in population over the two years.)
A4 demonstrate an understanding of ratio and rate [C, CN, V]	❑ express a two-term ratio from a given context in the forms 3:5 or 3 to 5 ❑ express a three-term ratio from a given context in the forms 4:7:3 or 4 to 7 to 3 ❑ express a part to part ratio as a part to whole fraction (e.g., frozen juice to water; 1 can concentrate to 4 cans of water can be represented as $\frac{1}{5}$, which is the ratio of concentrate to solution, or $\frac{4}{5}$, which is the ratio of water to solution) ❑ identify and describe ratios and rates from real-life examples, and record them symbolically ❑ express a given rate using words or symbols (e.g., 20 L per 100 km or 20 L/100 km) ❑ express a given ratio as a percent and explain why a rate cannot be represented as a percent
A5 solve problems that involve rates, ratios, and proportional reasoning [C, CN, PS, R]	❑ explain the meaning of $\frac{a}{b}$ within a given context ❑ provide a context in which $\frac{a}{b}$ represents a: - fraction - rate - ratio - quotient - probability ❑ solve a given problem involving rate, ratio, or percent

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

A6 demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically [C, CN, ME, PS]	❑ identify the operation required to solve a given problem involving positive fractions ❑ provide a context that requires the multiplying of two given positive fractions ❑ provide a context that requires the dividing of two given positive fractions ❑ estimate the product of two given positive proper fractions to determine if the product will be closer to 0, $\frac{1}{2}$, or 1 ❑ estimate the quotient of two given positive fractions and compare the estimate to whole number benchmarks ❑ express a given positive mixed number as an improper fraction and a given positive improper fraction as a mixed number ❑ model multiplication of a positive fraction by a whole number concretely or pictorially and record the process ❑ model multiplication of a positive fraction by a positive fraction concretely or pictorially using an area model and record the process ❑ model division of a positive proper fraction by a whole number concretely or pictorially and record the process ❑ model division of a positive proper fraction by a positive proper fraction pictorially and record the process ❑ generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers ❑ solve a given problem involving positive fractions, taking into consideration order of operations (limited to problems with positive solutions)
A7 demonstrate an understanding of multiplication and division of integers, concretely, pictorially, and symbolically [C, CN, PS, R, V]	❑ identify the operation required to solve a given problem involving integers ❑ provide a context that requires multiplying two integers ❑ provide a context that requires dividing two integers ❑ model the process of multiplying two integers using concrete materials or pictorial representations and record the process ❑ model the process of dividing an integer by an integer using concrete materials or pictorial representations and record the process ❑ solve a given problem involving the multiplication of integers (2-digit by 1-digit) without the use of technology ❑ solve a given problem involving the division of integers (2-digit by 1-digit) without the use of technology ❑ solve a given problem involving the division of integers (2-digit by 2-digit) with the use of technology ❑ generalize and apply a rule for determining the sign of the product and quotient of integers

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

PATTERNS AND RELATIONS (PATTERNS)

General Outcome: Use patterns to describe the world and solve problems.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
B1 graph and analyse two-variable linear relations [C, ME, PS, R, T, V]	❑ solve a given problem involving integers taking into consideration the order of operations ❑ determine the missing value in an ordered pair for a given equation ❑ create a table of values by substituting values for a variable in the equation of a given linear relation ❑ construct a graph from the equation of a given linear relation (limited to discrete data) ❑ describe the relationship between the variables of a given graph

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

General Outcome: Represent algebraic expressions in multiple ways.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
B2 model and solve problems using linear equations of the form – $ax = b$ – $\frac{x}{a} = b, a \neq 0$ – $ax + b = c$ – $\frac{x}{a} + b = c, a \neq 0$ – $a(x + b) = c$ concretely, pictorially, and symbolically, where a, b, and c are integers [C, CN, PS, V]	❑ model a given problem with a linear equation and solve the equation using concrete models (e.g., counters, integer tiles) ❑ verify the solution to a given linear equation using a variety of methods, including concrete materials, diagrams, and substitution ❑ draw a visual representation of the steps used to solve a given linear equation and record each step symbolically ❑ solve a given linear equation symbolically ❑ identify and correct an error in a given incorrect solution of a linear equation ❑ apply the distributive property to solve a given linear equation (e.g., $2(x + 3) = 5; 2x + 6 = 5; \ldots$) ❑ solve a given problem using a linear equation and record the process

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

SHAPE AND SPACE (MEASUREMENT)

General Outcome: Use direct or indirect measurement to solve problems.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
C1 develop and apply the Pythagorean theorem to solve problems [CN, PS, R, T, V]	❑ model and explain the Pythagorean theorem concretely, pictorially, or using technology ❑ explain, using examples, that the Pythagorean theorem applies only to right triangles ❑ determine whether or not a given triangle is a right triangle by applying the Pythagorean theorem ❑ determine the measure of the third side of a right triangle, given the measures of the other two sides, to solve a given problem ❑ solve a given problem that involves Pythagorean triples (e.g., 3, 4, 5 or 5, 12, 13)
C2 draw and construct nets for 3-D objects [C, CN, PS, V]	❑ match a given net to the 3-D object it represents ❑ construct a 3-D object from a given net ❑ draw nets for a given right circular cylinder, right rectangular prism, and right triangular prism, and verify by constructing the 3-D objects from the nets ❑ predict 3-D objects that can be created from a given net and verify the prediction
C3 determine the surface area of – right rectangular prisms – right triangular prisms – right cylinders to solve problems [C, CN, PS, R, V]	❑ explain, using examples, the relationship between the area of 2-D shapes and the surface area of a given 3-D object ❑ identify all the faces of a given prism, including right rectangular and right triangular prisms ❑ describe and apply strategies for determining the surface area of a given right rectangular or right triangular prism ❑ describe and apply strategies for determining the surface area of a given right cylinder ❑ solve a given problem involving surface area

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

C4	develop and apply formulas for determining the volume of right prisms and right cylinders [C, CN, PS, R, V]	❑ determine the volume of a given right prism, given the area of the base ❑ generalize and apply a rule for determining the volume of right cylinders ❑ explain the connection between the area of the base of a given right 3-D object and the formula for the volume of the object ❑ demonstrate that the orientation of a given 3-D object does not affect its volume ❑ apply a formula to solve a given problem involving the volume of a right cylinder or a right prism

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

SHAPE AND SPACE (3-D OBJECTS AND 2-D SHAPES)

General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyse the relationships among them.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
C5 draw and interpret top, front, and side views of 3-D objects composed of right rectangular prisms [C, CN, R, T, V]	❑ draw and label the top, front, and side views for a given 3-D object on isometric dot paper ❑ compare different views of a given 3-D object to the object ❑ predict the top, front, and side views that will result from a described rotation (limited to multiples of 90 degrees) and verify predictions ❑ draw and label the top, front, and side views that result from a given rotation (limited to multiples of 90 degrees) ❑ build a 3-D block object, given the top, front, and side views, with or without the use of technology ❑ sketch and label the top, front, and side views of a 3-D object in the environment with or without the use of technology

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

SHAPE AND SPACE (TRANSFORMATIONS)

General Outcome: Describe and analyse position and motion of objects and shapes.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
C6 demonstrate an understanding of tessellation by – explaining the properties of shapes that make tessellating possible – creating tessellations – identifying tessellations in the environment [C, CN, PS, T, V]	❑ identify, in a given set of regular polygons, those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices (e.g., squares, regular n-gons) ❑ identify, in a given set of irregular polygons, those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices ❑ identify a translation, reflection, or rotation in a given tessellation ❑ identify a combination of transformations in a given tessellation ❑ create a tessellation using one or more 2-D shapes, and describe the tessellation in terms of transformations and conservation of area ❑ create a new tessellating shape (polygon or non-polygon) by transforming a portion of a given tessellating polygon (e.g., one by M. C. Escher), and describe the resulting tessellation in terms of transformations and conservation of area ❑ identify and describe tessellations in the environment

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

STATISTICS AND PROBABILITY (DATA ANALYSIS)

General Outcome: Collect, display, and analyse data to solve problems.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
D1 critique ways in which data is presented [C, R, T, V]	❑ compare the information that is provided for the same data set by a given set of graphs, including circle graphs, line graphs, bar graphs, double bar graphs, and pictographs, to determine the strengths and limitations of each graph ❑ identify the advantages and disadvantages of different graphs, including circle graphs, line graphs, bar graphs, double bar graphs, and pictographs, in representing a specific given set of data ❑ justify the choice of a graphical representation for a given situation and its corresponding data set ❑ explain how the format of a given graph, such as the size of the intervals, the width of bars, and the visual representation, may lead to misinterpretation of the data ❑ explain how a given formatting choice could misrepresent the data ❑ identify conclusions that are inconsistent with a given data set or graph, and explain the misinterpretation

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

STATISTICS AND PROBABILITY (CHANCE AND UNCERTAINTY)

General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
D2 solve problems involving the probability of independent events [C, CN, PS, T]	❑ determine the probability of two given independent events and verify the probability using a different strategy ❑ generalize and apply a rule for determining the probability of independent events ❑ solve a given problem that involves determining the probability of independent events

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

STUDENT ACHIEVEMENT

Grade 9

GRADE 9: ASSESSMENT OVERVIEW TABLE

The purpose of this table is to provide teachers with suggestions and guidelines for types of formative and summative classroom-based assessment and grading of Mathematics 9.

Curriculum Organizers	Suggested Assessment Activities	Suggested Weight for Grading	Number of Outcomes	Number of Outcomes by Domain*		
				K	U&A	HMP
Number	• journals • observations • pictorial representations • Venn diagrams • Carroll diagrams • student interviews • Frayer models • sort and classify • interviews • similarities and differences • student centres • portfolio • charts • projects • self-evaluation • posters • open-ended problems • error correction • graphic organizers • technology • problem solving • concept maps • peer evaluation • sketches • student work • drawing • concrete representations • symbolic representations • geometric constructions • self-directed assignments • presentations • constructions • experiments • oral reports	**40-50%**	**6**	**2**	**3**	**1**
Patterns and Relations		**20-30%**	**7**	**2**	**3**	**2**
Shape and Space		**10-20%**	**5**	**2**	**1**	**2**
Statistics and Probability		**10-20%**	**4**	**0**	**1**	**3**
Totals		**100%**	**22**	**6**	**8**	**8**

* The following abbreviations are used to represent the three cognitive levels within the cognitive domain:
K = Knowledge; U&A = Understanding and Application; HMP = Higher Mental Processes.

KEY ELEMENTS: GRADE 9

MATHEMATICAL PROCESS (INTEGRATED)
The following mathematical processes have been integrated within the Prescribed Learning Outcomes and Suggested Achievement Indicators for the grade: communication, connections, mental mathematics and estimation, problem solving, reasoning, technology, and visualization.

NUMBER – develop number sense
- powers with integral bases and whole number exponents
- operations on powers
- rational numbers
- square root and approximate square root of positive rational numbers

PATTERNS AND RELATIONS – use patterns to describe the world and solve problems
Patterns
- generalize a pattern using linear equations
- graph linear relations for interpolation and extrapolation

Variables and Equations
- linear equations
- single variable linear inequalities with rational coefficients
- addition, subtraction, multiplication, and division of polynomials

SHAPE AND SPACE – use direct and indirect measurement to solve problems
Measurement
- circle properties

3-D Objects and 2-D Shapes
- surface area of composite 3-D objects
- similarity of polygons

Transformations
- scale diagrams
- line and rotation symmetry

STATISTICS AND PROBABILITY – collect, display, and analyse data to solve problems
Data Analysis
- factors affecting the collection of data
- population and sample data
- project plan for data collection and analysis

Chance and Uncertainty
- role of probability in society

NUMBER

General Outcome: Develop number sense.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
A1 demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by – representing repeated multiplication using powers – using patterns to show that a power with an exponent of zero is equal to one – solving problems involving powers [C, CN, PS, R]	❑ demonstrate the differences between the exponent and the base by building models of a given power, such as 2^3 and 3^2 ❑ explain, using repeated multiplication, the difference between two given powers in which the exponent and base are interchanged (e.g., 10^3 and 3^{10}) ❑ express a given power as a repeated multiplication ❑ express a given repeated multiplication as a power ❑ explain the role of parentheses in powers by evaluating a given set of powers (e.g., $(-2)^4$, (-2^4) and -2^4) ❑ demonstrate, using patterns, that a^0 is equal to 1 for a given value of a ($a \neq 0$) ❑ evaluate powers with integral bases (excluding base 0) and whole number exponents
A2 demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents [C, CN, PS, R, T]	❑ explain, using examples, the exponent laws of powers with integral bases (excluding base 0) and whole number exponents: - $\left(a^m\right)\left(a^n\right) = a^{m+n}$ - $a^m \div a^n = a^{m-n}, m > n$ - $(a^m)^n = a^{mn}$ - $(ab)^m = a^m b^m$ - $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$ ❑ evaluate a given expression by applying the exponent laws ❑ determine the sum of two given powers (e.g., $5^2 + 5^3$) and record the process ❑ determine the difference of two given powers (e.g., $4^3 - 4^2$) and record the process ❑ identify the error(s) in a given simplification of an expression involving powers

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

A3 demonstrate an understanding of rational numbers by – comparing and ordering rational numbers – solving problems that involve arithmetic operations on rational numbers [C, CN, PS, R, T, V]	❑ order a given set of rational numbers, in fraction and decimal form, by placing them on a number line (e.g., $\frac{3}{5}$, - 0.666 … , 0.5, - $\frac{5}{8}$) ❑ identify a rational number that is between two given rational numbers ❑ solve a given problem involving operations on rational numbers in fraction form and decimal form
A4 explain and apply the order of operations, including exponents, with and without technology [PS, T]	❑ solve a given problem by applying the order of operations without the use of technology ❑ solve a given problem by applying the order of operations with the use of technology ❑ identify the error in applying the order of operations in a given incorrect solution
A5 determine the square root of positive rational numbers that are perfect squares [C, CN, PS, R, T]	❑ determine whether or not a given rational number is a square number and explain the reasoning ❑ determine the square root of a given positive rational number that is a perfect square ❑ identify the error made in a given calculation of a square root (e.g., Is 3.2 the square root of 6.4?) ❑ determine a positive rational number given the square root of that positive rational number
A6 determine an approximate square root of positive rational numbers that are non-perfect squares [C, CN, PS, R, T]	❑ estimate the square root of a given rational number that is not a perfect square, using the roots of perfect squares as benchmarks ❑ determine an approximate square root of a given rational number that is not a perfect square using technology (e.g., calculator, computer) ❑ explain why the square root of a given rational number as shown on a calculator may be an approximation ❑ identify a number with a square root that is between two given numbers

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

PATTERNS AND RELATIONS (PATTERNS)

General Outcome: Use patterns to describe the world and solve problems.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
B1 generalize a pattern arising from a problem-solving context using linear equations and verify by substitution [C, CN, PS, R, V]	❑ write an expression representing a given pictorial, oral, or written pattern ❑ write a linear equation to represent a given context ❑ describe a context for a given linear equation ❑ solve, using a linear equation, a given problem that involves pictorial, oral, and written linear patterns ❑ write a linear equation representing the pattern in a given table of values and verify the equation by substituting values from the table
B2 graph linear relations, analyse the graph, and interpolate or extrapolate to solve problems [C, CN, PS, R, T, V]	❑ describe the pattern found in a given graph ❑ graph a given linear relation, including horizontal and vertical lines ❑ match given equations of linear relations with their corresponding graphs ❑ extend a given graph (extrapolate) to determine the value of an unknown element ❑ interpolate the approximate value of one variable on a given graph given the value of the other variable ❑ extrapolate the approximate value of one variable from a given graph given the value of the other variable ❑ solve a given problem by graphing a linear relation and analysing the graph

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

General Outcome: Represent algebraic expressions in multiple ways.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
B3 model and solve problems using linear equations of the form – $ax = b$ – $\frac{x}{a} = b, a \neq 0$ – $ax + b = c$ – $\frac{x}{a} + b = c, a \neq 0$ – $ax = b + cx$ – $a(x + b) = c$ – $ax + b = cx + d$ – $a(bx + c) = d(ex + f)$ – $\frac{a}{x} = b, x \neq 0$ where $a, b, c, d, e,$ and f are rational numbers [C, CN, PS, V]	❑ model the solution of a given linear equation using concrete or pictorial representations, and record the process ❑ determine, by substitution, whether a given rational number is a solution to a given linear equation ❑ solve a given linear equation symbolically ❑ identify and correct an error in a given incorrect solution of a linear equation ❑ represent a given problem using a linear equation ❑ solve a given problem using a linear equation and record the process
B4 explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context [C, CN, PS, R, V]	❑ translate a given problem into a single variable linear inequality using the symbols $\geq, >, <,$ or $\leq$ ❑ determine if a given rational number is a possible solution of a given linear inequality ❑ generalize and apply a rule for adding or subtracting a positive or negative number to determine the solution of a given inequality ❑ generalize and apply a rule for multiplying or dividing by a positive or negative number to determine the solution of a given inequality ❑ solve a given linear inequality algebraically and explain the process orally or in written form ❑ compare and explain the process for solving a given linear equation to the process for solving a given linear inequality ❑ graph the solution of a given linear inequality on a number line ❑ compare and explain the solution of a given linear equation to the solution of a given linear inequality ❑ verify the solution of a given linear inequality using substitution for multiple elements in the solution ❑ solve a given problem involving a single variable linear inequality and graph the solution

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

B5	demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2) [C, CN, R, V]	❑ create a concrete model or a pictorial representation for a given polynomial expression ❑ write the expression for a given model of a polynomial ❑ identify the variables, degree, number of terms, and coefficients, including the constant term, of a given simplified polynomial expression ❑ describe a situation for a given first degree polynomial expression ❑ match equivalent polynomial expressions given in simplified form (e.g., $4x - 3x^2 + 2$ is equivalent to $-3x^2 + 4x + 2$)
B6	model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2) [C, CN, PS, R, V]	❑ model addition of two given polynomial expressions concretely or pictorially and record the process symbolically ❑ model subtraction of two given polynomial expressions concretely or pictorially and record the process symbolically ❑ apply a personal strategy for addition and subtraction of given polynomial expressions, and record the process symbolically ❑ identify equivalent polynomial expressions from a given set of polynomial expressions, including pictorial and symbolic representations ❑ identify the error(s) in a given simplification of a given polynomial expression
B7	model, record, and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially, and symbolically [C, CN, R, V]	❑ model multiplication of a given polynomial expression by a given monomial concretely or pictorially and record the process symbolically ❑ model division of a given polynomial expression by a given monomial concretely or pictorially and record the process symbolically ❑ apply a personal strategy for multiplication and division of a given polynomial expression by a given monomial ❑ provide examples of equivalent polynomial expressions ❑ identify the error(s) in a given simplification of a given polynomial expression

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

SHAPE AND SPACE (MEASUREMENT)

General Outcome: Use direct or indirect measurement to solve problems.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
C1 solve problems and justify the solution strategy using circle properties, including – the perpendicular from the centre of a circle to a chord bisects the chord – the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc – the inscribed angles subtended by the same arc are congruent – a tangent to a circle is perpendicular to the radius at the point of tangency [C, CN, PS, R, T, V]	❑ provide an example that illustrates - the perpendicular from the centre of a circle to a chord bisects the chord - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc - the inscribed angles subtended by the same arc are congruent - a tangent to a circle is perpendicular to the radius at the point of tangency ❑ solve a given problem involving application of one or more of the circle properties ❑ determine the measure of a given angle inscribed in a semicircle using the circle properties ❑ explain the relationship among the centre of a circle, a chord, and the perpendicular bisector of the chord

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

SHAPE AND SPACE (3-D OBJECTS AND 2-D SHAPES)

General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyse the relationships among them.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
C2 determine the surface area of composite 3-D objects to solve problems [C, CN, PS, R, V]	❑ determine the area of overlap in a given concrete composite 3-D object, and explain its effect on determining the surface area (limited to right cylinders, right rectangular prisms, and right triangular prisms) ❑ determine the surface area of a given concrete composite 3-D object (limited to right cylinders, right rectangular prisms, and right triangular prisms) ❑ solve a given problem involving surface area
C3 demonstrate an understanding of similarity of polygons [C, CN, PS, R, V]	❑ determine if the polygons in a given pre-sorted set are similar and explain the reasoning ❑ draw a polygon similar to a given polygon and explain why the two are similar ❑ solve a given problem using the properties of similar polygons

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

SHAPE AND SPACE (TRANSFORMATIONS)

General Outcome Describe and analyse position and motion of objects and shapes.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
C4 draw and interpret scale diagrams of 2-D shapes [CN, R, T, V]	❑ identify an example in print and electronic media (e.g., newspapers, the Internet) of a scale diagram and interpret the scale factor ❑ draw a diagram to scale that represents an enlargement or reduction of a given 2-D shape ❑ determine the scale factor for a given diagram drawn to scale ❑ determine if a given diagram is proportional to the original 2-D shape and, if it is, state the scale factor ❑ solve a given problem that involves a scale diagram by applying the properties of similar triangles

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

C5 demonstrate an understanding of line and rotation symmetry [C, CN, PS, V]	❑ classify a given set of 2-D shapes or designs according to the number of lines of symmetry ❑ complete a 2-D shape or design given one half of the shape or design and a line of symmetry ❑ determine if a given 2-D shape or design has rotation symmetry about the point at the centre of the shape or design and, if it does, state the order and angle of rotation ❑ rotate a given 2-D shape about a vertex and draw the resulting image ❑ identify a line of symmetry or the order and angle of rotation symmetry in a given tessellation ❑ identify the type of symmetry that arises from a given transformation on the Cartesian plane ❑ complete, concretely or pictorially, a given transformation of a 2-D shape on a Cartesian plane, record the coordinates, and describe the type of symmetry that results ❑ identify and describe the types of symmetry created in a given piece of artwork ❑ determine whether or not two given 2-D shapes on the Cartesian plane are related by either rotation or line symmetry ❑ draw, on a Cartesian plane, the translation image of a given shape using a given translation rule, such as R2, U3, or ↑ ↑, ↑↑↑, label each vertex and its corresponding ordered pair, and describe why the translation does not result in line or rotation symmetry ❑ create or provide a piece of artwork that demonstrates line and rotation symmetry, and identify the line(s) of symmetry and the order and angle of rotation

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

STATISTICS AND PROBABILITY (DATA ANALYSIS)

General Outcome: Collect, display, and analyse data to solve problems.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
D1 describe the effect of – bias – use of language – ethics – cost – time and timing – privacy – cultural sensitivity on the collection of data [C, CN, R, T]	❑ analyse a given case study of data collection, and identify potential problems related to bias, use of language, ethics, cost, time and timing, privacy, or cultural sensitivity ❑ provide examples to illustrate how bias, use of language, ethics, cost, time and timing, privacy, or cultural sensitivity may influence the data
D2 select and defend the choice of using either a population or a sample of a population to answer a question [C, CN, PS, R]	❑ identify whether a given situation represents the use of a sample or a population ❑ provide an example of a situation in which a population may be used to answer a question and justify the choice ❑ provide an example of a question where a limitation precludes the use of a population and describe the limitation (e.g., too costly, not enough time, limited resources) ❑ identify and critique a given example in which a generalization from a sample of a population may or may not be valid for the population
D3 develop and implement a project plan for the collection, display, and analysis of data by – formulating a question for investigation – choosing a data collection method that includes social considerations – selecting a population or a sample – collecting the data – displaying the collected data in an appropriate manner – drawing conclusions to answer the question [C, PS, R, T, V]	❑ create a rubric to assess a project that includes the assessment of - a question for investigation - the choice of a data collection method that includes social considerations - the selection of a population or a sample and justifying the choice - the display of the collected data - the conclusions to answer the question ❑ develop a project plan that describes - a question for investigation - the method of data collection that includes social considerations - the method for selecting a population or a sample - the method to be used for collection of the data - the methods for analysis and display of the data ❑ complete the project according to the plan, draw conclusions, and communicate findings to an audience ❑ self-assess the completed project by applying the rubric

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

STATISTICS AND PROBABILITY (CHANCE AND UNCERTAINTY)

General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

PRESCRIBED LEARNING OUTCOMES	SUGGESTED ACHIEVEMENT INDICATORS
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
D4 demonstrate an understanding of the role of probability in society [C, CN, R, T]	❑ provide an example from print and electronic media (e.g., newspapers and the Internet), where probability is used ❑ identify the assumptions associated with a given probability and explain the limitations of each assumption ❑ explain how a single probability can be used to support opposing positions ❑ explain, using examples, how decisions based on probability may be a combination of theoretical probability, experimental probability, and subjective judgment

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

GLOSSARY

Mathematics 8 and 9

The Alberta Ministry of Education has developed an online glossary to support the implementation of the revised Kindergarten to Grade 9 Program of Studies. This glossary is based on the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework (CCF) for K to 9 Mathematics and therefore also supports the British Columbia Mathematics 8 and 9 IRP.

This online glossary provides additional supports for teachers that can not be provided through a conventional print glossary. As a result, the Ministry of Education encourages educational stakeholders to access the glossary through a link provided on the British Columbia Ministry of Education web site.

To access the glossary, follow the links for curriculum support material from the mathematics IRP main page at www.bced.gov.bc.ca/irp/irp_math.htm