This document represents an updating of the 2000 IRP. This updating has been undertaken for the purpose of

- clarifying the Prescribed Learning Outcomes
- introducing Suggested Achievement Indicators
- addressing content overload

Resources previously recommended for the 2000 version of the curriculum, where still valid, continue to support this updated IRP. (See the Learning Resources section in this IRP for additional information.)

Essentials of Mathematics 10 to 12

Integrated Resource Package 2006

BRITISH COLUMBIA
The Best Place on Earth

Ministry of Education

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ESSENTIALS OF MATHEMATICS 10 TO 12 IRP REFINEMENT TEAM

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This Integrated Resource Package (IRP) provides basic information teachers will require in order to implement Essentials of Mathematics 10 to 12. This document supersedes the Essentials of Mathematics 10 to 12 portion of the *Mathematics 10 to 12 Integrated Resource Package* (2000).

The information contained in this document is also available on the Internet at www.bced.gov.bc.ca/irp/irp.htm

The following paragraphs provide brief descriptions of the components of the IRP.

Introduction

The Introduction provides general information about Essentials of Mathematics 10 to 12, including special features and requirements.

Included in this section are

- a rationale for teaching Essentials of Mathematics 10 to 12 in BC schools
- the curriculum goals
- descriptions of the curriculum organizers – groupings for prescribed learning outcomes that share a common focus
- a suggested timeframe for each curriculum organizer
- a graphic overview of the curriculum content

Considerations For Program Delivery

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners.

Prescribed Learning Outcomes

This section contains the *prescribed learning outcomes*, the legally required content standards for the provincial education system. The learning outcomes define the required knowledge, skills, and attitudes for each subject. They are statements of what students are expected to know and be able to do by the end of the course.

Student Achievement

This section of the IRP contains information about classroom assessment and measuring student achievement, including sets of specific achievement indicators for each prescribed learning outcome. Achievement indicators are statements that describe what students should be able to do in order to demonstrate that they fully meet the expectations set out by the prescribed learning outcomes. Achievement indicators are not mandatory; they are provided to assist in the assessment of how well students achieve the prescribed learning outcomes.

Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of the prescribed learning outcomes.

Learning Resources

This section contains general information on learning resources, and provides a link to titles, descriptions, and ordering information for the recommended learning resources in the Essentials of Mathematics 10 to 12 Grade Collections.

Glossary

The glossary defines selected terms used in this Integrated Resource Package.

Introduction

Essentials of Mathematics 10 to 12

This Integrated Resource Package (IRP) sets out the provincially prescribed curriculum for Essentials of Mathematics 10 to 12. The development of this IRP has been guided by the principles of learning:

- Learning requires the active participation of the student.
- People learn in a variety of ways and at different rates.
- Learning is both an individual and a group process.

In addition to these three principles, this document recognizes that British Columbia's schools include young people of varied backgrounds, interests, abilities, and needs. Wherever appropriate for this curriculum, ways to meet these needs and to ensure equity and access for all learners have been integrated as much as possible into the learning outcomes and achievement indicators.

The Achievement Indicators were developed, in part, using the following documents:

- *Mathematics 10 to 12 Integrated Resource Package* (British Columbia Ministry of Education, 2000)
- *Essentials of Mathematics 10 Provincial Examination Specifications* (British Columbia Ministry of Education, 2004)

This document represents an updating of the 2000 IRP. This updating has been undertaken for the purpose of

- clarifying the prescribed learning outcomes
- introducing suggested achievement indicators
- addressing content overload

Resources previously recommended for the 2000 version of the curriculum continue to support this updated IRP. (See the Learning Resources section later in this IRP for additional information.)

Essentials of Mathematics 10 to 12, in draft form, was available for public review and response from November to December, 2005. Feedback from educators, students, parents, and other educational partners informed the development of this updated IRP.

RATIONALE

Mathematics is increasingly important in our technological society. Students today require the ability to reason and communicate, to solve problems, and to understand and use mathematics. Development of these skills helps students become numerate.

> Numeracy can be defined as the combination of mathematical knowledge, problem solving and communication skills required by all persons to function successfully within our technological world. Numeracy is more than knowing about numbers and number operations. (British Columbia Association of Mathematics Teachers, 1998)

Becoming numerate involves developing the ability to explore, conjecture, reason logically, and use a variety of mathematical methods to solve problems. It also involves the development of self-confidence and the ability to use quantitative and spatial information in problem solving and decision-making. As students develop their numeracy skills and concepts, they generally grow more confident and motivated in their mathematical explorations. This growth occurs as they learn to enjoy and value mathematics, to think analytically, and to understand and appreciate the role of mathematics in everyday life.

The provincial mathematics curriculum emphasizes the development of numeracy skills and concepts and their practical application in higher education and the workplace. The curriculum places emphasis on probability and statistics, reasoning and communication, measurement, and problem solving. To ensure that students are prepared for the demands of both further education and the workplace, the graduate years of the mathematics curriculum (Grades 10 to 12) help students develop a more sophisticated sense of numeracy.

Requirements and Graduation Credits

Essentials of Mathematics 10 and 11 *or* 12 are two of the courses available for students to satisfy the Graduation Program mathematics requirement.

Essentials of Mathematics 10, 11, and 12 are each designated as four-credit courses, and must be reported as such to the Ministry of Education for transcript purposes. Letter grades and percentages must be reported for these courses. It is not possible to obtain partial credit for these courses.

The course codes for Essentials of Mathematics 10 to 12 are EMA 10, EMA 11, and EMA 12. These courses are also available in French (Mathématiques de base 10, Mathématiques de base 11, Mathématiques de base 12; course codes EMAF 10, EMAF 11, EMAF 12).

Graduation Program Examination

Essentials of Mathematics 10 has a Graduation Program examination, worth 20% of the final course mark. Students are required to take this exam to receive credit for the course. There is no provincial exam for Essentials of Mathematics 12.

For more information, refer to the Ministry of Education examinations web site: www.bced.gov.bc.ca/exams/

Goals for Essentials of Mathematics 10 to 12

In order to meet the challenges of society, high school graduates must be numerate. Students following this pathway will have opportunities to improve their numeracy skills and concepts. Developing a sense of numeracy will help them to understand how mathematical concepts permeate daily life, business, industry, and government. Students need to be able to use mathematics not just in their work lives, but also in their personal lives as citizens and consumers. It is intended that students will learn to value mathematics and become confident in their mathematical abilities.

Goals for Essentials of Mathematics 10 to 12

The aim of Essentials of Mathematics 10 to 12 is to provide students with the necessary numeracy skills and concepts to be successful in their daily lives, business, industry, and government.

Students will

- become numerate citizens with the ability to explore, conjecture, reason logically, and use a variety of mathematical methods to solve problems
- develop self-confidence and the ability to use quantitative and spatial information in problem solving and decision making
- learn to enjoy and value mathematics, to think analytically, and to understand and appreciate the role of mathematics in everyday life
- be prepared for the demands of both further education and the workplace and develop a more sophisticated sense of numeracy

Curriculum Organizers

A curriculum organizer consists of a set of prescribed learning outcomes that share a common focus. The prescribed learning outcomes for Essentials of Mathematics 10 to 12 progress in age-appropriate ways, and are grouped under the following curriculum organizers and suborganizers.

Note that the ordering of these organizers and suborganizers is not intended to imply an order of instruction.

Curriculum Organizers and Suborganizers

Essentials of Mathematics

Number	Patterns and Relations	Shape and Space	Statistics and Probability
• Business Plan • Government Finances • Income and Debt • Investments • Owning and Operating a Vehicle • Personal Banking • Personal Finance • Personal Income Tax • Spreadsheets • Wages, Salaries, and Expenses	• Rate, Ratio, and Proportion • Relations and Formulas • Variation and Formulas	• Design and Measurement • Geometry Project • Measurement Technology • Trigonometry	• Probability and Sampling • Data Analysis and Interpretation

Number

In this organizer students apply a range of number operations and problem solving skills to solve problems involving personal and business finances.

The Number organizer includes the following suborganizers:

- Business Plan – research and design a business plan
- Government Finances – understand government finances and implication on payroll deductions and taxes
- Income and Debt – understand procedures and apply mathematics related to money management
- Investments – make sound financial decisions taking into account a variety of factors
- Owning and Operating a Vehicle – make sound decisions related to acquiring and maintaining a motor vehicle
- Personal Banking – understand and use the services and facilities of banks, trust companies, and credit unions
- Personal Finance – analyse various types of insurance and mortgage scenarios
- Personal Income Tax – understand how to prepare an income tax return
- Spreadsheets – design and use spreadsheets to make and justify decisions, solve problems, and support projections
- Wages, Salaries, and Expenses – calculate gross pay, net pay, and percentage changes in income

Patterns and Relations

Students need to recognize, extend, create, and use patterns as a routine aspect of their lives. This organizer provides opportunities for students to look for relationships among physical things, as well as the data used to describe those things. These relationships will be described visually, symbolically, orally, and in written form.

The Patterns and Relations organizer includes the following suborganizers:

- Rate, Ratio, and Proportion – make financial decisions based on calculations of unit rate, percent taxes, and discounts
- Relations and Formulas – interpret linear relations to solve problems
- Variation and Formulas – identify relationships between various quantities to solve problems

Shape and Space

It is important that students look for and use similarity and patterns in the solution of a range of problems. This organizer provides opportunities for students to apply geometry and measurement skills to solve realistic problems.

The Shape and Space organizer includes the following suborganizers:

- Design and Measurement – use geometric ideas in design and measurement of 3-D objects
- Geometry Project – acquire and apply a sense of proportion and shape to solve 2-D and 3-D problems
- Measurement Technology – convert between systems of measurement and use measurement devices to solve problems
- Trigonometry – use ratio, proportion, and trigonometry to solve problems involving unknown lengths or angles

Statistics and Probability

Students must be able to solve problems involving data sets presented in relevant contexts. Students should also be able to select appropriate data sampling methods and analyse the validity of data representations. Finally, students should also be able to model data graphically and symbolically so that patterns can be identified and used to solve problems.

The Statistics and Probability organizer includes the following suborganizers:

- Probability and Sampling – collect, display, and interpret data
- Data Analysis and Interpretation – display, analyse, and manipulate the presentation of data

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The following seven mathematical processes should be integrated within Essentials of Mathematics 10 to 12.

Communication

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students need to be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine, 1991, p. 5).

Mental Mathematics and Estimation

Mental mathematics is a combination of cognitive strategies that enhances flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility.

Even more important than performing computational procedures or using calculators is the greater facility that students need – more than ever before – with estimation and mental mathematics (NCTM, May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001).

Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating.

Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type "How would you...?" or "How could you...?" the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

Reasoning

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels.

Visualization

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with the opportunity to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to decide when to measure, when to estimate and to know several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Essentials of Mathematics 10 to 12: At a Glance

Essentials of Mathematics 10	Essentials of Mathematics 11	Essentials of Mathematics 12
Number		
Spreadsheets • design and use spreadsheets to make and justify decisions, solve problems, and support projections **Personal Banking** • prepare bank forms including cheques, deposit slips, chequebook activity record, and reconciliation statements **Wages, Salaries, and Expenses** • solve problems involving wages, salaries, and expenses	**Income and Debt** • demonstrate an awareness of selected forms of personal income and debt **Personal Income Tax** • prepare a simple income tax form **Owning and Operating a Vehicle** • analyse the cost of acquiring and operating a vehicle **Business Plan** • prepare a plan to operate a successful business	**Personal Finance** • solve consumer problems involving insurance, mortgages, and loans **Investments** • demonstrate and recognize the differences concerning different types of financial investments **Government Finances** • demonstrate an awareness of the income and expenditures of federal, provincial, and municipal governments
Patterns and Relations		
Rate, Ratio, and Proportion • apply the concepts of rate, ratio, and proportion to solve problems	**Relations and Formulas** • represent and interpret relations in a variety of contexts	**Variation and Formulas** • use algebraic and graphical models to generate patterns, make predictions, and solve patterns
Shape and Space		
Trigonometry • demonstrate an understanding of ratio and proportion, and apply these concepts in solving triangles **Geometry Project** • complete a project that includes a 2-D plan and a 3-D model of some physical structure	**Measurement Technology** • determine measurements in Système International (SI) and Imperial systems using different measuring devices	**Design and Measurement** • analyse objects, shapes, and processes to solve cost and design problems
Statistics and Probability		
Probability and Sampling • develop and implement a plan for the collection, display, and analysis of data using technology as required	**Data Analysis and Interpretation** • analyse data with a focus on the validity of its presentation and the inferences made	

Suggested Timeframe

Provincial curricula are developed in accordance with the amount of instructional time recommended by the Ministry of Education for each subject area. Teachers may choose to combine various curricula to enable students to integrate ideas and make meaningful connections.

In each of Grades 10, 11, and 12, a minimum of 100 hours of instructional time is recommended for the study of Essentials of Mathematics. Although a four-credit course is typically equivalent to 120 hours, this timeframe allows for flexibility to address local needs.

The following table shows the average number of hours suggested to deliver the prescribed learning outcomes in each curriculum suborganizer.

These estimations are provided as suggestions only; when delivering the prescribed curriculum teachers should adjust the instructional time as necessary.

Suggested Timeframe for Essentials of Mathematics 10 to 12

Curriculum Organizer (Suborganizer)	Suggested Timeframe	Curriculum Organizer (Suborganizer)	Suggested Timeframe	Curriculum Organizer (Suborganizer)	Suggested Timeframe
Essentials of Mathematics 10		**Essentials of Mathematics 11**		**Essentials of Mathematics 12**	
Number		**Number**		**Number**	
(Spreadsheets)	5 - 10 hrs	(Income and Debt)	15 - 20 hrs	(Personal Finance)	20 - 25 hrs
(Personal Banking)	10 - 15 hrs	(Personal Income Tax)	5 - 10 hrs	(Investments)	20 - 25 hrs
(Wages, Salaries, and Expenses)	15 - 20 hrs	(Owning and Operating a Vehicle)	15 - 20 hrs	(Government Finances)	15 - 20 hrs
		(Business Plan)	20 - 25 hrs		
Patterns and Relations		**Patterns and Relations**		**Patterns and Relations**	
(Rate, Ratio, and Proportion)	15 - 20 hrs	(Relations and Formulas)	15 - 20 hrs	(Variation and Formulas)	15 - 20 hrs
Shape and Space		**Shape and Space**		**Shape and Space**	
(Trigonometry)	5 - 10 hrs	(Measurement Technology)	15 - 20 hrs	(Design and Measurement)	20 - 25 hrs
(Geometry Project)	20 - 25 hrs				
Statistics and Probability		**Statistics and Probability**			
(Probability and Sampling)	10 - 15 hrs	(Data Analysis and Interpretation)	5 - 10 hrs		

Considerations for Program Delivery

Essentials of Mathematics 10 to 12

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners. Included in this section is information about

- Alternative Delivery policy
- instructional focus
- fostering the development of positive attitudes
- applying mathematics
- involving parents and guardians
- confidentiality
- inclusion, equity, and accessibility for all learners
- working with the school and community
- working with the Aboriginal community
- information and communications technology
- copyright and responsibility

Alternative Delivery Policy

The Alternative Delivery policy does not apply to Essentials of Mathematics 10 to 12.

The Alternative Delivery policy outlines how students, and their parents or guardians, in consultation with their local school authority, may choose means other than instruction by a teacher within the regular classroom setting for addressing prescribed learning outcomes contained in the Health curriculum organizer of the following curriculum documents:

- Health and Career Education K to 7, and Personal Planning K to 7 Personal Development curriculum organizer (until September 2008)
- Health and Career Education 8 and 9
- Planning 10

The policy recognizes the family as the primary educator in the development of children's attitudes, standards, and values, but the policy still requires that all prescribed learning outcomes be addressed and assessed in the agreed-upon alternative manner of delivery.

It is important to note the significance of the term "alternative delivery" as it relates to the Alternative Delivery policy. The policy does not permit schools to omit addressing or assessing any of the prescribed learning outcomes within the health and career education curriculum. Neither does it allow students to be excused from meeting any learning outcomes related to health. It is expected that students who arrange for alternative delivery will address the health-related learning outcomes and will be able to demonstrate their understanding of these learning outcomes.

For more information about policy relating to alternative delivery, refer to www.bced.gov.bc.ca/policy/

Instructional Focus

The Essentials of Mathematics 10 to 12 courses are arranged into a number of organizers with problem solving integrated throughout. Decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations allows more time for concept development.

In addition to problem solving, other critical thinking processes – reasoning and making connections – are vital to increasing students' mathematical power and must be integrated throughout the program. A minimum of half the available time within all organizers should be dedicated to activities related to these processes.

Instruction should provide a balance between estimation and mental mathematics, paper and pencil exercises, and the appropriate use of technology, including calculators and computers. (It is assumed that all students have regular access to appropriate technology such as graphing calculators, or computers with graphing software and standard spreadsheet programs.) Concepts should be introduced using manipulatives, and gradually developed from the concrete to the pictorial to the symbolic.

Fostering the Development of Positive Attitudes

Students should be exposed to experiences that encourage them to enjoy and value mathematics, develop mathematical habits of mind, and understand and appreciate the role of mathematics in human affairs. They should be encouraged to

explore, take risks, exhibit curiosity, and make and correct errors, so they gain confidence in their abilities to solve complex problems. The assessment of attitudes is indirect, and based on inferences drawn from students' behaviour. We can see what students do and hear what they say, and from these observations make inferences and draw conclusions about their attitudes.

Applying Mathematics

For students to view mathematics as relevant and useful, they must see how it can be applied to a wide variety of real-world applications. Mathematics helps students understand and interpret their world and solve problems that occur in their daily lives.

Involving Parents and Guardians

The family is essential in the development of students' attitudes and values. The school plays a supportive role by focussing on the prescribed learning outcomes in the Grades 10 to 12 Mathematics curriculum. Parents and guardians are encouraged to support, enrich, and extend the curriculum at home.

It is highly recommended that schools inform parents and guardians about the Essentials of Mathematics curriculum. Teachers (along with school and district administrators) may choose to do so by

- informing parents/guardians and students, via a course outline at the beginning of the course, of the prescribed learning outcomes for the course
- responding to parent and guardian requests to discuss course unit plans, learning resources, etc.

Confidentiality

The *Freedom of Information and Protection of Privacy Act* (FOIPPA) applies to students, to school districts, and to all curricula. Teachers, administrators, and district staff should consider the following:

- Be aware of district and school guidelines regarding the provisions of FOIPPA and how it applies to all subjects, including Essentials of Mathematics 10 to 12.
- Do not use students' Personal Education Numbers (PEN) on any assignments that students wish to keep confidential.
- Ensure students are aware that if they disclose personal information that indicates they are at risk for harm, then that information cannot be kept confidential.
- Inform students of their rights under FOIPPA, especially the right to have access to their own personal information in their school records. Inform parents of their rights to access their children's school records.
- Minimize the type and amount of personal information collected, and ensure that it is used only for purposes that relate directly to the reason for which it is collected.
- Inform students that they will be the only ones recording personal information about themselves unless they, or their parents, have consented to teachers collecting that information from other people (including parents).
- Provide students and their parents with the reason(s) they are being asked to provide personal information in the context of the Essentials of Mathematics 10 to 12 curriculum.
- Inform students and their parents that they can ask the school to correct or annotate any of the personal information held by the school, in accordance with Section 29 of FOIPPA.
- Ensure students are aware that their parents may have access to the schoolwork they create only insofar as it pertains to students' progress.
- Ensure that any information used in assessing students' progress is up-to-date, accurate, and complete.

For more information about confidentiality, refer to www.mser.gov.bc.ca/privacyaccess/

Inclusion, Equity, and Accessibility for All Learners

British Columbia's schools include young people of varied backgrounds, interests, and abilities. The Kindergarten to Grade 12 school system focusses on meeting the needs of all students. When selecting specific topics, activities, and resources to support the implementation of Essentials of Mathematics 10 to 12, teachers are encouraged to ensure that these

choices support inclusion, equity, and accessibility for all students. In particular, teachers should ensure that classroom instruction, assessment, and resources reflect sensitivity to diversity and incorporate positive role portrayals, relevant issues, and themes such as inclusion, respect, and acceptance.

Government policy supports the principles of integration and inclusion of students for whom English is a second language and of students with special needs. Most of the prescribed learning outcomes and suggested achievement indicators in this IRP can be met by all students, including those with special needs and/or ESL needs. Some strategies may require adaptations to ensure that those with special and/or ESL needs can successfully achieve the learning outcomes. Where necessary, modifications can be made to the prescribed learning outcomes for students with Individual Education Plans.

For more information about resources and support for students with special needs, refer to www.bced.gov.bc.ca/specialed/

For more information about resources and support for ESL students, refer to www.bced.gov.bc.ca/esl/

Working with the School and Community

This curriculum addresses a wide range of skills and understandings that students are developing in other areas of their lives. It is important to recognize that learning related to this curriculum extends beyond the Mathematics classroom.

Community organizations may also support the curriculum with locally developed learning resources, guest speakers, workshops, and field studies. Teachers may wish to draw on the expertise of these community organizations and members.

Working with the Aboriginal Community

The Ministry of Education is dedicated to ensuring that the cultures and contributions of Aboriginal peoples in BC are reflected in all provincial curricula. To address these topics in the classroom in a way that is accurate and that respectfully reflects Aboriginal concepts of teaching and learning, teachers are strongly encouraged to seek the advice and support of local Aboriginal communities. Aboriginal communities are diverse in terms of language, culture, and available resources, and each community will have its own unique protocol to gain support for integration of local knowledge and expertise. To begin discussion of possible instructional and assessment activities, teachers should first contact Aboriginal education co-ordinators, teachers, support workers, and counsellors in their district who will be able to facilitate the identification of local resources and contacts such as elders, chiefs, tribal or band councils, Aboriginal cultural centres, Aboriginal Friendship Centres, and Métis or Inuit organizations.

In addition, teachers may wish to consult the various Ministry of Education publications available, including the "Planning Your Program" section of the resource, *Shared Learnings.* This resource was developed to help all teachers provide students with knowledge of, and opportunities to share experiences with, Aboriginal peoples in BC.

For more information about these documents, consult the Aboriginal Education web site: www.bced.gov.bc.ca/abed/welcome.htm

Information and Communications Technology

The study of information and communications technology is increasingly important in our society. Students need to be able to acquire and analyse information, to reason and communicate, to make informed decisions, and to understand and use information and communications technology for a variety of purposes. Development of these skills is important for students in their education, their future careers, and their everyday lives.

Literacy in the area of information and communications technology can be defined as the ability to obtain and share knowledge through investigation, study, instruction, or transmission of information by means of media technology.

Becoming literate in this area involves finding, gathering, assessing, and communicating information using electronic means, as well as developing the knowledge and skills to use and solve problems effectively with the technology. Literacy also involves a critical examination and understanding of the ethical and social issues related to the use of information and communications technology. When planning for instruction and assessment in Essentials of Mathematics 10 to 12, teachers should provide opportunities for students to develop literacy in relation to information and communications technology sources, and to reflect critically on the role of these technologies in society.

Copyright and Responsibility

Copyright is the legal protection of literary, dramatic, artistic, and musical works; sound recordings; performances; and communications signals. Copyright provides creators with the legal right to be paid for their work and the right to say how their work is to be used. The law permits certain exceptions for schools (i.e., specific things permitted) but these are very limited, such as copying for private study or research. The copyright law determines how resources can be used in the classroom and by students at home.

In order to respect copyright it is necessary to understand the law. It is unlawful to do the following, unless permission has been given by a copyright owner:

- photocopy copyrighted material to avoid purchasing the original resource for any reason
- photocopy or perform copyrighted material beyond a very small part – in some cases the copyright law considers it "fair" to copy whole works, such as an article in a journal or a photograph, for purposes of research and private study, criticism, and review
- show recorded television or radio programs to students in the classroom unless these are cleared for copyright for educational use (there are exceptions such as for news and news commentary taped within one year of broadcast that by law have record-keeping requirements – see the web site at the end of this section for more details)
- photocopy print music, workbooks, instructional materials, instruction manuals, teacher guides, and commercially available tests and examinations
- show videorecordings at schools that are not cleared for public performance
- perform music or do performances of copyrighted material for entertainment (i.e., for purposes other than a specific educational objective)
- copy work from the Internet without an express message that the work can be copied

Permission from or on behalf of the copyright owner must be given in writing. Permission may also be given to copy or use all or some portion of copyrighted work through a licence or agreement. Many creators, publishers, and producers have formed groups or "collectives" to negotiate royalty payments and copying conditions for educational institutions. It is important to know what licences are in place and how these affect the activities schools are involved in. Some licences may also require royalty payments that are determined by the quantity of photocopying or the length of performances. In these cases, it is important to assess the educational value and merits of copying or performing certain works to protect the school's financial exposure (i.e., only copy or use that portion that is absolutely necessary to meet an educational objective).

It is important for education professionals, parents, and students to respect the value of original thinking and the importance of not plagiarizing the work of others. The works of others should not be used without their permission.

For more information about copyright, refer to http://cmec.ca/copyright/indexe.stm

Prescribed Learning Outcomes

Essentials of Mathematics 10 to 12

Prescribed learning outcomes are content standards for the provincial education system; they are the prescribed curriculum. Clearly stated and expressed in measurable and observable terms, learning outcomes set out the required knowledge, skills, and attitudes – what students are expected to know and be able to do – by the end of the specified course.

Schools have the responsibility to ensure that all prescribed learning outcomes in this curriculum are met; however, schools have flexibility in determining how delivery of the curriculum can best take place.

It is expected that student achievement will vary in relation to the learning outcomes. Evaluation, reporting, and student placement with respect to these outcomes are dependent on the professional judgment and experience of teachers, guided by provincial policy.

Prescribed learning outcomes for Essentials of Mathematics 10 to 12 are presented by curriculum organizer and suborganizer, and are coded alphanumerically for ease of reference; however, this arrangement is not intended to imply a required instructional sequence.

Wording of Prescribed Learning Outcomes

All learning outcomes complete the stem, "It is expected that students will …."

When used in a prescribed learning outcome, the word "**including**" indicates that any ensuing item **must be addressed**. Lists of items introduced by the word "including" represent a set of minimum requirements associated with the general requirement set out by the outcome. The lists are not necessarily exhaustive, however, and teachers may choose to address additional items that also fall under the general requirement set out by the outcome.

Conversely, the abbreviation "**e.g.**" (for example) in a prescribed learning outcome indicates that the ensuing items are provided for illustrative purposes or clarification, and **are not required**. Presented in parentheses, the list of items introduced by "e.g." is neither exhaustive nor prescriptive, nor is it put forward in any special order of importance or priority. Teachers are free to substitute items of their own choosing that they feel best address the intent of the learning outcome.

Domains of Learning

Prescribed learning outcomes in BC curricula identify required learning in relation to one or more of the three domains of learning: cognitive, psychomotor, and affective. The following definitions of the three domains are based on Bloom's taxonomy.

The **cognitive domain** deals with the recall or recognition of knowledge and the development of intellectual abilities. The cognitive domain can be further specified as including three cognitive levels: knowledge, understanding and application, and higher mental processes. These levels are determined by the verb used in the learning outcome, and illustrate how student learning develops over time.

- *Knowledge* includes those behaviours that emphasize the recognition or recall of ideas, material, or phenomena.
- *Understanding and application* represents a comprehension of the literal message contained in a communication, and the ability to apply an appropriate theory, principle, idea, or method to a new situation.
- *Higher mental processes* include analysis, synthesis, and evaluation. The higher mental processes level subsumes both the knowledge and the understanding and application levels.

The **affective domain** concerns attitudes, beliefs, and the spectrum of values and value systems.

The **psychomotor domain** includes those aspects of learning associated with movement and skill demonstration, and integrates the cognitive and affective consequences with physical performances.

Domains of learning and, particularly, cognitive levels, inform the design and development of the Graduation Program examination for Essentials of Mathematics 10.

Prescribed Learning Outcomes: Essentials of Mathematics 10

It is expected that students will:

NUMBER

Spreadsheets

A1 design and use a spreadsheet to make and justify decisions
A2 create a spreadsheet using various formatting options
A3 use a spreadsheet template to solve problems
A4 create a spreadsheet using formulas and functions
A5 use a spreadsheet to answer "what-if" questions
A6 identify where spreadsheets could be effectively used

Personal Banking

A7 name and describe various types of commonly used consumer bank accounts, including
- value accounts
- self-serve accounts
- full-serve accounts
- savings accounts

A8 complete various banking forms, including
- deposit slips
- withdrawal slips
- cheques

A9 describe the use of a bank card for automated teller machines (ATMs) and debit payments
A10 identify different types of bank service charges and their relative costs, including
- monthly account fees
- transaction charges
- interest charges

A11 reconcile statements, including chequebooks and electronic bank transactions with bank statements

Wages, Salaries, and Expenses

A12 calculate hours worked and gross pay
A13 calculate net income using deduction tables (focus on weekly) with different pay periods
A14 calculate changes in income
A15 develop a budget that matches predicted income

PATTERNS AND RELATIONS

Rate, Ratio, and Proportion

B1 use the concept of unit rate to determine the best buy on a consumer item and justify the decision
B2 solve problems on the application of sales tax in Canada
B3 describe a variety of sales promotion techniques and their financial implications for the consumer
B4 solve rate, ratio, and proportion problems involving price, length, area, volume, time, and mass

Prescribed Learning Outcomes: Essentials of Mathematics 10

SHAPE AND SPACE

Trigonometry

C1 apply ratio and proportion in similar triangles
C2 use the trigonometric ratios sine, cosine, and tangent in solving right triangles

Geometry Project

C3 measure lengths in both SI and Imperial units
C4 estimate measurements of objects in SI and Imperial systems, including
- length
- area
- volume
- mass

C5 interpret drawings and use the information to solve problems
C6 draw top, front, and side views for both 3-D rod or block objects and their sketches
C7 sketch 3-D designs using isometric dot paper
C8 enlarge or reduce a dimensioned object according to a specified scale
C9 solve problems involving linear dimensions, area, and volume
C10 complete a project that includes a 2-D plan and a 3-D model of some physical structure

STATISTICS AND PROBABILITY

Probability and Sampling

D1 read and interpret graphs
D2 use suitable graph types to display data (by hand or using technology), including
- broken line graphs
- bar graphs
- histograms
- circle graphs

D3 determine and use measures of central tendency to support decisions, including
- mean
- median
- mode

D4 use sample data to make predictions and decisions
D5 critique ways in which statistical information and conclusions are presented by the media and other sources

Prescribed Learning Outcomes: Essentials of Mathematics 11

It is expected that students will:

NUMBER

Income and Debt

A1 solve problems involving performance-based income, including
- commission sales
- piece work
- salary plus commission

A2 use simple and compound interest calculations to solve problems

A3 solve consumer problems involving
- credit cards
- exchange rates
- personal loans

Personal Income Tax

A4 prepare an income tax form for an individual who is single, employed, and without dependents

Owning and Operating a Vehicle

A5 solve problems involving the acquisition and operation of a vehicle, including
- renting
- leasing
- buying
- licensing
- insuring
- operating (e.g., fuel and oil)
- maintaining (e.g., repairs, tune-ups)
- depreciation

Business Plan

A6 prepare a business plan to own and operate a business that includes the following information as appropriate:
- monthly cost of operation (e.g., inventory, rent, wages, insurance, advertising, loan payments)
- hours of operation
- estimated daily sales (average)
- gross/net profit
- hourly earnings
- a scale floor plan of the store

Prescribed Learning Outcomes: Essentials of Mathematics 11

PATTERNS AND RELATIONS

Relations and Formulas

B1 express a linear relation of the form $y = mx + b$
- in words
- as a formula
- with a table of values
- as a graph

B2 interpolate and extrapolate values from the graph of a linear relation

B3 determine the slope of a linear relation and describe it in words

B4 interpret the meaning of the slope of a linear relation in a problem context

B5 substitute appropriate values into formulas and evaluate the result

SHAPE AND SPACE

Measurement Technology

C1 select and use appropriate measuring devices (in Imperial and SI systems), including
- Vernier calipers
- micrometers

C2 perform basic conversions within and between the Imperial and SI systems, using technology as appropriate

C3 use measuring devices and unit conversions to solve problems

STATISTICS AND PROBABILITY

Data Analysis and Interpretation

D1 display and analyse data on
- line plots
- bar graphs
- circle graphs

D2 manipulate the presentation of data to stress a particular point of view

Prescribed Learning Outcomes: Essentials of Mathematics 12

It is expected that students will:

NUMBER

Personal Finance

A1 solve problems involving different types of insurance, including
- life
- property

A2 determine the costs involved in purchasing a home, including
- gross debt service ratio
- payment options
- insurance
- additional fees

A3 solve problems involving different types of mortgages, including
- closed
- open
- fixed-rate
- variable-rate

Investments

A4 determine a financial plan to achieve personal goals

A5 calculate net worth for an individual

A6 describe different investment vehicles, including
- GICs
- bonds
- mutual funds
- stocks
- real estate
- RRSPs and RESPs

A7 compare and contrast different investment vehicles in terms of
- risk factors
- rates of return
- costs
- lengths of term

A8 investigate how to purchase and sell stocks

Organizer 'Number' continued on page 25

Prescribed Learning Outcomes: Essentials of Mathematics 12

Organizer 'Number' continued from page 24

Government Finances

A9 describe and calculate federal revenues, including
- GST
- excise tax
- customs duties

A10 describe and calculate federal government expenditures, including
- social welfare benefits
- health care
- policing
- armed forces
- employee wages and salaries

A11 describe and calculate provincial revenues, including
- PST
- corporation capital
- licences
- gasoline

A12 describe and calculate provincial government expenditures, including
- education funding
- social services funding
- employee wages and salaries

A13 determine how selected municipal taxes are calculated

A14 determine how municipalities allocate funds

PATTERNS AND RELATIONS

Variation and Formulas

B1 plot and analyse examples of direct variation, partial variation, and inverse variation

B2 given data, graph, or a situation, identify the type of variation represented

B3 use formulas to solve problems

SHAPE AND SPACE

Design and Measurement

C1 analyse objects shown in "exploded" format

C2 draw objects in "exploded" format

C3 solve problems involving estimation and cost of materials for building an object when a design is given

C4 design an object within a specified budget

Student Achievement

Essentials of Mathematics 10 to 12

This section of the IRP contains information about classroom assessment and student achievement, including specific achievement indicators to assist in the assessment of student achievement in relation to each prescribed learning outcome. Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of prescribed learning outcomes.

Classroom Assessment and Evaluation

Assessment is the systematic gathering of information about what students know, are able to do, and are working toward. Assessment evidence can be collected using a wide variety of methods, such as

- observation
- student self-assessments and peer assessments
- quizzes and tests (written, oral, practical)
- samples of student work
- projects and presentation
- oral and written reports
- journals and learning logs
- performance reviews
- portfolio assessments

Assessment of student performance is based on the information collected through assessment activities. Teachers use their insight, knowledge about learning, and experience with students, along with the specific criteria they establish, to make judgments about student performance in relation to prescribed learning outcomes.

Three major types of assessment can be used in conjunction to support student achievement.

- Assessment **for** learning is assessment for purposes of greater learning achievement.
- Assessment **as** learning is assessment as a process of developing and supporting students' active participation in their own learning.
- Assessment **of** learning is assessment for purposes of providing evidence of achievement for reporting.

Assessment for Learning

Classroom assessment for learning provides ways to engage and encourage students to become involved in their own day-to-day assessment – to acquire the skills of thoughtful self-assessment and to promote their own achievement.

This type of assessment serves to answer the following questions:

- What do students need to learn to be successful?
- What does the evidence of this learning look like?

Assessment for learning is criterion-referenced, in which a student's achievement is compared to established criteria rather than to the performance of other students. Criteria are based on prescribed learning outcomes, as well as on suggested achievement indicators or other learning expectations.

Students benefit most when assessment feedback is provided on a regular, ongoing basis. When assessment is seen as an opportunity to promote learning rather than as a final judgment, it shows students their strengths and suggests how they can develop further. Students can use this information to redirect their efforts, make plans, communicate with others (e.g., peers, teachers, parents) about their growth, and set future learning goals.

Assessment for learning also provides an opportunity for teachers to review what their students are learning and what areas need further attention. This information can be used to inform teaching and create a direct link between assessment and instruction. Using assessment as a way of obtaining feedback on instruction supports student achievement by informing teacher planning and classroom practice.

Assessment as Learning

Assessment as learning actively involves students in their own learning processes. With support and guidance from their teacher, students take responsibility for their own learning, constructing meaning for themselves. Through a process of continuous self-assessment, students develop the ability to take stock of what they have already learned, determine what they have not yet learned, and decide how they can best improve their own achievement.

Although assessment as learning is student-driven, teachers can play a key role in facilitating how this assessment takes place. By providing regular opportunities for reflection and self-assessment, teachers can help students develop, practise, and become comfortable with critical analysis of their own learning.

Assessment of Learning

Assessment of learning can be addressed through summative assessment, including large-scale assessments and teacher assessments. These summative assessments can occur at the end of the year or at periodic stages in the instructional process.

Large-scale assessments, such as Foundation Skills Assessment (FSA) and Graduation Program exams, gather information on student performance throughout the province and provide information for the development and revision of curriculum. These assessments are used to make judgments about students' achievement in relation to provincial and national standards.

Assessment of learning is also used to inform formal reporting of student achievement.

For Ministry of Education reporting policy, refer to www.bced.gov.bc.ca/policy/policies/student_reporting.htm

Assessment *for* Learning	Assessment *as* Learning	Assessment *of* Learning
Formative assessment is *ongoing in the classroom* • teacher assessment, student self-assessment, and/or student peer assessment • criterion-referenced – criteria based on prescribed learning outcomes identified in the provincial curriculum, reflecting performance in relation to a specific learning task • involves both teacher and student in a process of continual reflection and review about progress • teachers adjust their plans and engage in corrective teaching in response to formative assessment	**Formative assessment is** *ongoing in the classroom* • self-assessment • provides students with information on their own achievement and prompts them to consider how they can continue to improve their learning • student-determined criteria based on previous learning and personal learning goals • students use assessment information to make adaptations to their learning process and to develop new understandings	**Summative assessment occurs** *at end of year or at key stages* • teacher assessment • may be either criterion-referenced (based on prescribed learning outcomes) or norm-referenced (comparing student achievement to that of others) • information on student performance can be shared with parents/guardians, school and district staff, and other education professionals (e.g., for the purposes of curriculum development) • used to make judgments about students' performance in relation to provincial standards

For more information about assessment for, as, and of learning, refer to the following resource developed by the Western and Northern Canadian Protocol (WNCP): *Rethinking Assessment with Purpose in Mind*.

This resource is available online at www.wncp.ca/

Criterion-Referenced Assessment and Evaluation

In criterion-referenced evaluation, a student's performance is compared to established criteria rather than to the performance of other students. Evaluation in relation to prescribed curriculum requires that criteria be established based on the learning outcomes.

Criteria are the basis for evaluating student progress. They identify, in specific terms, the critical aspects of a performance or a product that indicate how well the student is meeting the prescribed learning outcomes. For example, weighted criteria, rating scales, or scoring guides (reference sets) are ways that student performance can be evaluated using criteria.

Wherever possible, students should be involved in setting the assessment criteria. This helps students develop an understanding of what high-quality work or performance looks like.

Criterion-referenced assessment and evaluation may involve these steps:

Step 1	Identify the prescribed learning outcomes and suggested achievement indicators (as articulated in this IRP) that will be used as the basis for assessment.
Step 2	Establish criteria. When appropriate, involve students in establishing criteria.
Step 3	Plan learning activities that will help students gain the knowledge, skills, and attitudes outlined in the criteria.
Step 4	Prior to the learning activity, inform students of the criteria against which their work will be evaluated.
Step 5	Provide examples of the desired levels of performance.
Step 6	Conduct the learning activities.
Step 7	Use appropriate assessment instruments (e.g., rating scale, checklist, scoring guide) and methods (e.g., observation, collection, self-assessment) based on the particular assignment and student.
Step 8	Review the assessment data and evaluate each student's level of performance or quality of work in relation to criteria.
Step 9	Where appropriate, provide feedback and/or a letter grade to indicate how well the criteria are met.
Step 10	Communicate the results of the assessment and evaluation to students and parents/guardians.

Key Elements

Key elements provide an overview of content in each curriculum organizer. They can be used to determine the expected depth and breadth of the prescribed learning outcomes.

Note that some topics appear at multiple grade levels in order to emphasize their importance and to allow for developmental learning.

Achievement Indicators

To support the assessment of provincially prescribed curricula, this IRP includes sets of achievement indicators in relation to each learning outcome.

Achievement indicators, taken together as a set, define the specific level of knowledge acquired, skills applied, or attitudes demonstrated by the student in relation to a corresponding prescribed learning outcome. They describe what evidence to look for to determine whether or not a student has fully met the intent of the learning outcome. Since each achievement indicator defines only one aspect of the corresponding learning outcome, the entire set of achievement indicators should be considered when determining whether students have fully met the learning outcome.

In some cases, achievement indicators may also include suggestions as to the type of task that would provide evidence of having met the learning outcome (e.g., problem solving; a constructed response such as a list, comparison, analysis, or chart; a product created and presented such as a report, poster, or model; a particular skill demonstrated).

Achievement indicators support the principles of assessment *for* learning, assessment *as* learning, and assessment *of* learning. They provide teachers and parents with tools that can be used to reflect on what students are learning, as well as provide students with a means of self-assessment and ways of defining how they can improve their own achievement.

Achievement indicators are not mandatory; they are suggestions only, provided to assist in the assessment of how well students achieve the prescribed learning outcomes.

Achievement indicators may be useful to provincial examination development teams and inform the development of exam items. However, examination questions, item formats, exemplars, rubrics, or scoring guides will not necessarily be limited to the achievement indicators as outlined in the Integrated Resource Packages.

Specifications for provincial examinations are available online at www.bced.gov.bc.ca/exams/specs/

The following pages contain the suggested achievement indicators corresponding to each prescribed learning outcome for the Essentials of Mathematics 10 to 12 curriculum. The achievement indicators are arranged by curriculum organizer and suborganizer for each grade; however, this order is not intended to imply a required sequence of instruction and assessment.

STUDENT ACHIEVEMENT

Essentials of Mathematics 10

Key Elements: Essentials of Mathematics 10

Mathematical Process (Integrated)
The following mathematical processes have been integrated within the prescribed learning outcomes and achievement indicators for the course: communication, problem solving, connections, mental mathematics and estimation, reasoning, technology, and visualization.

NUMBER

Spreadsheets
- spreadsheet components, including cells, rows, columns, and cell contents
- spreadsheet formats, including types of cell entries
- benefits of spreadsheets to model problems, answer "what-if" situations and graph data

Personal Banking
- types of bank accounts
- bank forms, including cheques, deposit slips, chequebook activity records, and reconciliation statements
- electronic banking, including bank cards, ATMs, telephone banking, and Internet banking
- banking fees and interest

Wages, Salaries, and Expenses
- gross pay, including hours worked and overtime wages
- net income after deductions, including CPP, EI, and income tax
- balanced budget

PATTERNS AND RELATIONS

Rate, Ratio, and Proportion
- consumer decisions, including 'best buy,' unit price, discount, GST, PST, and sales promotion
- rate, ratio, and proportion involving price, length, area, volume, time, and mass

SHAPE AND SPACE

Trigonometry
- similar triangles
- primary trigonometric ratios

Geometry Project
- SI and Imperial measurements of length, perimeter, area, surface area, volume, and mass
- perspective drawing and construction of 2-D shapes and 3-D objects
- enlargement and reduction of dimensioned objects
- geometry project, including a 2-D plan and 3-D model

STATISTICS AND PROBABILITY

Probability and Sampling
- graph data, including broken line graphs, bar graphs, histograms, and circle graphs
- measures of central tendency
- sample and population
- data presentation and conclusions

NUMBER

Students develop safe, secure, and accurate financial skills related to personal banking practices, wage determinations, and budget calculations.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Spreadsheets	
A1 design and use a spreadsheet to make and justify decisions	❑ create and use a spreadsheet to enter and compare given data to – identify differences between the items and their properties – identify which item might better satisfy a given set of criteria such as cost, size, etc.
A2 create a spreadsheet using various formatting options	❑ identify, using examples, the components of a spreadsheet, including cells, rows, and columns ❑ identify, using examples, the three types of entries that can be entered into the cell of a spreadsheet ❑ create a spreadsheet which contains a given set of formatting options such as – numbers with different characteristics (e.g., different number of decimal places, currency values, dates, and percentages) – cells, rows, and columns of different types and sizes – merged cells – cells showing fonts of differing types and characteristics – cells with different background features
A3 use a spreadsheet template to solve problems	❑ use a spreadsheet that contains pre-existing formatting, formulas, and functions to solve problems such as analysis of survey data ❑ identify errors in a given spreadsheet such as formula calculation, duplicate cells, or number formatting
A4 create a spreadsheet using formulas and functions	❑ create and use a spreadsheet containing formulas for – adding the contents of individual cells or a series of cells – subtracting the contents of individual cells – determining the product of the contents of given cells – determining the quotient of the contents of given cells – determining the average of the contents of given cells ❑ create and use a spreadsheet containing functions for – counting the number of cells that have a given value – graphing, in an appropriate manner, the results of a set of data entered in the spreadsheet
A5 use a spreadsheet to answer "what-if" questions *Suborganizer 'Spreadsheets' continued on page 36*	❑ create and use a spreadsheet to determine the effect of changing a condition such as a constant number ❑ create and use a spreadsheet in which a cell or set of cells will give one value when one condition exists and a different value when another condition exists

Prescribed Learning Outcomes	Suggested Achievement Indicators
Suborganizer 'Spreadsheets' *continued from page 35* A6 identify where spreadsheets could be effectively used	❑ identify the benefit of using a spreadsheet when presented with a problem containing a quantity of statistical, financial, or repetitive data ❑ explain the benefits of using a spreadsheet for solving a given problem ❑ identify a situation when using a spreadsheet may not be beneficial
Personal Banking A7 name and describe various types of commonly used consumer bank accounts, including – value accounts – self-serve accounts – full-serve accounts – savings accounts	❑ determine the type of account available at a local financial institution that would best meet a given set of criteria, and justify the choice ❑ determine the type of account that a small personal business would establish and give reasons for the choice of account
A8 complete various banking forms, including – deposit slips – withdrawal slips – cheques	❑ complete a variety of bank forms such as – cheques – deposit slips – withdrawal slips – forms related to transferring money between accounts – forms that allow an individual to pay bills (i.e., pre-authorized payment plan) ❑ identify errors on a given banking form
A9 describe the use of a bank card for automated teller machines (ATMs) and debit payments	❑ identify safe procedures to follow when using ATMs and debit machines ❑ identify common mistakes made by people using ATMs and debit machines ❑ describe ways of recording and tracking transactions conducted using ATMs and debit machines
A10 identify different types of bank service charges and their relative costs, including – monthly account fees – transaction charges – interest charges	❑ create a transaction record which will accurately reflect all transactions, including – all charges and fees levied against the account by the financial institution – interest deposits earned from the financial institution
A11 reconcile statements, including chequebooks and electronic bank transactions with bank statements	❑ create a bank statement with a personal transaction record to determine the correct balance ❑ identify errors in a transaction record

Prescribed Learning Outcomes	Suggested Achievement Indicators
Wages, Salaries, and Expenses	
A12 calculate hours worked and gross pay	❑ record start and end times worked for a given pay period to determine the total time worked for each day and the entire pay period ❑ calculate hours worked and gross pay from a time card ❑ determine gross pay from given or calculated hours worked when the rate of pay is known ❑ identify errors in calculating hours worked and gross pay
A13 calculate net income using deduction tables (focus on weekly) with different pay periods	❑ use gross pay, the pay period, deduction information, and deduction tables to calculate a person's net income ❑ determine the CPP, EI, and income tax deductions for a given weekly salary ❑ identify errors in the calculation of net income given a statement of deductions
A14 calculate changes in income	❑ determine the effect on gross weekly income when working overtime hours paid at time and a half or double time (e.g., if an employee worked 35 hours at the regular rate of $8.50 per hour, 5 hours of overtime paid at time and a half and 4 hours on a statutory holiday paid at double time) ❑ determine the change of gross and net income resulting from a raise when the raise is a – percentage of current income – change in the rate of pay ❑ determine the percentage change in gross income when new and old gross income figures are given or calculated
A15 develop a budget that matches predicted income	❑ create a balanced budget based on expense and income data that is either given or determined by the student ❑ explain considerations that must be made when developing a budget (e.g., prioritizing, unexpected expenses, one-time income)

PATTERNS AND RELATIONS

Students analyse a variety of factors that affect their decisions as consumers and make accurate calculations with respect to those decisions.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Rate, Ratio, and Proportion	
B1 use the concept of unit rate to determine the best buy on a consumer item and justify the decision	❑ determine the "best buy" between several given products when the total cost and quantity of each product is known ❑ explain the advantages and disadvantages of purchasing a given product from different sources ❑ determine when buying a consumer item in bulk may not be a good decision
B2 solve problems on the application of sales tax in Canada	❑ calculate the GST and PST for a given purchase and determine the total cost of the item ❑ determine the effect on the final cost of an item if the sales tax is increased or decreased
B3 describe a variety of sales promotion techniques and their financial implications for the consumer	❑ explain how different sales promotion techniques affect the cost of an individual item ❑ describe how different sales promotion techniques affect buying practices
B4 solve rate, ratio, and proportion problems involving price, length, area, volume, time, and mass	❑ calculate the ratio of two given items to each other (e.g., ratio of length to width of rectangular shape) ❑ solve problems using ratios ❑ calculate the rate of travel given time and distance measurements ❑ solve problems involving rates ❑ determine the measurements of a model when the dimensions of a given object and the scale of the model are known ❑ determine the proportion of given substances that can be found in a product

SHAPE AND SPACE

Students estimate and measure dimensions, area, volume and mass using both Imperial and SI units and solve a variety of problems related to these measurements.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Trigonometry C1 apply ratio and proportion in similar triangles	❑ identify relationships between the sides of a given triangle and the sides of another given triangle when both triangles have angles of the same size ❑ calculate the dimensions of a given triangle when the dimensions of a second given triangle with the same angles and the ratio of one triangle to the other are known ❑ solve problems using similar triangles ❑ determine if two triangles are similar and justify
C2 use the trigonometric ratios sine, cosine, and tangent in solving right triangles	❑ determine the sine, cosine, or tangent of a given angle in a right triangle using a scientific calculator ❑ determine the length of one side of a given right triangle when one side and one additional angle are known ❑ determine an angle in a right triangle when the dimensions of two or more sides are given ❑ solve problems involving trigonometric ratios when a diagram is provided ❑ solve problems involving trigonometric ratios when a diagram is not provided
Geometry Project C3 measure lengths in both SI and Imperial units	❑ determine the dimensions of a given shape or object using a given measuring device
C4 estimate measurements of objects in SI and Imperial systems, including – length – area – volume – mass	❑ estimate the dimensions of a given regularly shaped object or shape using a referent (e.g., the height of the desk is about three rulers long so the desk is three feet high) ❑ estimate the area of a given surface (e.g., surface of the school parking lot) using a referent ❑ estimate the volume of a given object using a referent (e.g., the volume of a person) ❑ estimate the mass of a given object using a referent (e.g., the mass of a vending machine)
C5 interpret drawings and use the information to solve problems *Suborganizer 'Geometry Project' continued on page 40*	❑ use a scale drawing of a given area (e.g., classroom and its contents) to determine the most effective way to accommodate a change (e.g., rearranging furniture)

Prescribed Learning Outcomes	Suggested Achievement Indicators
Suborganizer 'Geometry Project' continued from page 39	
C6 draw top, front, and side views for both 3-D rod or block objects and their sketches	❑ create a scale drawing of the top, front, and side views of a given 3-D rod object ❑ create a scale drawing of the top, front, and side views of a given block object such as a house, desk, or room ❑ create a model of a 3-D rod object from its top, front, and side views ❑ create a model of a block object from its top, front, and side views
C7 sketch 3-D designs using isometric dot paper	❑ create a representative drawing of a given object or design using isometric dot paper
C8 enlarge or reduce a dimensioned object according to a specified scale	❑ determine the dimensions of a given object from a scale drawing or model ❑ determine the dimensions of a given model when the dimensions of the object and the scale are known
C9 solve problems involving linear dimensions, area, and volume	❑ determine the measurements necessary to calculate the perimeter, area, surface area, and volume of a given object ❑ calculate the perimeter, area, surface area, and volume of a given object from the dimension measurements taken by the student
C10 complete a project that includes a 2-D plan and a 3-D model of some physical structure	❑ sketch a 2-D plan from a given 3-D model ❑ create a 3-D model from a given 2-D plan ❑ create a 2-D plan (e.g., an ideal home recreation room including furnishings) and use the plan to create a 3-D model

STATISTICS AND PROBABILITY

Students become familiar with sampling methods and the collection and analysis of data for use in making predictions and suggestions for change.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Probability and Sampling D1 read and interpret graphs	❑ use the information represented on a given graph to solve problems ❑ determine the scale used on a given graph ❑ justify a given interpretation from a given graph
D2 use suitable graph types to display data (by hand or using technology), including – broken line graphs – bar graphs – histograms – circle graphs	❑ select and draw graphs to display different given types of data (e.g., a bar graph to represent quantities of fish in different bodies of water, or a circle graph to represent quantities in each part of a whole) ❑ represent a given data set on a broken line graph, bar graph, histogram, or circle graph
D3 determine and use measures of central tendency to support decisions, including – mean – median – mode	❑ calculate the mean, median, and mode of a given set of data ❑ determine the median, mean, and mode of the results of a given survey (e.g., survey that asks students to identify the subject that they like the least/most) ❑ solve problems involving the calculation of measures of central tendency (e.g., determine the average number of respondents in favour of a particular choice and make a reasonable decision or recommendation based on this finding)
D4 use sample data to make predictions and decisions	❑ explain the relationship between a given sample and the total population ❑ collect sample data to answer a question and make predictions with respect to the whole population (e.g., student population across the school district) ❑ make and justify decisions based on sample data collected
D5 critique ways in which statistical information and conclusions are presented by the media and other sources	❑ explain, using examples, how data can be misrepresented and should not be taken as the complete truth (e.g., a health issue presented in the media should be considered carefully and not taken as the sole basis for making dramatic changes in lifestyle)

STUDENT ACHIEVEMENT

Essentials of Mathematics 11

Key Elements: Essentials of Mathematics 11

Mathematical Process (Integrated)
The following mathematical processes have been integrated within the prescribed learning outcomes and achievement indicators for the course: communication, problem solving, connections, mental mathematics and estimation, reasoning, technology, and visualization.

NUMBER

Income and Debt
- performance-based income, including commission, piece work, and salary plus commission
- simple and compound interest, including principal, interest rate, time, and 'Rule of 72'
- consumer problems, including credit cards, loans, and exchange rates

Personal Income Tax
- income and deductions
- income tax forms for a single employed individual with no dependents

Owning and Operating a Vehicle
- cost of owning a vehicle, including payment, insurance, operation, and maintenance costs
- cost of leasing a vehicle, including payment, insurance, operation, and maintenance costs
- cost of renting a vehicle, including rent, insurance, and operation costs
- depreciation

Business Plan
- operation costs for a small business, including wages, inventory, rent, insurance, advertising, etc.
- revenue generated from a small business
- floor plan for a small business

PATTERNS AND RELATIONS

Relations and Formulas
- linear relations expressed in words, tables of values, graphs, and formulas
- interpolation and extrapolation of linear relations
- slope of linear relation, including formula substitution and manipulation

SHAPE AND SPACE

Measurement Technology
- SI and Imperial measurements, including conversion within and between systems
- Vernier calipers and micrometers

STATISTICS AND PROBABILITY

Data Analysis and Interpretation
- line plots, bar graphs, and circle graphs
- data sets, including cluster, gap, outlier, range, mean, median, and mode
- data manipulation to stress a particular point of view

NUMBER

Students solve problems related to personal and business finance, including income derived in different ways, costs related to financial decisions such as borrowing, travel, and foreign purchases, saving, borrowing, and acquiring a vehicle.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Income and Debt	
A1 solve problems involving performance-based income, including – commission sales – piece work – salary plus commission	❑ determine the gross income of a given individual who earns – only commission when the commission rate is fixed or graduated and the sales are given – income based on unit production when the rate and number of units have been identified – a base salary plus commission based on a graduated scale of production or sales ❑ determine the preferred method of payment when two or more options are provided and justify
A2 use simple and compound interest calculations to solve problems	❑ calculate the interest paid given the principal, interest rate, and time ❑ calculate the simple interest rate given the principal, interest amount, and time ❑ calculate the principal given the rate, interest amount, and time ❑ calculate the time given the principal, interest amount, and interest rate ❑ calculate factors related to compound interest based on the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ ❑ determine the time to double a given investment using the 'Rule of 72'
A3 solve consumer problems involving – credit cards – exchange rates – personal loans	❑ explain, using examples, the terms associated with credit cards, including previous balance, unpaid balance, new balance, minimum payment, available credit, and payment due ❑ describe how credit cards can be used to manage personal finances ❑ calculate the balance owing on a given credit card when full payment is not made and the interest rate and term are given ❑ explain the advantages and disadvantages of paying for a given product using different purchasing options (e.g., purchasing a major piece of furniture and making monthly payments, credit card, cash, or deferred payments, and 'interest free' periods) ❑ calculate the actual cost of an item sold using a foreign currency but paid for in Canadian funds and vice versa ❑ determine the actual cost of repaying a given loan when different conditions apply (e.g., different amortization periods, fixed versus variable interest rates, different interest compounding periods, and different terms)

Prescribed Learning Outcomes	Suggested Achievement Indicators
Personal Income Tax A4 prepare an income tax form for an individual who is single, employed, and without dependents	❑ prepare a personal income tax return for a person who is single with no dependents and who is not self-employed ❑ explain, using examples, the difference between income and deductions as it applies to income tax
Owning and Operating a Vehicle A5 solve problems involving the acquisition and operation of a vehicle, including – renting – leasing – buying – licensing – insuring – operating (e.g., fuel and oil) – maintaining (e.g., repairs) – depreciation	❑ identify the costs associated with renting a given vehicle and determine the best rental conditions ❑ compare costs related to buying and leasing a given vehicle and explain the benefits ❑ calculate the full cost of ownership of a given vehicle over a specified time period and under various circumstances ❑ calculate the full cost of acquiring and operating a given leased vehicle over a specified time period and under various circumstances ❑ calculate the depreciation over a given period of time
Business Plan A6 prepare a business plan to own and operate a business that includes the following information as appropriate: – monthly cost of operation (e.g., inventory, rent, wages, insurance, advertising, loan payments, etc.) – hours of operation – estimated daily sales (average) – gross/net profit – hourly earnings – a scale floor plan of the store	❑ prepare a business plan for a given small personal business, following an acceptable format, that includes – monthly cost of operation (e.g., inventory, rent, wages, insurance, advertising, loan payments, etc.) – hours of operation – estimated daily sales (average) – gross/net profit – hourly earnings – a scale floor plan of the store – assessing any competition

PATTERNS AND RELATIONS

Students analyse data found in a variety of formats by means of words, charts, tables, graphs, and formulas.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Relations and Formulas B1 express a linear relation of the form $y = mx + b$ – in words – as a formula – with a table of values – as a graph	❑ calculate a person's income (e.g., if they earn $8 an hour) and make a table of values from this information ❑ construct a graph in the form $y = mx + b$ from words, table, and a formula ❑ express "base salary coupled with commission based on a given rate of income" as a formula of the form $y = mx+b$, in words, as values in a table using different volumes of production and rates of commission, and as a graph showing different volumes of production
B2 interpolate and extrapolate values from the graph of a linear relation	❑ use graphs to support predictions related to x and y values that are contained within a graph (e.g., the cost of vehicle ownership over the full time of ownership when variables such as the cost of fuel and insurance change) ❑ use graphs to support predictions related to x and y values that are not shown on a graph (e.g., the cost of borrowing money if a given loan is paid off early) ❑ interpolate the approximate value of one variable on a given graph, given the value of the other variable ❑ extend a given graph (extrapolate) to determine the value of an unknown element ❑ extrapolate the approximate value of one variable from a given graph, given the value of the other variable ❑ solve a given problem by graphing a linear relation and analysing the graph
B3 determine the slope of a linear relation and describe it in words	❑ describe, in words, the relationship between two variables (e.g., given income and expenses) ❑ calculate the slope of a given graph
B4 interpret the meaning of the slope of a linear relation in a problem context	❑ work from two or more given graphs that model linear relations and compare the data with respect to identification of linear variables and the slopes of the data lines
B5 substitute appropriate values into formulas and evaluate the result	❑ substitute values into a formula to solve for an unknown, and evaluate the reasonableness of the results (e.g., $I = Prt$ used for calculating simple interest to determine the interest on money borrowed or invested when one or more variables is changed) ❑ manipulate a given formula to solve for an unknown

SHAPE AND SPACE

Students take measurements using appropriate tools such as rulers, calipers, and micrometers in both Imperial and SI measurements and convert measurements between the two systems in order to solve a variety of problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Measurement Technology	
C1 select and use appropriate measuring devices (in Imperial and SI systems), including – Vernier calipers – micrometers	❑ use appropriate rulers and tape measures to measure dimensions of given objects (e.g., items found around the school) ❑ use Vernier calipers to find inside and outside measurements of given objects (e.g., nut on a desk) ❑ use a micrometer to find outside measurements of given objects (e.g., coins) ❑ express measurements taken in appropriate Imperial or SI units (e.g., do not use miles or kilometres to indicate the dimensions of a desk)
C2 perform basic conversions within and between the Imperial and SI systems, using technology as appropriate	❑ convert a given Imperial measurement into SI units (e.g., centimetres and inches, kilometres and miles, kilograms and pounds, square metres and square yards) ❑ convert a given SI measurement into Imperial units ❑ solve problems that require conversions within the SI or Imperial systems (e.g., determining the volume of an object when all of the measurements are not in the same units)
C3 use measuring devices and unit conversions to solve problems	❑ identify the appropriate unit conversions required to solve a problem ❑ solve problems involving the conversion of given measurements between SI and Imperial, e.g.: – a recipe calls for half a pound of meat but the meat is sold in grams – a carpet is sold by the square metre but the room is measured in square feet

STATISTICS AND PROBABILITY

Students represent data effectively and make predictions and inferences based on these representations.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Data Analysis and Interpretation D1 display and analyse data on – line plots – bar graphs – circle graphs	❑ explain a given data set using terms such as – cluster – gap – outlier – range – mean – median – mode ❑ construct line plots, bar graphs, and circle graphs from a given set of data and use them to analyse the data
D2 manipulate the presentation of data to stress a particular point of view	❑ explain how the selection of a particular graph type can provide misleading information ❑ explain how different types of graphs can be created to provide biased information (e.g., selection of scale, inconsistent use of intervals) ❑ create a graph of a given data set to stress a particular point of view ❑ find examples of graphs in which a particular point of view is stressed in print and electronic media, such as newspapers, magazines, and the Internet, and explain why the presentation type may have been chosen

STUDENT ACHIEVEMENT

Essentials of Mathematics 12

Key Elements: Essentials of Mathematics 12

Mathematical Process (Integrated)
The following mathematical processes have been integrated within the prescribed learning outcomes and achievement indicators for the course: communication, problem solving, connections, mental mathematics and estimation, reasoning, technology, and visualization.

NUMBER

Personal Finance
- property and life insurance, including premium, beneficiary, policy, surrender value, market value, and replacement value
- home purchase costs, including mortgage, insurance, taxes, heating costs, etc.
- mortgages, including gross debt service ratio, eligibility, payment options
- types of mortgages, including closed, open, fixed, fixed rate, and variable closed

Investments
- statement of net worth
- financial plan based on a budget
- GIC, bond, mutual fund, stock, real estate, RRSP, and RESP investment options
- investment factors, including risk, rates of return, cost, and length of term
- trading stocks

Government Finances
- federal revenues, including GST, excise tax, and customs duties
- federal government expenditures, including social welfare, health care, policing, armed forces, and wages
- provincial revenue, including PST, corporation capital, licences, and gasoline taxes
- provincial government expenditures, including education, social services, and wages
- municipal revenues, including property tax and mill rate
- municipal expenditures

PATTERNS AND RELATIONS

Variations and Formulas
- direct variation, partial variation, and inverse variation
- variation patterns in tables of values and graphs
- formula substitution and manipulation

SHAPE AND SPACE

Design and Measurement
- "exploded" format for objects
- cost of an object based on design
- design for a specified budget

NUMBER

Students identify the types and functions of insurance, the preparation of a personal financial plan, the types of taxes imposed by different levels of government, and the costs associated with foreign purchasing and currency exchange.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Personal Finance A1 solve problems involving different types of insurance, including – life – property	❑ explain, using examples, the meaning of terms associated with life and property insurance, including – premium – beneficiary – policy – surrender values – market value – replacement value ❑ determine the most cost-effective life insurance using tables (e.g., when the purchaser is between the ages of 20 and 60) ❑ calculate the cost of different types of life insurance policies that are dependent on the type, value, practices, and characteristics of the insured person ❑ identify the restrictions and areas of coverage provided by different types of given property insurance and calculate the costs ❑ determine the cost of insuring a home and its contents for the replacement value
A2 determine the costs involved in purchasing a home, including – gross debt service ratio – payment options – insurance – additional fees *Suborganizer 'Personal Finance'* *continued on page 54*	❑ explain the meaning of the term "gross debt service ratio" (GSDR) and factors affecting the calculation of GSDR ❑ calculate GSDR using given data ❑ determine if a purchaser is eligible for purchasing a home in a given case study ❑ explain different given payment options that are available for mortgages and the advantages and disadvantages of each ❑ explain the additional fees and costs, over and above the purchase price, associated with purchasing a given house

Prescribed Learning Outcomes	Suggested Achievement Indicators
Suborganizer 'Personal Finance' *continued from page 53* A3 solve problems involving different types of mortgages, including – closed – open – fixed-rate – variable-rate	❑ create a one-year amortization schedule for a given mortgage type when the amount of the mortgage, down payment, interest rate, and term of the mortgage are provided ❑ determine the payments to be made on each given type of mortgage when different repayment schedules are used ❑ determine the costs associated with carrying a second mortgage on a given property ❑ determine the total mortgage costs when carrying a first and second mortgage on a given home ❑ determine the payment changes that would occur by changing amortization periods and payment options of a given mortgage
Investments A4 determine a financial plan to achieve personal goals A5 calculate net worth for an individual	❑ develop a monthly budget based on given income and expenses ❑ develop a statement of net worth in order to calculate the debt equity ratio ❑ prepare a financial plan incorporating given personal budget, savings and investment plans in order to achieve established financial goals
A6 describe different investment vehicles, including – GICs – bonds – mutual funds – stocks – real estate – RRSPs and RESPs	❑ explain the characteristics and benefits of given types of investment vehicles ❑ recommend and justify the most appropriate investment vehicle(s) for an individual given the person's financial circumstances ❑ recommend choices related to given RRSP and RESP investments and the reasons for making each choice ❑ explain the advantages and disadvantages of investing in an RRSP ❑ explain the advantages and disadvantages of investing in an RESP
A7 compare and contrast different investment vehicles in terms of – risk factors – rates of return – costs – lengths of term	❑ recommend the most appropriate investment vehicle(s) for an individual given the person's financial circumstances and explain the choice ❑ explain the risk factors, rates of return, and costs associated with a given investment
A8 investigate how to purchase and sell stocks	❑ describe the processes involved in making decisions related to purchasing and selling given stocks ❑ describe the processes involved in purchasing and selling given stocks ❑ determine the value of a stock

Prescribed Learning Outcomes	Suggested Achievement Indicators
Government Finances	
A9 describe and calculate federal revenues, including – GST – excise tax – customs duties	❑ determine the sources of federal revenue, using media sources such as the Internet ❑ create a pie chart to represent the relative revenue collected by the federal government ❑ solve problems involving GST, excise tax, and customs duties, e.g.: – determine the customs duties on a given purchase (e.g., car purchased in Germany for a price of 18,000 (EUR) and brought into Canada) – calculate the GST due on a given purchase (e.g., a purchase valued at $2,500 (USD) made in the United States and imported into Canada) – calculate the cost, including exchange and all taxes of a given item (e.g., supply of electronics purchased in Japan for 100,000 (JPY) and brought into Canada)
A10 describe and calculate federal government expenditures, including – social welfare benefits – health care – policing – armed forces – employee wages and salaries	❑ describe the process of transfer payments between given levels of government ❑ describe the relative amounts the federal government spends on given services such as welfare, social security, policing, armed forces, recreational services, and environmental activities, services, and issues ❑ describe the relative amounts the federal government spends on employee wages and salaries
A11 describe and calculate provincial revenue, including – PST – corporation capital – licences – gasoline	❑ determine the sources of provincial revenue, using media sources such as the Internet ❑ create a pie chart to represent the relative revenue collected by the provincial government ❑ solve problems involving PST, corporation capital, licences, and gasoline taxes, e.g.: – calculate the PST due on a given purchase (e.g., clothing purchase, for an adult B.C. resident, which totals $260) – identify the provincial and federal taxes and tax rates on a given purchase (e.g., gasoline) and calculate the cost of the purchase if these taxes were removed – identify licences issued by the provincial government and the cost of each licence
A12 describe and calculate provincial expenditures, including – education funding – social services funding – employee wages and salaries *Suborganizer 'Government Finances'* *continued on page 56*	❑ determine the relative amounts the provincial government spends on given services such as education and social services ❑ describe the relative amounts the provincial government spends on employee wages and salaries

Prescribed Learning Outcomes	Suggested Achievement Indicators
Suborganizer 'Government Finances' continued from page 55 A13 determine how selected municipal taxes are calculated	❑ identify sources of revenue for given municipal governments ❑ calculate the property tax when the market value and mill rate are known for a given property ❑ determine the tax rate for a given property in terms of cents per dollar, percent rate, and mill rate ❑ identify and explain the components in a given property tax assessment
A14 determine how municipalities allocate funds	❑ identify sources of municipal expenditures, using media sources such as the Internet

PATTERNS AND RELATIONS

Students examine different types of variations using tables, graphs, and formulas to solve problems related to the data being compared.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Variation and Formulas	
B1 plot and analyse examples of direct variation, partial variation, and inverse variation	❑ express the relationship between given variables as a table of values and as a graph, e.g.: – the number of rolls of wallpaper needed and the area of wall to be covered, as a formula – the relationship between the amount sold and the gross pay when an employee receives a base salary plus commission for sales made, as a formula – the relationship between the amount of money owing on a loan and the amount of time that payments have been made, as a formula ❑ create an equation from given data ❑ identify the dependent and independent variables in a given relationship ❑ identify the constant of variation (if any) in a given relationship ❑ explain the meaning of the x- and y-intercepts in a given graph
B2 given data, graph, or a situation, identify the type of variation represented	❑ identify a given relationship as direct variation, partial variation, or inverse variation when the information is provided as a table of data or a graph, or is described in words
B3 use formulas to solve problems	❑ use a given formula to identify and solve missing values, e.g.: – solve a given problem such as the depreciation of the value of a car when the constant of the inverse variation is 'k' and the current value of the car is given as 'V' and the age of the car is given as 'a' – solve a given partial variation problem such as determining the cost of having a stereo repaired when the service centre charges a basic fee in addition to an hourly rate for each hour or partial hour of time spent working on the system – solve a given direct variation problem such as determining the number of phone calls that can be received on one full charge of a cell phone

SHAPE AND SPACE

Students explore the components of design and measurement to create and interpret different views and determine the cost of the construction of an object.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Design and Measurement	
C1 analyse objects shown in "exploded" format	❑ describe a given object, as it would appear, based on what is shown in an "exploded" format drawing ❑ identify the parts of a given object based on its appearance in an "exploded" format drawing
C2 draw objects in "exploded" format	❑ create an "exploded" format drawing of a given object or an object shown in an accurate drawing
C3 solve problems involving estimation and cost of materials for building an object when a design is given	❑ estimate the cost of making a given object such as a student work station ❑ determine the cost of making a given object, such as a student work station, when the design and material costs are given
C4 design an object within a specified budget	❑ design a given object, such as a student work station, given the total budget and the costs of materials

Learning Resources

Essentials of Mathematics 10 to 12

This section contains general information on learning resources, and provides a link to the titles, descriptions, and ordering information for the recommended learning resources in the Essentials of Mathematics 10 to 12 Grade Collections.

What Are Recommended Learning Resources?
Recommended learning resources are resources that have undergone a provincial evaluation process using teacher evaluators and have Minister's Order granting them provincial recommended status. These resources may include print, video, software and CD-ROMs, games and manipulatives, and other multimedia formats. They are generally materials suitable for student use, but may also include information aimed primarily at teachers.

Information about the recommended resources is organized in the format of a Grade Collection. A Grade Collection can be regarded as a "starter set" of basic resources to deliver the curriculum. In many cases, the Grade Collection provides a choice of more than one resource to support curriculum organizers, enabling teachers to select resources that best suit different teaching and learning styles. Teachers may also wish to supplement Grade Collection resources with locally approved materials.

How Can Teachers Choose Learning Resources to Meet Their Classroom Needs?
Teachers must use either:

- provincially recommended resources OR
- resources that have been evaluated through a local, board-approved process

Prior to selecting and purchasing new learning resources, an inventory of resources that are already available should be established through consultation with the school and district resource centres. The ministry also works with school districts to negotiate cost-effective access to various learning resources.

What Are the Criteria Used to Evaluate Learning Resources?
The Ministry of Education facilitates the evaluation of learning resources that support BC curricula, and that will be used by teachers and/or students for instructional and assessment purposes. Evaluation criteria focus on content, instructional design, technical considerations, and social considerations.

Additional information concerning the review and selection of learning resources is available from the ministry publication, *Evaluating, Selecting and Managing Learning Resources: A Guide* (Revised 2002)
www.bced.gov.bc.ca/irp/resdocs/esm_guide.pdf

What Funding is Available for Purchasing Learning Resources?
As part of the selection process, teachers should be aware of school and district funding policies and procedures to determine how much money is available for their needs. Funding for various purposes, including the purchase of learning resources, is provided to school districts. Learning resource selection should be viewed as an ongoing process that requires a determination of needs, as well as long-term planning to co-ordinate individual goals and local priorities.

What Kinds of Resources Are Found in a Grade Collection?
The Grade Collection charts list the recommended learning resources by media format, showing links to the curriculum organizers and suborganizers. Each chart is followed by an annotated bibliography. Teachers should check with suppliers for complete and up-to-date ordering information. Most suppliers maintain web sites that are easy to access.

Essentials of Mathematics 10 to 12 Grade Collections

The Grade Collections for Essentials of Mathematics 10 to 12 list the recommended learning resources for these courses. Resources previously recommended for the 2000 version of the curriculum, where still valid, continue to support this updated IRP. The ministry updates the Grade Collections on a regular basis as new resources are developed and evaluated.

Please check the following ministry web site for the most current list of recommended learning resources in the Essentials of Mathematics 10 to 12 Grade Collections: www.bced.gov.bc.ca/irp_resources/lr/resource/gradcoll.htm

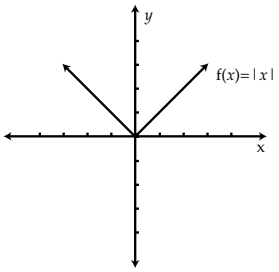
Glossary

Essentials of Mathematics 10 to 12

This appendix provides an illustrated glossary of terms used in this Integrated Resource Package. The terms and definitions are intended to be used by readers unfamiliar with mathematical terminology. For a more complete definition of each term, refer to a mathematical dictionary such as the *Nelson Canadian School Mathematics Dictionary* (ISBN 17-604800-6).

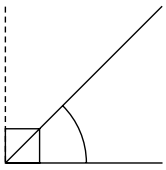
A

absolute value of a number
How far the number is from 0. Example: the absolute values of -4.2 and of 4.2 are each 4.2.



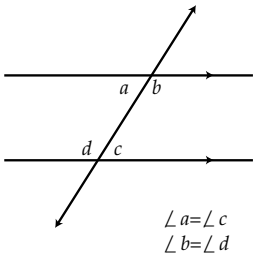
absolute value function
The function f defined by $f(x)=|x|$, where $|x|$ denotes the absolute value of x.

accuracy
A measure of how far an estimate is from the true value.

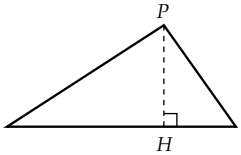


acute angle
An angle whose measure is between 0° and 90°.

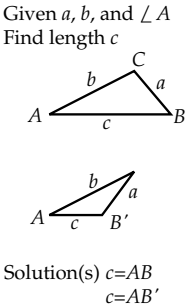
algorithm
A mechanical method for solving a certain type of problem, often a method in which one kind of step is repeated a number of times.



alternate interior angles
In the diagram to the left, the angles labelled a and c are alternate interior angles, as are the angles b and d.



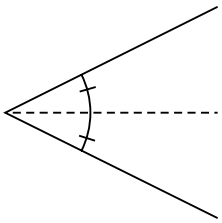
altitude of a triangle
A line segment PH, where P is a vertex of the triangle, H lies on the line through the other two vertices, and PH is perpendicular to that line.



ambiguous case
Two sides of a triangle and the angle opposite one of them are specified, and we want to calculate the remaining angles or side. There may be no solution, exactly one, or exactly two.

amplitude (of a periodic curve)
The maximum displacement from a reference level in either a positive or negative direction. That reference level is often chosen halfway between the biggest and smallest values taken on by the curve.

analytic geometry (coordinate geometry)
An approach to geometry in which position is indicated by using coordinates, lines, and curves, and other objects are represented by equations, and algebraic techniques are used to solve geometric problems.



angle bisector
A line that divides an angle into two equal parts.

antiderivative
If $f(x)$ is the derivative of $F(x)$, then $F(x)$ is an *antiderivative* of $f(x)$. *Indefinite integral* means the same thing.

antidifferentiation
The process of finding antiderivatives.

arc
A connected segment of a circle or curve.

arc sine (of x)
The angle (in radians) between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is x.
Notation: $\sin^{-1} x$ or $\arcsin x$.

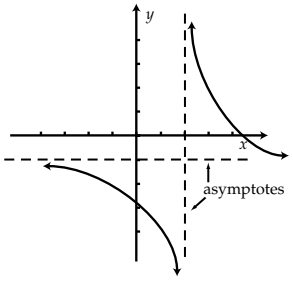
arc tangent
The angle (in radians) between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose tangent is x.
Notation: $\tan^{-1} x$ or $\arctan x$.

arithmetic operation
Addition, subtraction, multiplication, and division.

arithmetic sequence (arithmetic progression)
A sequence in which any term is obtained from the preceding term by adding a fixed amount, the *common difference*.

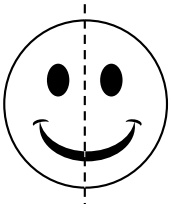
If a is the first term and d the common difference, then the sequence is $a, a + d, a + 2d, a + 3d, \ldots$. The n-th term t_n is given by the formula $t_n = a + (n - 1)d$.

arithmetic series
The sum S_n of the first n terms of an arithmetic sequence. If the sequence has first term a and common difference d, then
$S_n = \frac{1}{2}n\left[2a + \left(n - 1\right)d\right] = \frac{1}{2}n\left(a + l\right)$ where l is $a + (n - 1)d$, the "last" term.



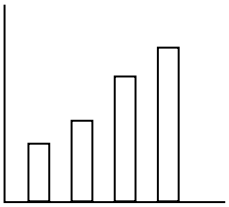
asymptote (to a curve)
A line l such that the distance from points P on the curve to l approaches zero as the distance of P from an origin increases without bound as P travels on a certain path along the curve.

average velocity
The net change in position of a moving object divided by the elapsed time.



axis of symmetry (of a geometric figure)
A line such that for any point P of the figure, the mirror image of P in the line is also in the figure.

B



bar graph
A graph using parallel bars (vertical or horizontal) that are proportional in length to the data they represent.

base
In the expression s^t, the number or expression s is called the base, and t is the exponent. In the expression $\log_a u$, the base is a.

binomial
The sum of two monomials.

binomial distribution
The probabilities associated with the number of successes when an experiment is repeated independently a fixed number of times. For example, the number of times a six is obtained when a fair die is tossed 100 times has a binomial distribution.

Binomial Theorem
A rule for expanding expressions of the form $(x + y)^n$.

bisect
To divide into two equal parts.

broken-line graph
A graph using line segments to join the plotted points to represent data.

C

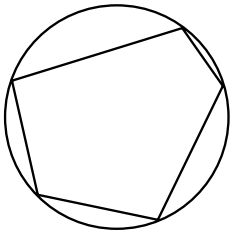
Cartesian (rectangular) coordinate system
A coordinate system in which the position of a point is specified by using its signed distances from two perpendicular reference lines (axes).

central angle
An angle determined by two radii of a given circle; equivalently, an angle whose vertex is at the centre of the circle.

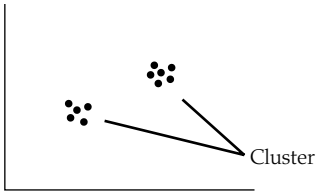
chain rule
A rule for differentiating composite functions.
If $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$.

chord
The line segment that joins two points on a curve, usually a circle.

circumference
The boundary of a closed curve, such as a circle; also, the measure (length) of that boundary. Please see *perimeter*.



circumscribed
The polygon *P* is *circumscribed* about the circle *C* if *P* is inside *C* and the edges of *P* are tangent to *C*. The circle *C* is *circumscribed* about the polygon *Q* if *Q* is inside *C* and the vertices of *Q* are on the boundary of *C*. The notion can be extended to other figures, and to three dimensions.



cluster
A collection of closely grouped data points.

coefficient
A numerical or constant multiplier in an algebraic expression. The coefficient of x^2 in $4x^2 - 2axy$ is 4, and the coefficient of xy is $-2a$.

collinear
Lying on the same line.

combination
A set of objects chosen from another set, with no attention paid to the order in which the objects are listed (see also *permutation*). The number of possible combinations of *r* objects selected from a set of *n* distinct objects is ${}_nC_r$, pronounced "*n* choose *r*."

common factor (CF)
A number that is a factor of two or more numbers. For example, 3 is a common factor of 6 and 12. *Common divisor* means the same thing. The term is also used with polynomials. For example, $x - 1$ is a common factor of $x^2 - x$ and $x^2 - 2x + 1$.

common fraction
A number written as $\frac{a}{b}$, where the numerator *a* and the denominator *b* are integers, and *b* is not zero. Examples: $\frac{4}{5}, \frac{-13}{6}, \frac{3}{1}$.

compass
An instrument for drawing circles or arcs of circles.

complementary angles
Two angles that add up to a right angle.

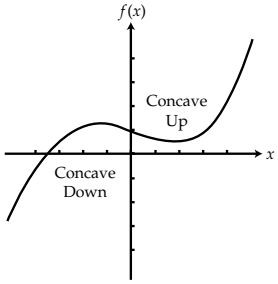
completing the square
Rewriting the quadratic polynomial $ax^2 + bx + c$ in the form $a(x - p)^2 + q$, perhaps to solve the equation $ax^2 + bx + c = 0$.

complex fraction
A fraction in which the numerator or the denominator, or both, contain fractions.

composite function
A function $h(x)$ obtained from two functions f and g by using the rule $h(x) = f(g(x))$ (first do g to x, then do f to the result).

composite number
An integer greater than 1 that is not prime, such as 9 or 14.

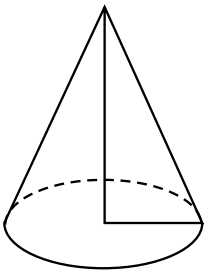
compound interest
The interest that accumulates over a given period when each successive interest payment is added to the principal in order to calculate the next interest payment.



concave down (or downward)
The function $f(x)$ is concave down on an interval if the graph of $y = f(x)$ lies below its tangent lines on that interval.

concave up (or upward)
The function $f(x)$ is concave up on an interval if the graph of $y = f(x)$ lies above its tangent lines on that interval.

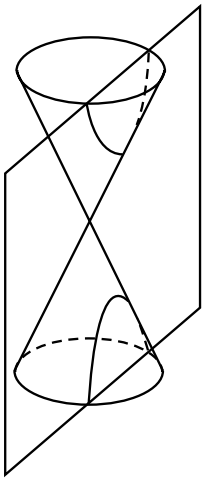
conditional probability
The probability of an event given that another event has occurred. The (conditional) probability that someone earns more than $200,000 a year, given that the person plays in the NHL, is different from the probability that a randomly chosen person earns more than $200,000.



cone (right circular)
The three-dimensional object generated by rotating a right triangle about one of its legs.

confidence interval(s)
An interval that is believed, with a preassigned degree of confidence, to include the particular value of some parameter being estimated.

congruent
Having identical shape and size.



conic section
A curve formed by intersecting a plane and the surface of a double cone. Apart from degenerate cases, the conic sections are the ellipses, the parabola, and the hyperbolas.

conjecture
A mathematical assertion that is believed, at least by some, to be true, but has not been proved.

constant
A fixed quantity or numerical value.

continuous data
Data that can, in principle, take on any real value in some interval. For example, the exact height of a randomly chosen individual, or the exact length of life of a U-235 atom can be modelled by a continuous distribution.

continuous function
Informally, a function $f(x)$ is *continuous* at a if $f(x)$ does not make a sharp jump at a. More formally, $f(x)$ is continuous at a if $f(x)$ approaches $f(a)$ as x approaches a.

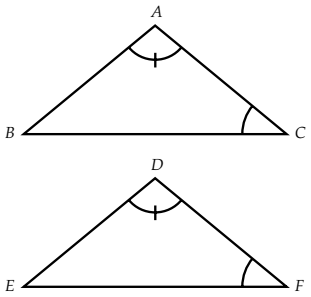
contrapositive
The contrapositive of "Whenever A is true, B must be true" is "Whenever B is false, then A must be false." Any assertion is logically equivalent to its contrapositive, so one strategy for proving an assertion is to prove its contrapositive.

converse (of a theorem)
The assertion obtained by interchanging the premise and the conclusion. If the theorem is "Whenever A happens, B must happen," then its converse is "Whenever B happens, A must happen." The converse of a theorem need not be true.

coordinate geometry
Please see *analytic* geometry.

coordinates
Numbers that uniquely identify the position of a point relative to a coordinate system.

correlation coefficient
A number (between -1 and 1) that measures the degree to which a collection of data points lies on a line.



corresponding angles and corresponding sides
Angles or sides that have the same relative position in geometric figures.

cosecant (of x)
This is $\frac{1}{\sin x}$. Notation: csc x.

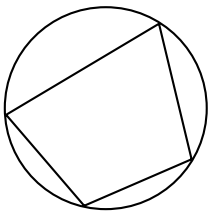
cosine law (law of cosines)
A formula used for solving triangles in plane geometry.
$c^2 = a^2 + b^2 - 2ab\cos C$

cosine (function)
See *primary trigonometric functions*.

cotangent (of x)
This is $\frac{1}{\tan x}$. Notation: cot x.

coterminal angles
Angles that are rotations between the same two lines, termed the initial and terminal arms. For example: 20°, –340°, 380° are coterminal angles.

critical number (of a function)
A number where the function is defined, and where the derivative of the function is equal to 0 or doesn't exist.



cyclic (inscribed) quadrilateral
A quadrilateral whose vertices all lie on a circle.

D

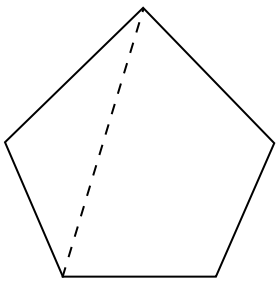
decimal fraction
In principle, a fraction $\frac{a}{b}$ where a is an integer and b is a power of 10.

For example, $\frac{1}{4} = \frac{25}{100}$, so $\frac{1}{4}$ can be expressed as a decimal fraction, usually written as 0.25.

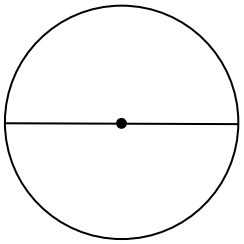
decreasing function
The function $f(x)$ is decreasing on an interval for any numbers s and t in that interval, if t is greater than s then $f(t)$ is less than $f(s)$.

deductive reasoning
A process by which a conclusion is reached from certain assumptions by the use of logic alone.

degree
The highest power or sum of powers that occurs in any term of a given polynomial or polynomial equation. For example, $6x + 17$ has degree 1, and $2 + x^3 + 7x$ has degree 3, as does $2 + 6x + 7y + xy^2$.



diagonal
A line segment that joins two non-adjacent vertices in a polygon or polyhedron.



diameter
A line segment that joins two points on a circle or sphere and passes through the centre. All diameters of a circle or sphere have the same length. That common length is called the diameter.

difference of squares
An expression of the form $A^2 - B^2$, where A and B are numbers, polynomials, or perhaps other mathematical expressions. We can factor $A^2 - B^2$ as $(A+B)(A-B)$.

differentiable
A function is *differentiable* at $x = a$ if under extremely high magnification, the graph of the function looks almost like a straight line near a. Most familiar functions are differentiable everywhere that they are defined.

differential equation
An equation that involves only two variable quantities, say x and y, and the first derivative, or higher derivatives, of y with respect to x.
Example: $3y^2 \frac{dy}{dx} = e^x$.

differentiate; differentiation
To find the derivative; the process of finding derivatives.

direct variation
The quantity Q varies directly with x if $Q = ax$ for some constant a. This can be contrasted with *inverse* variation, in which $Q = \frac{a}{x}$ for some a.

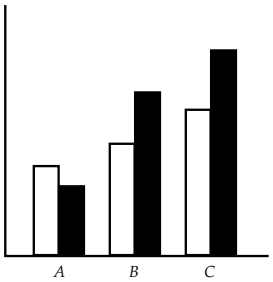
discrete data
Data arising from situations in which the possible outcomes lie in a finite or infinite sequence.

discriminant
The *discriminant* of the quadratic polynomial $ax^2 + bx + c$ (or of the equation $ax^2 + bx + c = 0$) is $b^2 - 4ac$.

displacement
Position, as measured from some reference point.

distance formula
The formula used in coordinate geometry to find the distance between two points. If A has coordinates (x_1,y_1) and B has coordinates (x_2,y_2), then the distance from A to B is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

domain (of a function)
The set of numbers where the function is defined. For example, if $f(x) = \frac{\sqrt{x-2}}{x-5}$, then the domain of $f(x)$ consists of all real numbers greater than or equal to 2, except for the number 5.



double bar graph
A bar graph that uses bars to represent two sets of data visually.

E

edge
The straight line segment that is formed where two faces of a polyhedron meet.

ellipse
A closed curve obtained by intersecting the surface of a cone with a plane. Please see *conic section*.

equation
A statement that two mathematical expressions are equal, such as $3x + y = 7$.

equidistant
Having equal distances from some specified object, point, or line.

estimate
v. To approximate a quantity, perhaps only roughly.
n. The result of estimating. Also, an approximation, based on sampling, to some number associated with a population, such as the average age.

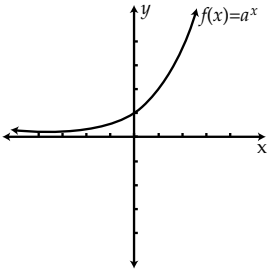
Euclidean geometry
Geometry based on the definitions and axioms set out in Euclid's Elements.

event
A subset of the sample space of all possible outcomes of an experiment.

experimental probability
An estimate of the probability of an event obtained by repeating an experiment many times. If the event occurred in k of the n experiments, it has experimental probability k / n.

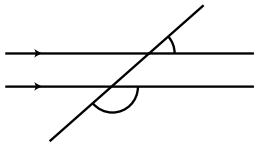
exponent
The number that indicates the power to which the base is raised. For example: 3^4: exponent is 4.

exponential decay
A quantity undergoes exponential decay if its rate of decrease at any time is proportional to its size at the time. Exponential decay models well the decay of radioactive substances.



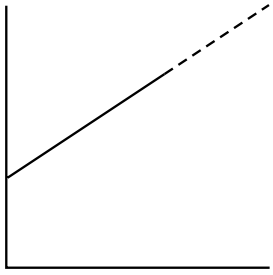
exponential function
An exponential function is a function of the form $f(x) = a^x$, where $a > 0$ and the variable x occurs as the exponent. The exponential function is the function $f(x) = e^x$, where e is a mathematical constant roughly equal to 2.7182818284.

exponential growth
A quantity undergoes exponential growth if its rate of increase at any time is proportional to its size at the time. Exponential growth models well the growth of a population of bacteria under ideal conditions.



exterior angles on the same side of the transversal
A transversal of two parallel lines forms two supplementary exterior angles.

extraneous root
A spurious root obtained by manipulating an equation. For example, if we square both sides of $1 - x + \sqrt{x - 1}$ and simplify, we obtain $(x - 1)(x - 2) = 0$, that is, $x = 1$ or $x = 2$. Since 2 is not a root of the original equation, it is sometimes called an extraneous root.



extrapolate
Estimate the value of a function at a point from values at places on one side of the point only.

extreme values
The highest and lowest numbers in a set.

F

face
One of the plane surfaces of a polyhedron.

factor
n. A *factor* of a number n is a number (usually taken to be positive) that divides n exactly. For example, the factors of 18 are 1, 2, 3, 6, 9, and 18. Similarly, a factor of a polynomial $P(x)$ is a polynomial that divides $P(x)$ exactly. Thus x and x - 1 are two of the factors of $x^3 - x$.
v. To *factor* a number or polynomial is to express it as a product of basic terms. For example, $x^3 - x$ factors as $x(x - 1)(x + 1)$.

Factor Theorem
If $P(x)$ is a polynomial, and a is a root of the equation $P(x) = 0$, then $x - a$ is a factor of $P(x)$.

factor(s)
Numbers multiplied to produce a specific product. For example: $2 \times 3 \times 3 = 18$: factors are 2 and 3; $(x - 2)$ and $(x + 1)$ are factors of $x^2 + x - 2$.

first-hand data
Data collected by an individual directly from observations or measurements.

flip
Another word for *reflection*.

frequency diagram
A diagram used to record the number of times various events occurred.

function
A rule that produces, for any element x of a certain set A, an object $f(x)$. The set A is the *domain* of the function; the set of values taken on by $f(x)$ is the *range* of the function.

More formally, a function is a collection of ordered pairs (x, y) such that the second entry y is completely determined by the first entry x.

function notation
If a quantity y is completely determined by a quantity x, then y is called a function of x. For example, the area of a circle of general radius x might be denoted by $A(x)$ (pronounced "A of x.) In this case, $A(x) = \pi x^2$.

fundamental counting principle
If an event can happen in x different ways, and for each of these ways a second event can happen in y different ways, then the two events can happen in $x \times y$ different ways.

G

general polynomial equation
An equation of the form $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n = 0$.

general term (of a sequence)
If n is unspecified, a_n is called the *general term* of the sequence $a_1, a_2, a_3, \ldots$ Sometimes there is an explicit formula for a_n in terms of n.

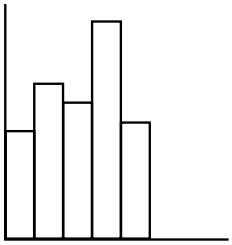
geometric sequence (progression)
A sequence in which each term except the first is a fixed multiple of the preceding term. If the first term is a, and each term is r times the previous one (r is the *common ratio*), then the general term t_n is given by $t_n = ar^{n-1}$.

geometric series
The sum S_n of the first n terms of a geometric sequence. If a is the first term, r the common ratio, where $r \neq 1$, then $S_n = \dfrac{a\left(1 - r^n\right)}{1 - r}$.
See also *infinite geometric series*.

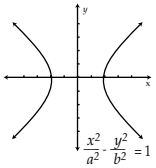
greatest common factor (GCF)
The largest positive integer that divides two or more given numbers. For example, the GCF of 12 and 18 is 6. The GCF is also called the greatest common divisor (GCD).

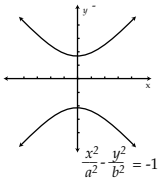
H

higher derivatives
The derivative of the derivative of $f(x)$, the derivative of the derivative of the derivative of $f(x)$, and so on.



histogram
A bar graph showing the frequency in each class using class intervals of the same length.





hyperbola
A curve with two branches where a plane and a circular conical surface meet. Please see *conic section*.

hypotenuse
The side opposite the right angle in a right triangle.

hypothesis
A statement or condition from which consequences are derived.

I

identity
A statement that two mathematical expressions are equal for all values of their variables.

if–then proposition
A mathematical statement that asserts that *if* certain conditions hold, *then* certain other conditions hold.

Imperial measure
The system of units (foot, pound, and so on) for measuring length, mass, and so on that was once the legal standard in Great Britain.

implicit function
A function y of x defined by a formula of shape $H(x, y) = 0$.
For example, $y^3 - x^2 + 1 = 0$ defines y implicitly as a function *of* x.
In this case, y is given *explicitly* by $y = \left(x^2 - 1\right)^{\frac{1}{3}}$. But often (example: $H(x, y) = y^7 + \left(x^2 + 1\right)y - 1$), it is not possible to give an explicit formula for y.

improper fraction
A *proper fraction* is a fraction whose numerator is less in absolute value than its denominator. An *improper fraction* is a fraction that is not a proper fraction.

increasing function
The function $f(x)$ is increasing on an interval for any numbers s and t in that interval, if t is greater than s then $f(t)$ is greater than $f(s)$.

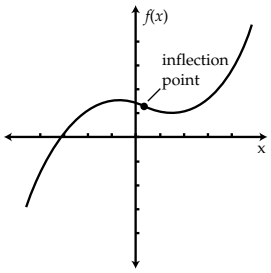
indefinite integral
Another word for *antiderivative.*

independent events
Two events are *independent* if whether or not one of them occurs has no effect on the probability that the other occurs.

inductive reasoning
A form of reasoning in which the truth of an assertion in some particular cases is used to leap to the (tentative) conclusion that the assertion is true in general.

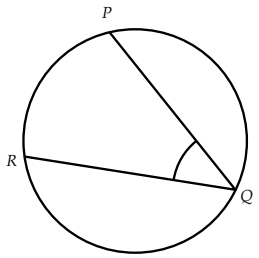
inequality
A mathematical statement that one quantity is greater than or less than the other. The statement $s > t$ means that s is greater than t, while $s < t$ means that s is less than t.

infinite geometric series
The sum $a + ar + ar^2 + \ + ar^{n-1} + \$ of all of the terms of a geometric sequence. If $|r| < 1$, then this sum is equal to $\frac{a}{1-r}$.



inflection point
A point on a curve that separates a part of the curve that is concave up from one that is concave down.

initial value problem
A function is described by specifying a differential equation that it satisfies, together with the value of the function at some "initial" point; the problem is to find the function.

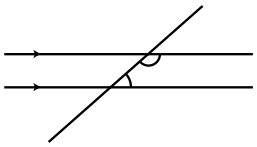


inscribed angle
The angle PQR, where P, Q, and R are three points on a curve, in most cases a circle.

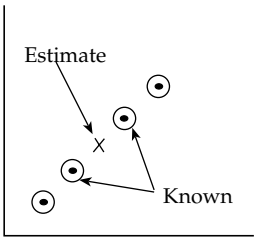
instantaneous velocity (at a particular time)
The exact rate at which the position is changing at that time.

integer
One of 0, 1, -1, 2, -2, 3, -3, 4, -4, and so on.

integration
In part, the process of finding antiderivatives.



interior angles on the same side of the transversal
The transversal of two parallel lines forms interior supplementary angles.



interpolate
Estimate the value of a function at a point from values of the function at places on both sides of the point.

intersection
The point or points where two curves meet.

interval
The set of all real numbers between two given numbers, which may or may not be included. The set of all real numbers from a given point on, or up to a given point, is also an interval, as is the set of all real numbers.

inverse (of a function)
The function $g(x)$ is the inverse of the function $f(x)$ if $f(g(x)) = x$ and $g(f(x)) = x$ for all x, or more informally if each function undoes what the other did.

inverse operations
Operations that counteract each other. For example, addition and subtraction are inverse operations.

inverse trigonometric functions
Inverses of the six basic trigonometric functions. For the two most commonly used, please see *arc sine* and *arc tan*.

irrational number
A number that cannot be expressed as a quotient of two integers. For example, $\sqrt{2}$, π, and e are irrational numbers.

irregular
Lacking in symmetry or pattern.

isosceles triangle
A triangle that has two or more equal sides. Occasionally defined as a triangle that has *exactly* two equal sides.

L

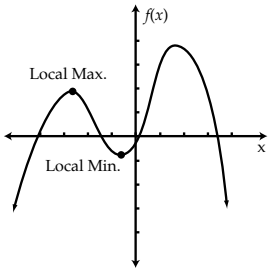
least squares
A criterion used to find the line of best fit, namely that the sum of the squares of the differences between "predicted values" and actual values should be as small as possible.

limit
The *limit* of $f(x)$ as x approaches a (notation: $\lim_{x \to a} f(x)$) is the number that $f(x)$ tends to as x moves closer and closer to a. There may not be such a number. For example, if x is measured in radians, $\lim_{x \to 0} \frac{\sin x}{x} = 1$, but $\lim_{x \to 0} \frac{1}{x}$ does not exist.

line of best fit
For a collection of points in the plane obtained from an experiment, a line that comes in some sense closest to the points. Please see *least squares*.

linear function
A function f given by a formula of the type $f(x) = ax + b$, where a and b are specific numbers.

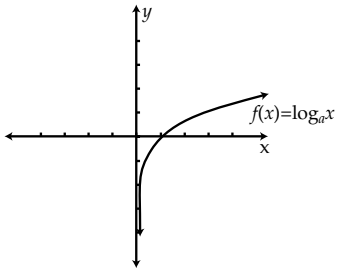
linear programming
Finding the largest or smallest value taken on by a given function $a_1x_1 + a_2x_2 + \ldots + a_nx_n$ (the *objective function*) given that $x_1, x_2, \ldots, x_n$ satisfy certain *linear constraints*. The constraints are inequalities of the form $b_1x_1 + b_2x_2 + \ldots + b_n \geq c$. Many applied problems, such as designing the cheapest animal feed that meets given nutritional goals, can be formulated as linear programming problems.



local maximum
The function $f(x)$ is said to reach a local maximum at $x = a$ if there is a neighbourhood of a such that $f(x) \leq f(a)$ for any x in the neighbourhood; informally, $(a, f(a))$ is at the top of a hill.

local minimum
The function $f(x)$ is said to reach a local minimum at $x = a$ if there is a neighbourhood of a such that $f(x) \geq f(a)$ for any x in the neighbourhood; informally, $(a, f(a))$ is at the bottom of a valley.

logarithmic differentiation
The process of differentiating a product/quotient of functions by finding the logarithm and then differentiating. For example, let $y = \frac{(1+x)^2}{(1+3x)}$. Then $\ln y = 2\ln(1+x) = \ln(1+3x)$ and $\frac{1}{y}\frac{dy}{dx} = \frac{2}{1+x} - \frac{3}{1+3x}$.



logarithmic function
Let a be positive and not equal to 1. The logarithm of x to the base a is the number u such that $a^u = x$, and is denoted by $\log_a x$. Any function of the form $f(x) = \log_a x$ is called a logarithmic function.

lowest common multiple (LCM)
The smallest positive integer that is a multiple of two or more given positive integers. For example, the LCM of 3, 4, and 6 is 12. The LCM is often called the *least* common multiple.

M

matrix
A rectangular array of numbers. For example:

$$\begin{pmatrix} 3 & 4 \\ -2 & 5 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$$

2×2 matrix 3×1 matrix

maximum point (or value)
The greatest value of a function.

mean (of a sequence of numerical data)
A measure of the average value, obtained by adding up the terms of the sequence and dividing by the number of items.

median (of a sequence of numerical data)
The "middle value" when the data are arranged in order of size. If there is an even number of data, then the average of the two middle values. For example, the median of 5, 3, 7.4, 5, 8, is 5, while the median of 5, 7.4, 5, and 8 is 6.2.

median (of a triangle)
The line segment that joins a vertex of the triangle to the midpoint of the opposite side.

minimum point (or value)
The lowest value of a function.

mixed number
A number that is expressed as the sum of a whole number and a fraction.
For example: $3\frac{2}{5}$

mode
The value that occurs most often in a sequence of data.

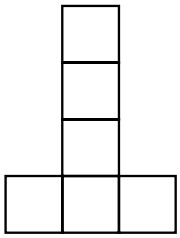
monomial
An algebraic expression that is a product of variables and constants.
Examples: $6x^2$, 1, $\frac{3}{4}x^2y$

multiple (of an integer)
The result obtained when the given integer is multiplied by some integer. Equivalently, an integer that has the given integer as a factor. (Often negative integers are not allowed.)

N

natural logarithm
Logarithm to the base e, where e is a fundamental mathematical constant roughly equal to 2.7182818284. The natural logarithm of x is usually written ln x.

natural number (counting number)
One of the numbers 1, 2, 3, 4, . . . *Positive integer* means the same thing.

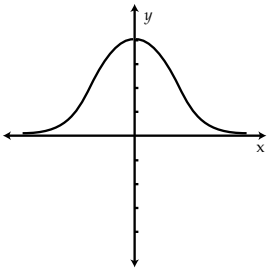


net
A flat diagram consisting of plane faces arranged so that it may be folded to form a solid.

Newton's Law of Cooling
The assertion that if a warm object is placed in a cool room, its temperature decreases at a rate proportional to the difference in temperature between the object and its surroundings.

Newton's Method
An often highly efficient iterative method for approximating the roots of $f(x) = 0$. If r_n is the current estimate, then the next estimate is the x-intercept of the tangent line to $y = f(x)$ at $x = r_n$.

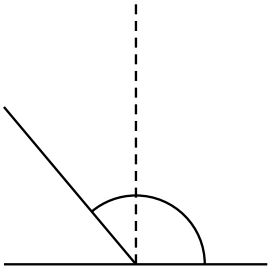
non-differentiable (function)
A function $f(x)$ is non-differentiable at $x = a$ if it does not have a derivative there. Example: if $f(x) = |x|$ then $f(x)$ is not differentiable at $x = 0$, basically because the curve $y = |x|$ has a sharp kink there.



normal distribution curve
The standard normal distribution curve has equation $y = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$.

The general normal is obtained by shifting the standard normal to the left or right, and/or rescaling. These curves are sometimes called bell-shaped curves. They figure importantly in probability, statistics, and signal processing.

O



obtuse angle
An angle whose measure is between 90° and 180°.

one-sided limit
Sometimes $f(x)$ exhibits different behaviour depending on whether x approaches a from the right (through values of x greater than a) or from the left. For example, let $f(x) = \frac{1}{\left(1 + 2^{\frac{1}{x}}\right)}$. As x approaches 0 from the right, $f(x)$ approaches 0 (notation: $\lim_{x \to 0^+} f(x) = 0$) while $f(x)$ approaches 1 as x approaches 0 from the left.

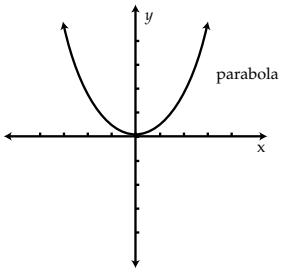
optimization problem
A problem, often of an applied nature, in which we need to find the largest or smallest possible value of a quantity; also called a *max/min problem*.

ordered pair
A sequence of length 2. Ordered pairs (x, y) of real numbers are used to indicate the x and y coordinates of a point in the plane.

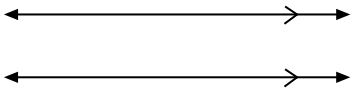
ordinal number
A number designating the place occupied by an item in an ordered sequence (e.g., first, second, and third).

origin
The point in a coordinate system at the intersection of the axes.

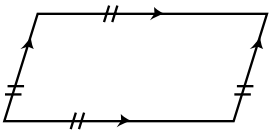
P



parabola
The intersection of a conical surface and a plane parallel to a line on the surface.



parallel lines
Two lines in the *plane* are *parallel* if they do not meet. In three-dimensional space, two lines are parallel if they do not meet *and* there is a plane that contains them both. Alternately, in the plane or in space, two lines are parallel if they stay a constant distance apart.



parallelogram
A quadrilateral such that pairs of opposite sides are parallel.

percentage
In a problem such as "Find 15% of 400," the number 400 is sometimes called the *base*, 15% or 0.15 is called the *rate*, and the answer 60 is sometimes called the *percentage*.

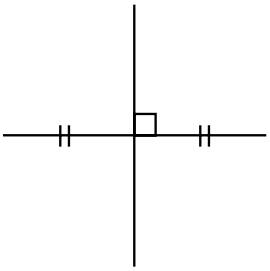
percent error
The relative error expressed in parts per hundred. Let A be an estimate of a quantity whose true value is T. Then $A - T$ is the *error*, and $(A - T)/T$ is the *relative error*.

percentile
The k-th *percentile* of a sequence of numerical data is the number x such that k percent of the data points are less than or equal to x. (Often x is not precisely determined, particularly if the data set is not large.)

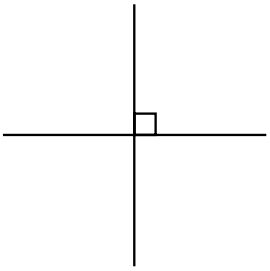
perimeter
The length of the boundary of a closed figure.

period
The interval taken to make one complete oscillation or cycle.

permutation
An ordered arrangement of objects. The number of ways of producing a permutation of r (distinct) objects from a collection of n objects is ${}_nP_r$, where ${}_nP_r = n(n - 1)(n\ \ 2)\ldots(n - r + 1)$.

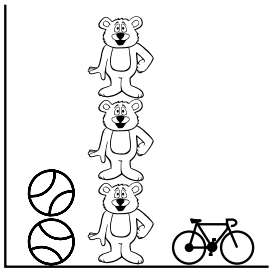


perpendicular bisector
A line that intersects a line segment at a right angle and divides the line segment into two equal parts.

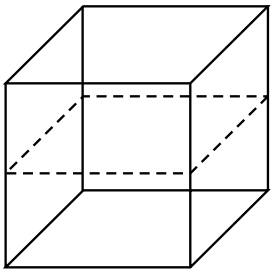


perpendicular line
Two lines that intersect at a right angle.

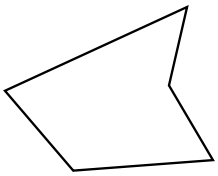
phase shift
A horizontal translation of a periodic function. For example, the function $\cos 2\left(x - \frac{\pi}{3}\right)$ is $\cos 2x$ with a phase shift of $\frac{\pi}{3}$.

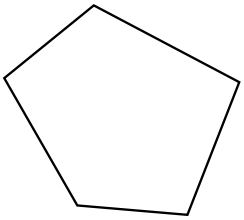


pictograph
A graph that uses pictures or symbols to represent similar data.



plane of symmetry
A 2-D flat surface that cuts through a 3-D object, forming two parts that are mirror images.





polygon
A closed curve formed by line segments that do not intersect other than at the vertices.

polygonal region
A part of a plane that has a polygon as a boundary.

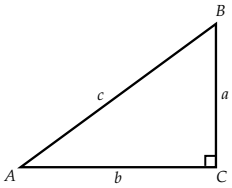
polyhedron
A solid bounded by plane polygonal regions.

polynomial
A mathematical expression that is a sum of monomials. Examples:
$4x^3 - 3x - \frac{1}{2}, \pi x^2 + 2\pi xy, xyz$

population
The items, actual or theoretical, from which a sample is drawn.

power
A power of q is any term of the form q^k. Often but not always, k is taken to be a positive integer.

precision
A measure of the estimated degree of repeatability of a measurement, often described by a phrase such as correct to two decimal places.



Sine A = $\frac{a}{c}$

primary trigonometric functions

$$\sin A = \frac{a}{c} = \frac{opposite}{hypotenuse}$$

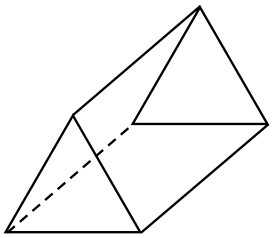
$$\cos A = \frac{b}{c} = \frac{adjacent}{hypotenuse}$$

$$\tan A = \frac{a}{b} = \frac{opposite}{adjacent}$$

Functions of angles defined, for an acute angle, as ratios of sides in a right triangle.

prime
A positive integer that is divisible by exactly two positive integers, namely 1 and itself. The first few primes are 2, 3, 5, 7, 11, and 13.

prime factorization (of a positive integer)
The given integer expressed as a product of primes. For example, $2 \times 5 \times 3 \times 2$ is a prime factorization of 60. Usually the primes are listed in increasing order. The standard prime factorization of 60 is $2^2 \times 3 \times 5$.



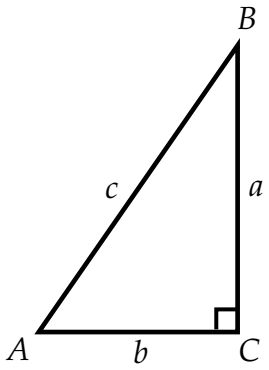
prism
A solid with two parallel and congruent bases in the shape of polygons; the other faces are parallelograms.

probability (of an event)
A number between 0 and 1 that measures the likelihood that the event will occur. P(A) often denotes the probability of the event A.

product
The *product* of two or more objects (numbers, functions, etc.) is the result of multiplying these objects together.

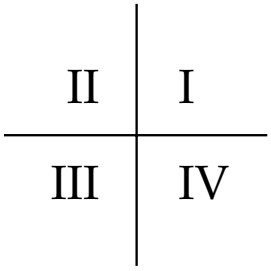
product rule
The rule for finding the derivative of a product of two functions. If $p(x) = f(x)g(x)$ then $p'(x) = f(x)g'(x) + g(x)f'(x)$.

pyramid
A polyhedron one of whose faces is an arbitrary polygon (called the *base*) and whose remaining faces are triangles with a common vertex called the *apex*).



Pythagorean theorem
In a right-angled triangle, the sum of the squares of the lengths of the sides containing the right angle is equal to the square of the hypotenuse ($a^2 + b^2 = c^2$).

Q

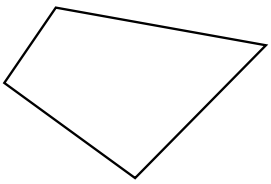


quadrant
One of the four regions that the plane is divided into by two perpendicular lines. When these lines are the usual coordinate axes, the quadrants are called the first quadrant, the second quadrant, and so on as in the diagram.

quadratic formula
A formula used to determine the roots of a quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

quadratic function
A function of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. The graph of such a function is a parabola.



quadrilateral
A polygon with four sides.

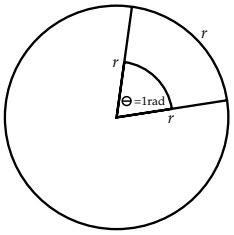
quartile
The 25th percentile is the first quartile, the 50th percentile is the second quartile (or median), and the 75th percentile is the third quartile. Please see *percentile*.

quotient
The result of dividing one object (number, function) by another.

quotient rule
The rule for differentiating the quotient of two functions.

If $q(x) = \dfrac{f(x)}{g(x)}$ then $q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{\left(g(x)\right)^2}$.

R



radian
Equal to the central angle subtended by an arc of unit length at the centre of a circle of unit radius.

radical
The square root, or cube root, and so on of a quantity. For example, the cube root of the quantity Q is the quantity R such that R^3 (the cube of R) is equal to Q. The square root of Q is written $\sqrt{Q}$ ($\sqrt{\ }$ is the *radical sign*). The cube root of Q is written $\sqrt[3]{Q}$.

radius
A line segment that joins the centre of a circle or sphere to a point on the boundary. All radii of a circle or sphere have the same length. That common length is called the *radius*.

range
A measure of variability of a sequence of data, defined to be the difference between the extremes in the sequence. For example, if the data are 27, 22, 27, 20, 35, and 34, then the range is 15.

range (of a function)
The set of values taken on by a function. Please see *function*.

rank ordering
Ordering (of a sample) according to the value of some statistical characteristic.

rate
A comparison of two measurements with different units. For example, the speed of an object measured in kilometres per hour.

rate of change (of a function at a point)
How fast the function is changing. If $f(x)$ is the function, its rate of change with respect to x at $x = a$ is the derivative of $f(x)$ at $x = a$.

ratio
Another word for quotient. Also, an indication of the relative size of two quantities. We say that P and Q are in the ratio a:b if the size of A divided by the size of B is a / b.

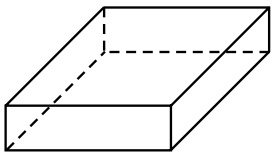
rational expression
The quotient of two polynomials.

rational number
A number that can be expressed as a / b, where a and b are integers.

rationalize the denominator
Transform a quotient P / Q where the denominator Q involves radicals into an equivalent expression with the denominator free of radicals.
For example: $\dfrac{4}{4-\sqrt{7}} = \dfrac{(4)(4+\sqrt{7})}{(4-\sqrt{7})(4+\sqrt{7})} = \dfrac{(4)(4+\sqrt{7})}{9}$.

real number
An indicator of location on a line with respect to an origin; a quantity represented by an arbitrary decimal expansion.

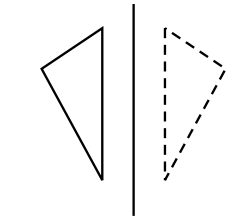
reciprocal
The number or expression produced by dividing 1 by a given number or expression.



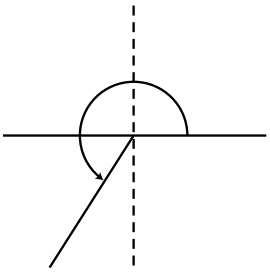
rectangular prism
A prism whose bases are congruent rectangles.

recursive definition (of a sequence)
A way of defining a sequence by possibly specifying some terms directly, and giving an algorithm by which any term can be obtained from its predecessors. For example, the Fibonacci sequence is defined recursively by the rules $F_0 = F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$.

reference angle
The acute angle between the ray line and the x-axis. For example, the reference angles of a 165° and of a 195° angle are each 15° angles.



reflection (in a line)
The transformation that takes any 2-dimensional object to the object that is symmetrical to it with respect to the line, that is, to its mirror image in the line. *Flip* means the same thing. In three-dimensions, we can define analogously *reflection in a plane*.



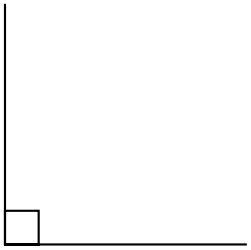
reflex angle
An angle greater than 180° and less than 360°.

relative maximum, minimum
Please see *local maximum*, *local minimum*.

Remainder Theorem
If we divide the polynomial $P(x)$ by $x - a$, the remainder is equal to $P(a)$.

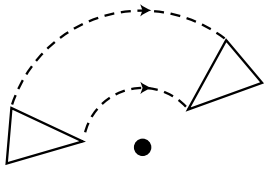
repeating decimal
A decimal expansion that has a block of digits that ultimately cycles forever. For example, $\frac{23}{22}$ has the decimal expansion 1.0454545 . . ., with the block 45 ultimately cycling forever. A terminating decimal like 0.25 is usually viewed as being a repeating decimal, indeed in two ways: as 0.25000 . . . and 0.24999 . . .

resultant
The sum of two or more vectors.



right angle
An angle whose measure is 90°.

root of an equation (in one variable)
If the equation has the form $F(x) = G(x)$, a root of the equation is a number a such that $F(a) = G(a)$.



rotation (in the plane)
A transformation in which an object is turned through some angle about a point. An analogous notion can be defined in three dimensions; there the turn is about a line.

rounding
A process to follow when making an approximation to a given number by using fewer significant figures.

S

sample
A selection from a population.

sample space
The set of all possible outcomes of an experiment.

scalar
A number. Usually used in contexts where there are also vectors around, or functions. Examples of usage: "the length of a vector is a scalar"; "-3 sin x is a scalar multiple of sin x."

scatter plot
If each item in a sample yields two measurements, such as the height x and weight y of the individual chosen, the point with coordinates (x,y) is plotted. If this is repeated for all members of the sample, the resulting collection of points is a scatter plot.

secant (of x)
This is $\frac{1}{\cos x}$. Notation: sec x.

secant line
A line that passes through two points on a curve.

second derivative
The second derivative of $f(x)$ is the derivative of the derivative of $f(x)$. Two common notations: $f''(x)$ and $\frac{d^2 f}{dx^2}$.

second derivative test
Suppose that $f(a) = 0$. The second derivative test gives a way of checking whether at $x = a$ the function $f(x)$ reaches a local minimum or a local maximum.

second-hand data
Data not collected directly by the researcher. For example: encyclopedia.

semicircle
A half-circle; any diameter cuts a circle into two semicircles.

sequence
A finite ordered list $t_1, t_2, \ldots, t_n$ of terms (finite sequence) or a list $t_1, t_2, \ldots, t_n, \ldots$ that goes on forever (an infinite sequence).

series
Any sum of $t_1 + t_2 + \ldots + t_n$, the first n terms of a sequence. The sum $t_1 + t_2 + \ldots + t_n + \ldots$ of all the terms of an infinite sequence is an *infinite series*. The concept of limit is required to define the sum of infinitely many terms.

SI measure
Abbreviation for Système International d'Unités – International System of Units – kilogram, second, ampere, kelvin, candela, mole, radian, and so on.

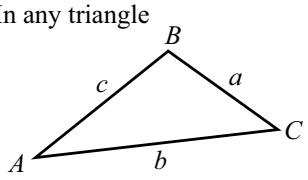
side (of a polygon)
Any of the line segments that make up the boundary of the polygon.

sigma notation
The use of the sign (Greek capital sigma) to denote sum.
For example: $\sum_{i=1}^{5} a_i = a_1 + a_2 + a_3 + a_4 + a_5$.

simple interest
Interest computed only on the original principal of a loan or bank deposit.

sine (function)
Please see *primary trigonometric functions*.

In any triangle

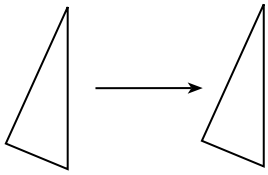


$$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$$

sine law (law of sines)
A formula used for solving triangles in plane trigonometry.

skeleton
A representation of the edges of a polyhedron.

skip counting
Counting by multiples of a number. For example: 2, 4, 6, 8.

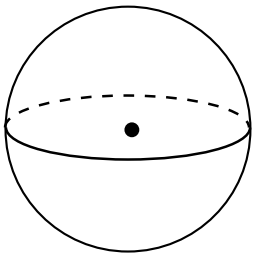


slide
A transformation of a figure by moving it up/down and/or left/right without any rotation. The word is a synonym for the more standard mathematical term translation.

slope
The slope of a (non-vertical) line is a measure of how fast the line is climbing. It can be defined as the change in the y-coordinate of a point on the line when the x-coordinate is increased by 1. If a curve has a (non-vertical) tangent line at the point, the slope of the curve at the point is defined to be the slope of that tangent line.

slope-intercept form (of the equation of a line)
An equation of the form $y = mx + b$. All lines in the plane except for vertical lines can be written in this form. The number m is the slope and b is the y-intercept.

solution (of a differential equation)
A function that satisfies the differential equation. For example, for any constant C, the function given $y = \left(x^2 + C\right)^{\frac{1}{3}}$ is a solution of the differential equation $3y^2 \frac{dy}{dx} = 2x$.

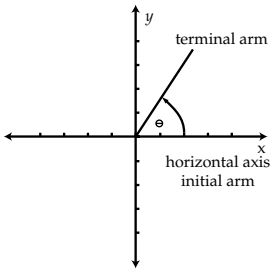


sphere
A solid whose surface is all points equidistant from a centre point.

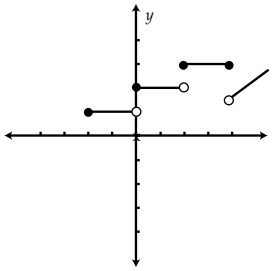
square root
The square root of x is the non-negative number that when multiplied by itself produces x. For example, 5 is the square root of 25. In general the square root of x^2 is $|x|$, the absolute value of x.

standard deviation
Sample standard deviation is the square root of the sample variance. Population standard deviation is the square root of the population variance.

standard form
The usual form of an equation. For example, the standard form of the equation of a circle is $(x-a)^2+(y-b)^2=r^2$, because it reveals geometrically important features, the centre and the radius.



standard position (angle in)
The initial arm of the angle is the positive horizontal axis (x-axis.) Counterclockwise rotation gives a positive angle.



step function
A function whose graph is flat except at a finite number of points, where it takes a sudden jump.

supplementary angles
Two angles whose sum is 180°.

symmetrical (has symmetry)
A geometrical figure is symmetrical if there is a rotation reflection, or combination of these that takes the figure to itself but moves some points. For example, a square has symmetry because it is taken to itself by a rotation about its centre through 90°.

system of equations
A set of equations. A *solution* of the system is an assignment of values to the variables such that all of the equations are (simultaneously) satisfied. For example, $x = 1$, $y = 2$, $z = -3$ is a solution of the system $x + y + z = 0$, $x - y - 4x = 11$.

T

tangent (function)
Please see *primary trigonometric functions.*

tangent (to a curve)
A line is tangent to a curve at the point P if under very high magnification, the line is nearly indistinguishable from the curve at points close to P. A tangent line to a circle can be thought of as a line that meets the circle at only one point.

tangent line approximation
If P is a point on a curve, then close to P the curve can be approximated by the tangent line at P. In symbols, if x is close to a, then $f(x)$ is very closely approximated by $f(a) + (x - a)f'(a)$.

tangram
A square cut into seven shapes: two large triangles, one medium triangle, two small triangles, one square, and one parallelogram.

term
Part of an algebraic expression. For example, x^3 and $5x$ are terms of the polynomial $x^3 + 3x^2 + 5x - 1$.

terminating decimal
A decimal expansion that (ultimately) ends. Example: 3.73.

tesselation
A covering of a surface (usually the entire plane) without overlap or bare spots, by copies of a given geometric figure or of a finite number of given geometric figures. The word comes from *tessala*, the Latin word for a small tile.

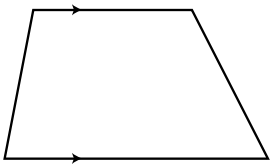
theoretical probability (of an event)
A numerical measure of the likelihood that the event will occur, based on a probability model. If, as can happen with dice or coins, an experiment has only a finite number n of possible outcomes, all equally likely, and in k of these the event occurs, then the theoretical probability of the event is k / n.

tolerance (interval)
The set of numbers that are considered acceptable as the dimension of an item. Example: a manufacturer's tolerance interval for the weight of a "400 gram" box of cereal might be from 395 grams to 420 grams.

transformation
A change in the position of an object, and/or a change in size, and related changes. Also, a change in the form of a mathematical expression.

translation
Please see *slide*.

transversal
A line that intersects two or more lines at different points.



trapezoid
A quadrilateral that has two parallel sides. Some definitions require that the remaining two sides not be parallel.

tree diagram
A pictorial way of representing the outcomes of an experiment that involves more than one step.

trigonometry
The branch of mathematics concerned with the properties and applications of the trigonometric functions, in particular their use in "solving" triangles, in surveying, in the study of periodic phenomena, and so on.

trinomial
A polynomial that has three terms. For example: $ax^2 + bx + c$.

turn
Please see *rotation*.

U

unbiased
A sampling procedure for estimating a population parameter (like the proportion of BC teenagers who smoke) is unbiased if on average it should yield the correct value. At a more informal level, a polling procedure is unbiased if proper randomization procedures are used to select the sample, the wording of the questions is neutral, and so on.

unit circle
A circle of radius 1.

unit vector
A vector of length 1.

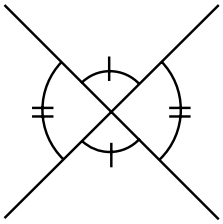
V

variable
A mathematical entity that can stand for any of the members of a given set.

variance
Sample variance is a measure of the variability of a sample, based on the sum of the squared deviations of the data values about the mean. *Population variance* is a theoretical measure of the variability of a population.

vector
A directed line segment (arrow) used to describe a quantity that has direction as well as magnitude.

vertex (pl. vertices)
In a polygon, a point of intersection of two sides. In a polyhedron, a vertex of a face.



vertically opposite angles
Opposite (and equal) angles resulting from the intersection of two lines.

W

whole number
One of the counting numbers 0, 1, 2, 3, 4, and so on; a non-negative integer.

X

***x*-intercept(s)**
The point(s) at which a curve meets the x-axis (horizontal axis).

Y

***y*-intercept(s)**
The point(s) at which a curve meets the y-axis (vertical axis).

Z

***z*-score**

If x is the numerical value of one observation in a sample, the z-score of x is $\frac{\left(x-\bar{x}\right)}{s}$, where $\bar{x}$ is the sample mean and s is the sample standard deviation. The z-score measures how far x is from the mean.

zero (root) of *f*(*x*)
Any number a such that $f(a)=0$.